CHOICE OF METRICS FOR THE REFERENCE FRAME IN THE THREE-DIMENSIONAL CHRONOMETRICALLY INVARIANT TWO-METRIC FORMALISM

V. A. Barynin

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The choice of temporal and spatial metrics for the reference frame in the three-dimensional chronometrically invariant two-metric formalism is discussed in connection with various approaches for describing gravitation.

The three-dimensional chronometrically invariant two-metric formalism proposed in [1-3] is characterized by specifying temporal and spatial metrics, τ_α and (ε_{ik}), for the reference frame which determined infinitesimal distances and time intervals:

$$dT = \tau_\alpha dx^\alpha, \quad dL = \sqrt{\varepsilon_{ik} dx^i dx^k}, \tag{1}$$

where the Greek indices run over the values 0, 1, 2, 3; and the Latin indices run over 1, 2, 3. These metrics are independent of the metric tensor $g_{\alpha\beta}$.

The law for transforming τ_α and ε_{ik} for arbitrary coordinate and time transformations is defined as such that dT and dL are airy four-dimensional scalars, i.e., three-dimensional chronometrically invariant scalars which do not depend on the choice of reference frame. Otherwise, these metrics can be chosen completely arbitrarily if there are no additional physical limitations associated with the particular approach used to describe gravitation.

We first consider the purely geometric approach: in a fixed reference frame the components of the spatial and temporal metrics are related to the components of metric tensor $g_{\alpha\beta}$ by

$$\tau_\alpha = \frac{g_{\alpha 0}}{\sqrt{g_{00}}}, \quad (2a) \qquad \varepsilon_{ik} = -g_{ik} + \frac{g_{i0} g_{k0}}{g_{00}}, \tag{2b}$$

which hold over all space at any time.

The quantities dL and dT determined for these τ_α and ε_{ik}, values represent "observable" distances and time intervals [4, 5].

Let us consider the motion of a particle in a gravitational field in a reference frame having these methods. In general, the equation describing this motion, written in the spirit of this formalism is

$$\frac{D}{dT}(M\overset{*}{U}{}^\lambda) = -M\overset{*}{\Pi}{}^\lambda_{\mu\lambda} \overset{*}{U}{}^\mu \overset{*}{U}{}^\nu, \tag{3}$$

where D/dt denotes absolute four-dimensional chronometrically invariant differentiation with respect to T, and $\overset{*}{U}{}^\mu$ are the components of the four-dimensional chronometrically invariant velocity of the particle [1]. With this choice of τ_α and ε_{ik} the quantities $\overset{*}{\Pi}{}^\lambda_{\mu\nu}$, which are the components of the gravitational field intensity, vanish identically. There is thus no gravitational field in this reference frame, and the particle moves along a geodesic, experiencing new forces.

We note that Eq. (3) can be written in a different form. From the point of view of the three-dimensional modification of this formalism [3], it is

$$\frac{\Delta}{dT}(Mv^l) = -M[\overset{*}{P}{}^l_{00} + (\overset{*}{P}{}^l_{m0} + \overset{*}{P}{}^l_{0m})v^m + \overset{*}{P}{}^l_{mn}v^m v^n]. \tag{4}$$

The three-dimensional chronometrically invariant absolute derivative with respect to the invariant time is on the left side of this equation, while $v^l \equiv \overset{*}{U}{}^l$ is the three-dimensional chronometrically invariant velocity of the particle. When τ_α and ε_{ik} are chosen in the form (2), this equation becomes equal to the corresponding equation derived by the Zel'manov [5]. Only the quantity $\overset{*}{P}{}^l_{mn}$ on the right side vanishes.

However, the nonvanishing quantities $\overset{*}{P}{}^l_{00}, \overset{*}{P}{}^l_{m0}, \overset{*}{P}{}^l_{0m}$, and the quantities $\overset{*}{P}{}^0_{00}, \overset{*}{P}{}^0_{m0}, \overset{*}{P}{}^0_{0m}, \overset{*}{P}{}^0_{mn}$ cannot be interpreted as components of the gravitational field intensity, since they are not expressed in terms of chronometrically invariant time derivatives and chronometrically invariant ε-covariant coordinate derivatives of the chronometrically invariant gravitational-field potentials $\overset{*}{g}_{\mu\nu}$; instead they depend only on the chronometrically invariant partial derivatives of the components of the metrics selected. For this reason the nonvanishing terms on the right side of the equation can be interpreted as constituting some fictitious force, related to the deformation and noninertial motion of the reference frame.

In any modification of the three-dimensional chronometrically invariant two-metric formalism, the purely geometric to describing gravitation is thus characterized by a situation in which the gravitational field intensity vanishes identically in all space.

We emphasize that all this discussion holds only in a single reference frame, since Eqs. (2) are violated in a transformation to a new frame: their left and right sides transformed in different manners.

Another, purely field approach to describing gravitational employs this choice of spatial and temporal methods for the reference frame; in this case the temporal method permits synchronization of time in all space, while the spatial method is planar and independent of the time period.

In this approach both these modifications of this formalism coincides [3]; all the quantities $\overset{*}{P}^{\lambda}_{\mu\lambda}$ are components of the gravitational field intensity since they are expressed in terms of chronometrically invariant time derivatives and chronometrically invariant, ε-covariant derivatives with respect to the coordinates of the chronometrically invariant potentials $\overset{*}{g}_{\mu\nu}$ of the gravitational field. In Eqs. (3) or (4) there are no fictitious forces in this case, due to the limitations imposed on the choice of metrics. We note that these limitations lead to the existence of integral conservation laws which do not exist in all other cases.

Within the framework of this purely field approach to describing gravitation, however, the choice of metrics can remain fairly arbitrary; for any particular reference at a given time and at a given spatial point all quantities can be transformed into "observables" by choosing those metrics for which the temporal metrics at this space-time point satisfies (2a) and for which the spatial metric satisfies (2b), along with its first chronometrically invariant partial derivatives with respect to the coordinate.

In this case the quantities $\overset{*}{P}^{l}_{mn}$, which are expressed in terms of chronometrically invariant ε-covariant derivatives with respect to the coordinates of the chronometrically invariant gravitational potential γ_{ik}, derived in [2], vanish at this space-time point. By transforming to a different reference frame, moving with respect to the first without deformation, we can cause the quantity $\overset{*}{P}^{l}_{00}$ to vanish at this point. With $0 = {}_{\omega}a$, the right side of Eq. (4) or (3) then vanishes; i.e., the force acting on the particle at rest vanishes. However, this result cannot be interpreted as implying the disappearance of the gravitational field at this space-time point, since the other components ($\overset{*}{P}^{0}_{00}$, $\overset{*}{P}^{0}_{k0}$, $\overset{*}{P}^{l}_{k0}$, and $\overset{*}{P}^{0}_{ik}$) do not vanish.

This result again emphasizes that the so-called equivalency principle does not hold even locally, strictly speaking, since gravitation and inertia are described [2] by potentials which differ in nature.

The specification for all space of those metrics for which the part of the gravitational field described by $\overset{*}{P}^{l}_{mn}$ turn out to be the "transformed" part of the gravitational field is in a certain sense (with the reservations above) analogous to constructing a planar tangential space in Rylov's theory of a relative gravitational field [6].

Finally, a general approach can be taken to describe gravitation, characterized by complete arbitrariness in the choice of metrics. In this case a difference between τ_{α} and ε_{ik} and the values given by (2) leads to nonvanishing $\overset{*}{\Pi}^{\lambda}_{\mu\lambda}$, which describes the intensity of some "difference" field. None of the terms in Eq. (4) vanish, but, in contrast with the purely field approach, the right side of this equation also contains fictitious forces which arise because τ_{α} and ε_{ik} are not constants.

On the one hand, therefore, complete arbitrariness in the choice of spatial and temporal metrics for the reference frame in the three-dimensional chronometrically invariant two-metric formalism can always be limited by a local or global approach to the "observables"; on the other hand, this arbitrariness makes it possible to use various approaches to describe gravitation.

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Department of Theoretical Physics