RADIATION BY A PARTICLE IN A LOW-LYING ENERGY LEVEL IN A STRONG MAGNETIC FIELD

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Equations are derived for the total power and the polarization of the radiation emitted by an electron or by a scalar particle in the second excited state and moving in a superstrong magnetic field.

Quantum effects are important for an electron for low values of the main quantum number as well as for high energies [1, 2]. A study of an electron moving in a superstrong field was reported in [1], where it was found that the intensity of the radiation emitted differs from both the classical result and from the "ultraquantum" result.

Direct solution of the Klein-Gordon equation for a scalar particle in a uniform and constant magnetic field yields an energy

$$E = mc^2\sqrt{1+2\left(n+\frac{1}{2}\right)\frac{H}{H_0}}; \quad n = 0, 1, 2, \ldots, \tag{1}$$

where

$$H_0 = \frac{m^2c^3}{e\hbar} = 4{,}41 \cdot 10^{13} \text{ Oe.}$$

The boson wave function is (for the case in which the magnetic field is directed along the z axis)

$$\Psi(\vec{r}, t) = \sqrt[4]{\gamma}\,[\sqrt{\pi}\,n!\,2^n]^{-\frac{1}{2}}\,\frac{e^{i(k_1x+k_3z-cKt)}}{\sqrt{2K}}\,H_n(\xi), \tag{2}$$

where

$$K = \frac{E}{\hbar c}, \quad \gamma = \frac{eH}{\hbar c}, \quad \xi = \sqrt{\gamma}\left(y+\frac{k_1}{\gamma}\right),$$

k_1 and k_3 are the components of the boson wave vector, and $H_n(\xi)$ are the Hermite polynomials.

The intensity of the radiation emitted by a boson in a transition from state K and K' is found from quantum theory to be [3]

$$W_\lambda = \frac{ce^2}{2\pi}\int d^3\varkappa\,\delta(K-K'-\varkappa)\,\Phi_\lambda, \tag{3}$$

where $\lambda = \sigma$ corresponds to the two polar linearizations of the emitted radiation, and

$$\Phi_\sigma = |\bar{P}_x \sin\varphi - \bar{P}_y \cos\varphi|^2, \tag{3'}$$

$$\Phi_\pi = |\bar{P}_x \sin\varphi + \bar{P}_y \cos\varphi|^2 \cos\theta. \tag{3''}$$

Here $\bar{P}_x$ and $\bar{P}_y$ are matrix elements of the generalized momentum, calculated with the help of functions (2), while θ and φ are the polar angles showing the photon direction.

Substituting Eqs. (3') and (3'') into Eq. (3) and integrating over $\varkappa$ and φ, we find

$$W_\lambda = \frac{W_0}{1-\beta^2}\cdot\frac{x_0^2}{2(2n+1)(1+x_0^2)}\int_0^1 \frac{(1+x_0t)\,dt}{\sqrt{(1-t)(1-x_0^2t)}}\,Q_\lambda, \tag{4}$$

where

$$\sin^2\theta = \frac{(1+x_0^2)\,t}{(1+x_0t)^2},$$

$$X_0 = \frac{\left(\frac{2\nu}{2n+1}\right)\beta^2}{1+\sqrt{1-\frac{2\nu}{2n+1}\beta^2}}, \quad W_0 = \frac{m^2e^2c^3}{\hbar^2}; \quad \nu = n-n',$$

$$Q_\sigma = [\sqrt{n+1}\,I_{n+1,n'}(\nu x_0 t) - \sqrt{n}\,I_{n-1,n'}(\nu x_0 t)]^2,$$

$$Q_\pi = [\sqrt{n+1}\,I_{n+1,n'}(\nu x_0 t) + \sqrt{n}\,I_{n-1,n'}(\nu x_0 t)]^2\left[1-\frac{(1+x_0)^2\,t}{(1+x_0t)^2}\right],$$

and the functions $I_{n,n'}(x)$ are related to the Laguerre polynomials by [4]

$$I_{n,n'}(x) = \frac{1}{\sqrt{n!n'!}} e^{-x/2} x^{n-n'/2} L_{n'}^{n-n'}(x).$$

For the case of superstrong fields ($H \gg H_0$), we see from Eq. (1) that, even for small values of the main quantum number, the boson is relativistic ($\beta \sim 1$). Accordingly, in strong fields, the discrete nature of the boson's energy spectrum should be reflected in the radiation intensity.

The case in which the initial state of the particle corresponds to the n = 1 level was analyzed in [1]; below we take up transitions from the n = 2 level and compare the results with those of [1].

The transition in exact equation (4) to the limiting case of superstrong fields (i.e., $\beta \sim 1$), leads to integrals which can be evaluated only numerically. Carrying out this evaluation, we find the following results for σ and π, the components of the linear polarization for the boson: for $2 \to 0$ transitions,

$$W_\sigma^{(1)} = 0{,}38 W^{(1)}, \quad W_\pi^{(1)} = 0{,}62 W^{(1)},$$
$$W^{(1)} = 0{,}12 \frac{e^2 m^2 c^3}{\hbar^2} \left(\frac{E}{mc^2}\right)^2, \tag{5}$$

for $2 \to 1$ transitions

$$W_\sigma^{(2)} = 0{,}82 W^{(2)}, \quad W_\pi^{(2)} = 0{,}18 W^{(2)}, \quad W^{(2)} = 0{,}11 \frac{e^2 m^2 c^3}{\hbar^2} \left(\frac{E}{mc^2}\right)^2. \tag{6}$$

Evalutating the total radiation intensity for the transition $2 \to 1$ and $2 \to 0$, we find

$$W_\sigma = W_\sigma^{(1)} + W_\sigma^{(2)} = 0{,}60 W,$$
$$W_\pi = W_\pi^{(1)} + W_\pi^{(2)} = 0{,}40 W,$$
$$W = W^{(1)} + W^{(2)} = 0{,}23 \frac{e^2 m^2 c^3}{\hbar^2} \left(\frac{E}{mc^2}\right)^2. \tag{7}$$

Comparison with the results [1] shows that the total intensity for the $2 \to 1$ and $2 \to 0$ transitions is nearly four times that for the $1 \to 0$ transition. The radiation becomes less polarized as the energy level increases: for the $1 \to 0$ we have $W_\sigma / W_\pi = 2$, [1], while for the $2 \to 1$ and $2 \to 0$ transitions we have $W_\sigma / W_\pi = 1{,}5$.

Qualitatively new values are found for the polarizations for the partial transitions (see (5)): for the $2 \to 0$ transition, the σ component is less than the π component, but the σ component of the total intensity is greater than the corresponding π component.

Similar calculations for electrons lead to the following equations: where $2 \to 0$,

$$W_\sigma^{(1)} = 0{,}69 W^{(1)}, \quad W_\pi^{(1)} = 0{,}31 W^{(1)},$$
$$W^{(1)} = 0{,}33 \frac{e^2 m^2 c^3}{\hbar^2} \left(\frac{E}{mc^2}\right)^2; \tag{8}$$

for the $2 \to 1$ transition,

$$W_\sigma^{(2)} = 0{,}47 W^{(2)}, \quad W_\pi^{(2)} = 0{,}53 W^{(2)},$$
$$W^{(2)} = 0{,}39 \frac{e^2 m^2 c^3}{\hbar^2} \left(\frac{E}{mc^2}\right)^2. \tag{9}$$

The components for the total radiation intensity for the $2 \to 0$ and $2 \to 1$ transitions are

$$W_\sigma = W_\sigma^{(1)} + W_\sigma^{(2)} = 0{,}57 W,$$
$$W_\pi = W_\pi^{(1)} + W_\pi^{(2)} = 0{,}43 W,$$
$$W = 0{,}72 \frac{e^2 m^2 c^3}{\hbar^2} \left(\frac{E}{mc^2}\right)^2. \tag{10}$$

Comparison of Eqs. (10) with results of [1] reveals that the intensity for $2 \to 0$ and $2 \to 1$ transitions is approximately twice that of the $1 \to 0$ transition. While the polarization for the $1 \to 0$ transition is such that we have $W_\sigma / W_\pi = 2{,}87$,, we find $2 \to 1$ and $2 \to 0$ for the $W_\sigma / W_\pi = 1{,}32$ transitions. For the $2 \to 1$ transition we find that the σ component for the electron is smaller than the π component.

Qualitatively new values are thus found for the polarization of the radiation emitted during transitions of the particle from the second excited level to the lower levels.

We could solve the analogous problem through the use of the Dirac equation rather than the Pauli equation; we would find explicitly the contribution of the magnetic field to the radiation for the case of an electron moving in a strong magnetic field.

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