MASS SPECTRUM OF PARTICLES IN NONLINEAR QUARK THEORY

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Vestnik Moskovskogo Universiteta, Fizika, Vol. 25, No. 3, pp. 256-261, 1970
UDC 593.12.01

Equations are derived for the mass spectrum of baryons and mesons within the framework of nonlinear quark theory on the basis of the relativistic, SU(3)-invariant equations derived previously for these particles. The equations derived are the same as those found by the ordinary (purely group) approach in nonrelativistic SU(6)-invariant theory.

The quark model has had many successes in theory of elementary particles: mesons and baryons have been classified, the magnetic moments of the baryons have been determined, and several relations have been determined among their strong-coupling constants [1, 2]. On the other hand, no acceptable dynamic application has been made of the quark model in the equations of field theory. Below we continue a study [3, 4] of this model for nonlinear field equations.

The equations found in [3] for the baryon and meson fields, Ψ_{ABC} and Φ_A^B, respectively, are

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - IM_0\right)_{AA'} \Psi_{A'BC} =$$

$$= \frac{l^2}{10}\{\Phi_A^D \Psi_{DBC} + \Phi_B^D \Psi_{ADC} + \Phi_C^D \Psi_{ABD} - 3\Phi_D^D \Psi_{ABC}\}, \tag{1}$$

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - Im_0\right)_{AA'} \Phi_{A'B} = \frac{2l^2}{3}\{\Phi_A^D \Phi_D^B - \Phi_D^D \Phi_A^B\}, \tag{2}$$

where

$A \equiv (\alpha, l)$; $B \equiv (\beta, m)$, $c \equiv (\gamma, n)$, (l, m, n) are unitary indices, and (α, β, γ) are spinor indices. Equations (1) and (2), which are invariant with respect to P_4 (the Poincaré group) and SU(3), are a system of equations for interacting fields.

Writing Ψ_{ABC} and Φ_A^B in equations (1) and (2) as

$$\Psi_{ABC} = D_{ABC}\left(\frac{3}{2}\right) + B_{ABC}\left(\frac{1}{2}\right), \tag{3}$$

$$\Phi_A^B = \frac{1}{4}\left\{I\varphi_0 + \gamma_5\varphi_5 + i\gamma_\mu\gamma_5\varphi_{\mu 5} + \gamma_\mu\varphi_\mu + \frac{1}{2}\sigma_{\mu\nu}\varphi_{\mu\nu}\right\}_A^B, \tag{4}$$

where $\varphi_{5,l}^m$, $\varphi_{\mu,l}^m$ describe the meson octets $(S=0^-)$ and nonets $(S=1^-)$, and $D(^3/_2)$, $B(^1/_2)$ describe the baryon decuplets $(S={}^3/_2{}^+)$ and octets $(S={}^1/_2{}^+)$, we find equations for real particles. We previously [3, 4] analyzed the interaction terms in these equations and determined the coupling constants and magnetic moments of these particles. Below we will analyze the linear terms in these fields and we will determine the mass spectrum of the particles (after violations of the SU(3) symmetry of the equations).

Curiously, the mass spectrum of baryons and mesons which is found in the ordinary (purely group) approach only within the framework of SU(6) nonrelativistic) symmetry is found within the framework of relativistic SU(3) invariant equations (1) and (2).

From equations (2) and (4) for scalar field $\varphi_{0,l}^m$ we find $\partial_\mu\varphi_0 = 0$. To determine the magnetic moments and coupling constant we assume $\varphi_{0,l}^m = 0$ ($\varphi_{0,l}^m = \text{const} \neq 0$ does not affect these results). In determining the rest masses of the particles, however, the relation $\varphi_{0,l}^m = \text{const} \neq 0$ is of fundamental importance. When the equations have SU(3) symmetry, the field $\varphi_{0,l}^m$ has nine components.

Substituting equation (4) into equation (1), and discarding all the interaction terms involving real mesons, we find

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - IM_0\right)\Psi_{lmn} = \frac{l^2}{10}\{\varphi_{0,l}^\lambda \Psi_{\lambda mn} + \varphi_{0,m}^\lambda \Psi_{l\lambda n} + \varphi_{0,n}^\lambda \Psi_{lm\lambda} - 3\varphi_{0,\lambda}^\lambda \Psi_{lmn}\}. \tag{5}$$

Equation (2) relates fields $(\varphi_5, \varphi_{\mu 5})$ and $(\varphi_\mu, \varphi_{\mu\nu})$, even in the linear approximation. To find the equations containing only φ_5 and φ_μ, we must go to second-order equations. Ommitting all the interaction terms involving real mesons in the equations, we find

$$-\Box\varphi_l^m = m_0^2\varphi_l^m + \frac{m_0 l^2}{3}[\varphi_0\varphi]_{+l}^m + \frac{l^4}{36}[\varphi_0[\varphi_0\varphi]_+]_{+l}^m, \tag{6}$$

where $\varphi\equiv\varphi_5$ and $\varphi\equiv\varphi_\mu$. In the case of the baryons, we thus find relations among masses, while in the case of the mesons we find relations among the squares of the masses. This result is of course adopted as an empirical result within the framework of the purely group approach.

Below we analyze the splitting of the multiplet masses: the question of the basic mass of the multiplet remains open.

SPECTRUM OF BARYON AND MESON MASSES

Equations (1) and (2) were derived on the basis of an SU(3)-invariant for quarks; if we evaluate the SU(3) invariance of the equations (but retaining the SU(2) invariance)in the initial equation for the quarks, and if we construct on the basis of this new equation new equations for the mesons and baryons analogous to equations (1) and (2), the resulting equations will have SU(2) invariants (in addition to P_4 invariants). Derivation and analysis of these equations is of independent interest, but for our purposes here it is sufficient to consider a violation of symmetry in equations (1)-(4) as a so-called spontaneous violation of symmetry [5]. For this purpose, we restrict the discussion to a constant scalar field $\varphi_{0,l}^m$:

$$\varphi_0 = \alpha + \beta\lambda_8 + g\chi = \text{const.} \tag{7}$$

$$(\chi_l^m) = \begin{pmatrix} \frac{a_3}{\sqrt{2}} + \frac{ia_0}{3\sqrt{2}}, & a_1, & 0 \\ a_2, & -\frac{a_3}{\sqrt{2}} + \frac{ia_0}{3\sqrt{2}}, & 0 \\ 0 & 0 & -\frac{i2a_0}{3\sqrt{2}} \end{pmatrix}, \tag{8}$$

where α, β, and g are constants. This form of $\varphi_{0,l}^m$ provides SU(3) invariants for equations (3) and (4).

SPECTRUM OF BARYON MASSES $(S=1/2^+, S=3/2^+)$

Substituting (7) and (8) into equation (5), and first considering the case g = 0, we find

$$M^*_{(lmn)} = \left(M_{0,s} - \frac{3l^2}{5}\alpha\right) + \frac{l^2}{10}\beta(\lambda_8^{ll} + \lambda_8^{mm} + \lambda_8^{nn}) \equiv M_1 + M_2 Y_{(lmn)}, \tag{9}$$

where

$$M_{0,s} \equiv (M_{0,1/2}, M_{0,3/2}),\ Y_{(lmn)} = \frac{1}{\sqrt{3}}(\lambda_8^{ll} + \lambda_8^{mm} + \lambda_8^{nn}).$$

For g ≠ 0 and for the case of the baryon octet, we find from equation (3) the following for the baryon isosinglet Λ:

$$(\gamma p)\Lambda = M^*_\Lambda\Lambda,\ p_\mu = i\frac{\partial}{\partial x_\mu} \tag{10}$$

For the baryon isodoublet, we have $N \equiv \begin{pmatrix} n \\ p \end{pmatrix}$ and $\Xi = \begin{pmatrix} \Xi^0 \\ -\Xi^- \end{pmatrix}$:

$$(\gamma p)N = \left\{M^*_N + \frac{l^2}{10\sqrt{2}}g(\vec{\tau}\vec{a} + ia_0)\right\}N, \tag{11}$$

where $\vec{\tau}\vec{a} = \sqrt{2}(\tau^- a_1 + \tau^+ a_2) + \tau^3 a_3$.

and $\vec{\tau}$ are the Pauli matrices (we can write an analgous equation for). For the baryon isotriplet we have

$$(\gamma p)\Sigma = \left\{M^*_\Sigma + \frac{l^2}{10\sqrt{2}}g\vec{\omega}\vec{a}\right\}\Sigma,$$

$$\vec{\omega}\vec{a} = \sqrt{2}(\omega^- a_1 + \omega^+ a_2) + \omega^3 a_3,$$

$$\omega^+ = \sqrt{2}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},\quad \omega^- = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\omega^3 = 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{12}$$

We consider the fields a_i to be operators having the properties

$$a_i = b_i + b_i^*,\ a_2 = a_1^*,$$

$$[b_i b_j^*]_- = \delta_{ij},\quad [b_i b_j]_- = [b_i^* b_j^*]_- = 0,$$
$$\langle N(p)|a_i|N(p)\rangle = 0,\ \langle N(p)|a^2|N(p)\rangle = 1, \tag{13}$$

where $\langle N(p)|a_i|N(p)\rangle$ is the average value of a_i when there is a single nucleon N(p) in the initial and final states.

We define the rest mass as

$$M = \sqrt{\langle N(p)|p^2|N(p)\rangle}. \tag{14}$$

Then from equations (11) and (12), and using equations (13) and (14), we find

$$M_\Lambda = M_\Lambda^*,\ M_N = M_N^* + \frac{1}{2}\left(\frac{gl^2}{10}\right)^2 + \ldots,$$

$$M_\Sigma = M_\Sigma^* + 2\left(\frac{gl^2}{10}\right)^2 + \ldots,\ M_\Xi = M_\Xi^* + \frac{1}{2}\left(\frac{gl^2}{10}\right)^2 + \ldots. \tag{15}$$

Mass spectrum (10) can be written [1, 5]

$$M = M_1 + M_2 Y + M_3\left[I(I+1) - \frac{1}{4}Y^2\right], \tag{16}$$

$$M_1 \equiv \left(M_{0,1/2}^* - \frac{3l^2}{5}\alpha\right),\ M_2 \equiv \sqrt{3}\frac{l^2}{10}\beta,\ M_3 \equiv \left(\frac{gl^2}{10}\right)^2. \tag{17}$$

Analogously, for the case of the baryon decuplet (S = 3/2+) we have

$$(\gamma p)\,\Omega_- = \left\{M_{\Omega_-}^* - \sqrt{2}\left(\frac{gl^2}{10}\right) i a_0\right\}\Omega_-, \tag{18}$$

$$(\gamma p)\,\Delta = \left\{M_\Delta^* + \frac{1}{\sqrt{2}}\left(\frac{gl^2}{10}\right)(\vec{\omega}'\vec{a} + i a_0)\right\}\Delta, \tag{19}$$

where

$$\Delta \equiv \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \\ \Delta^{-} \end{pmatrix},\quad \vec{\omega}'\vec{a} = \sqrt{2}(\omega'_- a_1 + \omega'_+ a_2) + \omega'_3 a_3,$$

$$\omega'_+ = \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix},\quad \omega'_- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix},$$

$$\omega'_3 = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

and equations analogous to equations (10) and (11) for $\Xi^* \equiv \begin{pmatrix}\Xi^{*0} \\ \Xi^{*-}\end{pmatrix}$ and $\Sigma^* \equiv \begin{pmatrix}\Sigma^{*+} \\ \Sigma^{*0} \\ \Sigma^{*-}\end{pmatrix}$, respectively.

Hence, using equations (13) and (14) we find

$$M_{\Omega_-} = M_{\Omega_-}^* - \left(\frac{gl^2}{10}\right)^2 + \ldots,\ M_{\Sigma^*} = M_{\Sigma^*}^* + 2\left(\frac{gl^2}{10}\right)^2 \ldots,$$

$$M_{\Xi^*} = M_{\Xi^*}^* + \frac{1}{2}\left(\frac{gl^2}{10}\right)^2 + \ldots,\quad M_\Delta = M_\Delta^* + \frac{7}{2}\left(\frac{gl^2}{10}\right)^2 + \ldots. \tag{20}$$

Spectrum (10) is also included in equation (16), but for a different value of the basic mass, $M_{0,3/2}$.

MASS SPECTRUM OF MESONS

We first consider the mass spectrum for the mesons of the octet ($S = 0^-$, $\varphi \equiv \varphi_5$, $\varphi_{5,l}^l = 0$). Equation (6) in the form given is not compatible with the condition $\varphi_{5,l}^l = 0$; to resolve this problem we add to (4) a term

$$-\frac{1}{3}\left\{\frac{m_0 l^2}{3}\,\mathrm{sp}\,[\varphi_0\varphi_5]_+ + \frac{l^4}{36}\,\mathrm{sp}\,[\varphi_0[\varphi_0\varphi_5]_+]_+\right\}\delta_{lm}.$$

$$-\Box\varphi^m_{5,l} = m_0^2\varphi^m_{5,l} + \frac{m_0 l^2}{3}\left\{[\varphi_0\varphi_5]^m_{+l} - \frac{1}{3}[\varphi_0\varphi_5]^l_{+l}\delta_{lm}\right\} + + \frac{l^4}{36}\left\{[\varphi_0\,]\varphi_0\varphi_5]_+]^m_{+l} - \frac{1}{3}[\varphi_0[\varphi_0\varphi_5]_+]^l_{+l}\delta_{lm}\right\}. \tag{21}$$

Substituting Eqs. (8) and (9) into (21), we find, for the case g = 0,

$$m^{*2}_{(lmn)} = C_0^2 + C_1(\lambda_8^{ll} + \lambda_8^{mm}),$$
$$C_0 = m^2_{0,s} + \frac{2m_{0,s}l^2}{3}\alpha + \frac{l^4}{9}\left(\alpha^2 + \frac{4}{3}\beta^2\right),$$
$$C_1 = \frac{2l^2}{3}\beta\left\{m_0 + \frac{l^2}{3}\left(\alpha - \frac{1}{\sqrt{3}}\beta\right)\right\}. \tag{22}$$

For g ≠ 0, we find from (21) ($k \equiv (k^+, k^0, k^-, \bar{k}^0)$)

$$p^2\pi^\pm = \left\{m^{*2}_\pi + \frac{2}{36}g^2l^4\left(a_1a_2 - \frac{1}{9}a_0^2\right)\right\}\pi^\pm,$$
$$p^2\pi^0 = \left\{m^{*2}_{\pi^0} + \frac{2}{36}g^2l^4\left(a_3^2 - \frac{1}{9}a_0^2\right)\right\}\pi^0,$$
$$p^2\eta = \left\{m^{*2}_\eta + \frac{2}{108}g^2l^4(a_3^2 + 2a_1a_2 - a_0^2)\right\}\eta,$$
$$p^2k = \left\{m^{*2}_k + \frac{1}{36}g^2l^4\left(\frac{1}{2}a_3^2 - \frac{1}{18}a_0^2 + a_1a_2\right)\right\}k. \tag{23}$$

Using Eqs. (13) and (14), we then find

$$m^2_\pi = m^{*2}_\pi + \frac{16}{9}\left(\frac{g^2l^4}{36}\right),\quad m^2_\eta = m^{*2}_\eta + \frac{12}{9}\left(\frac{g^2l^4}{36}\right),\quad m^2_k = m^{*2}_k + \frac{13}{9}\left(\frac{g^2l^4}{36}\right). \tag{24}$$

Mass spectrum (24) can be written

$$m^2 = m_1 + m_2Y + m_3\left[I(I+1) - \frac{1}{4}Y^2\right]; \tag{25}$$

$$m_1 \equiv C_0^2 + \frac{4}{3}\left(\frac{g^2l^4}{36}\right),\quad m_2 \equiv \sqrt{3}\,C_1,$$
$$m_3 \equiv \frac{2}{9}\left(\frac{g^2l^4}{36}\right). \tag{26}$$

Let us consider the mass spectrum of the nonet mesons ($S = 1^-$) in the approximation $m_\rho = m_\omega$. In the form given above, Eq. (6) is incompatible with the condition $m_\rho = m_\omega$; to resolve this problem we must add to (6) the term

$$2\left(\frac{g^2l^4}{36}\right)\varphi^2_{0,1}\varphi^2_{0,2}(\varphi^1_{\mu,1} + \varphi^2_{\mu,2})\delta_{lm}. \tag{27}$$

Then we find

$$p^2\rho_\mu = \left\{m^{*2}_\rho + 2\left(\frac{g^2l^4}{36}\right)\left(a_1a_2 - \frac{1}{2}a_0^2\right)\right\}\rho_\mu,$$
$$p^2k^*_\mu = \left\{m^{*2}_{k^*} + \left(\frac{g^2l^4}{36}\right)\left(a_1a_2 + \frac{1}{2}a_3^2 - \frac{1}{18}a_0^2\right)\right\}k^*_\mu,$$
$$p^2\Phi_\mu = \left\{m^{*2}_\Phi + \left(\frac{g^2l^4}{36}\right)\left(-\frac{8}{9} + 2a_1a_2\right)\right\}\Phi_\mu. \tag{28}$$

In deriving the equation Φ_μ we used the notation

$$\varphi^1_{\mu,1} + \varphi^2_{\mu,2} = \sqrt{6}\,\omega_{8,\mu} + \frac{2}{\sqrt{3}}\Phi_\mu.$$

From (18), and using (13) and (14), we find

$$m^2_\rho = m^{*2}_\rho + \frac{16}{9}\left(\frac{g^2l^4}{36}\right),$$
$$m^2_{k^*} = m^{*2}_{k^*} + \frac{13}{9}\left(\frac{g^2l^4}{36}\right),$$
$$m^2_\Phi = m^{*2}_\Phi + \frac{10}{9}\left(\frac{g^2l^4}{36}\right). \tag{29}$$

Mass spectrum (29) (for the octet $S = 1^-$) is also included in Eq. (25), but for a different value of the basic mass $m_{0,1}$.

We have thus obtained results within the framework of relativistic and SU(3)-invariant equations (1) and (3) which are usually obtained only in nonrelativistic SU(6)-invariant theory.

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5 May 1969
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