BREMSSTRAHLUNG DURING THE SCATTERING OF A NEUTRINO BY AN ELECTRON WITH AN ACCOUNT OF RADIATION CORRECTIONS

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The angular and energetic distributions are calculated for the bremsstrahlung emitted during the scattering of a neutrino by an electron. The radiation corrections of order α to the effective total cross section for these scattering is calculated; the expression for this correction also takes into account the emission of a hard photon.

Elastic scattering of neutrinos by electrons has not yet been detected experimentally under laboratory conditions, but the measurement of the corresponding cross section is clearly important for the physics of the weak interactions of leptons. There have been several studies of $\nu_e e$ scattering [1-6]; it was shown in [2-4], e.g., that the use of a polarized electron target will aid the detection of this scattering. Differential cross sections for $\nu_e e$ and $\bar{\nu}_e e$ scattering were found in [7, 8] with an account of radiation corrections of the order of α.

Below we study the emission of photons during neutrino-electron scattering,

$$\nu_e + e \rightarrow \nu_e' + e' + \gamma, \tag{1}$$

which is of clear importance for two reasons. This radiation can be used to detect $\nu_e e$ scattering, since it is possible that bremsstrahlung γ's rather than recoil electrons will be detected most easily in experiments with high-energy neutrinos. The cross section for the electromagnetic radiation accompanying $\nu_e e$ scattering is approximately α^{-1} times smaller than the cross section for the basic process for neutrino energies of the order of m_e. As the energy of the incident neutrinos increases, however, the cross section for reaction (1) increases more rapidly than that for $\nu_e e \rightarrow \nu_e' e'$, and the two cross sections are of the same order of magnitude at a neutrino energy of $E_\nu \sim 1$ (in the laboratory coordinate system).

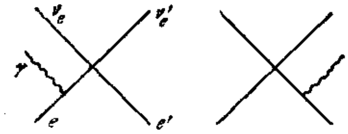
The bremsstrahlung emitted during $\nu_e e$ scattering is also important is connection with the electromagnetic effects for this reaction. The cross section for reaction (1) integrated over the momentum of the photon is added to that for the basic process $\nu_e e \rightarrow \nu_e' e'$, which contains radiation corrections of the order of α. In the calculations of the radiation corrections of the order of α in [7, 8], account was made of the radiation of only a soft photon, having an energy $\varepsilon_\gamma \ll \Delta E_\gamma$, in addition to the differential cross sections for $\nu_e e(\bar{\nu}_e e)$ scattering. It was assumed that the maximum energy of the soft photon satisfied $\Delta E_\gamma \ll m_e$. Here the radiation correction depends on ΔE_γ. In this case it is essential and possible to compare the correction with experiment. The emission of a hard photon ($\varepsilon_\gamma \gg \Delta E_\gamma$) must also be taken into account: in this case the radiation correction in the order of α will not depend on ΔE_γ and will take into account all possible effects of electromagnetic origin.

We start from the interaction Hamiltonian

$$H_{\text{in}} = \frac{G}{\sqrt{2}} [\bar{\psi}_{e'} \gamma_\rho (1 + \gamma_5) \psi_{\nu_e}] [\bar{\psi}_{\nu_e'} \gamma_\rho (1 + \gamma_5) \psi_e] + ie\bar{\psi}_e \hat{A} \psi_e, \tag{2}$$

where G is the coupling constant for the weak four-fermion V-A interaction, $G \simeq 1{,}01 \cdot 10^{-5} \frac{1}{m_p^2}$, $\hat{A} = A_\mu \gamma_\mu$, and A_μ are the operators for the electromagnetic field.

Two Feynman diagrams correspond to the emission of a photon:



We denote the particles in reaction (1) in the following manner:

$$1 = e',\ 2 = e,\ 3 = \nu_e',\ 4 = \nu_e.$$

The matrix element for the action (1) is

$$M = i \frac{Ge}{\sqrt{2}} \bar{u}_1 \left[\gamma_\rho (1 + \gamma_5) \frac{i(\hat{p}_2 - \hat{k}) - m}{(p_2 - k)^2 + m^2} \frac{\hat{e}}{\sqrt{2k_0}} + \right.$$
$$\left. + \frac{\hat{e}}{\sqrt{2k_0}} \frac{i(\hat{p}_1 + \hat{k}) - m}{(p_1 + k)^2 + m^2} \gamma_\rho (1 + \gamma_5) \right] u_2 (\bar{u}_3 \gamma_\rho (1 + \gamma_5) u_4). \tag{3}$$

where $\hat{e} = e_\mu \gamma_\mu$, and e_μ is the polarization vector of the photon.

The differential cross section is found from

$$d\sigma_\gamma = (2\pi)^{-5} \frac{1}{-(p_2 p_4)} |M|^2 \delta(p_2 + p_4 - p_1 - p_3 - k) \frac{d\vec{p}_1}{E_1} \frac{d\vec{p}_3}{E_3} \frac{d\vec{k}}{k_0}. \tag{3'}$$

Using standard calculation methods (see, e.g., [9]), we find the following cross section for reaction (1) in an arbitrary reference system:

$$d\sigma_\gamma = \frac{2G^2 e^2}{(2\pi)^5} \frac{\delta(p_2 + p_4 - p_1 - p_3 - k)}{-(p_2 p_4)} \Big\{ -\Big(\frac{m^2}{\varkappa^2} + \frac{m^2}{\varkappa'^2} + 2\frac{p_1 p_2}{\varkappa\varkappa'}\Big)(p_1 p_3)(p_2 p_4) +$$
$$+ \Big(\frac{1}{\varkappa} - \frac{1}{\varkappa'}\Big)(p_2 p_4)(p_1 p_3) + \Big(\frac{m^2}{\varkappa^2} - \frac{1}{\varkappa}\Big)(k p_4)(p_1 p_3) -$$
$$- \Big(\frac{m^2}{\varkappa'^2} + \frac{1}{\varkappa'}\Big)(k p_3)(p_2 p_4) + \frac{1}{\varkappa}(p_2 p_4)(p_2 p_3) - \frac{1}{\varkappa'}(p_1 p_4)(p_1 p_3) +$$
$$+ \frac{p_1 p_2}{\varkappa\varkappa'}(k p_4)(p_1 p_3) - \frac{p_1 p_2}{\varkappa\varkappa'}(p_2 p_4)(k p_3)\Big\} \frac{d\vec{k}}{k_0} \frac{d\vec{p}_1}{E_1} \frac{d\vec{p}_3}{E_3}, \tag{4}$$

where

$$\varkappa = (k p_2), \quad \varkappa' = (k p_1).$$

In Eq. (4), $p_\beta = (\vec{p}_\beta, iE_\beta)$, ($\beta = 1, 2, 3, 4$) are the 4-momenta of the final electron, the initial electron, the scattered neutrino, and the initial neutrino, respectively; $k = (\vec{k}, ik_0)$ is the 4-momentum of the bremsstrahlung photon; and m is the rest mass of the electron. Here we are using a unit system in which we have $c = \hbar = 1$ and the metric is $ab = a_\mu b_\mu = \vec{a}\vec{b} + a_4 b_4$, where $a = (\vec{a}, ia_0)$.

We integrate over the momenta of the scattered neutrino and electron by means of the integrals

$$I_n = \int \frac{\delta(q - p_1 - p_3)}{(\varkappa')^n} \frac{d\vec{p}_1}{E_1} \frac{d\vec{p}_3}{E_3},$$
$$n = 0, 1, 2; \quad q = p_2 + p_4 - k. \tag{5}$$

We find

$$I_0 = 2\pi\Big(1 + \frac{m^2}{q^2}\Big), \quad I_1 = \frac{2\pi}{(kq)} \ln\Big(-\frac{q^2}{m^2}\Big),$$
$$I_2 = -\frac{2\pi}{(kq)^2}\Big(1 + \frac{q^2}{m^2}\Big). \tag{5'}$$

All the other integrals are expressed in combinations of I_0, I_1 and I_2.

Transforming to the laboratory system ($\vec{p}_2 = 0$), we find from Eqs. (4) and (5) the angular and energetic distributions of the bremsstrahlung:

$$d\sigma_\gamma(\omega, \theta) = \frac{\alpha G^2 m^2}{2\pi^3} \omega d\omega d\Omega_\gamma \Big\{\Big(1 - \frac{2}{\omega\Delta} + 2\frac{1+\gamma}{\omega^2\Delta^2}\Big) \ln(1 + 2E - 2\omega E\Delta) +$$
$$+ \frac{1}{\omega^2}\Big(\frac{1}{2} - \frac{1}{4}\gamma - E\Big) - \frac{1}{2}\gamma - 4 - \frac{1}{\omega}\Big(3 - \frac{3}{4}\gamma + 4E\Big) +$$
$$+ \omega\Delta\Big[1 + \frac{1}{\omega^2}\Big(2E - \frac{1}{2}\Big) + \frac{3}{\omega}\Big(E + \frac{1}{2}\Big)\Big] - E(1 + \omega)\Delta^2 -$$
$$- \frac{6}{\omega^2\Delta^2} + \frac{2}{\omega\Delta}\Big(4 + \frac{1}{1 + 2E} + \frac{2E(1+E)}{\omega(1 + 2E)}\Big) +$$
$$+ \frac{1}{1 + \frac{1}{2}\gamma - \omega\Delta}\Big[\frac{1}{8}\frac{\gamma^2}{\omega^2} + \frac{1}{4}\gamma^2 - \frac{1}{2}\gamma + \frac{2}{1 + 2E} +$$
$$+ \frac{1}{\omega}\Big(\frac{3}{4}\gamma - \frac{3}{8}\gamma^2 - \frac{1}{1 + 2E}\Big)\Big]\Big\}. \tag{6}$$

Here E and ω are the energies of the initial neutrino and the emitted photon in units of the rest mass of the electron:

$$E = \frac{E_{\nu_e}}{m}, \quad \omega = \frac{k_0}{m}, \quad \gamma = \frac{1}{E},$$

$$\Delta = 1 + \gamma - \cos\theta, \quad \cos\theta = (\vec{p}^{\circ}_{\nu_e} \vec{p}^{\circ}_{\gamma}), \quad \vec{p}^{\circ}_{\nu_e} = \frac{\vec{p}_{\nu_e}}{|\vec{p}_{\nu_e}|}, \quad \vec{p}^{\circ}_{\gamma} = \frac{\vec{k}}{|\vec{k}|}.$$

The ω and θ values in Eq. (6) are limited by the curve $\omega(1+\gamma-\cos\theta)=1$ for $\omega \geqslant \frac{E}{1+2E}$. There is no such limitations for $\omega \leqslant \frac{E}{1+2E}$, and we have $|\cos\theta| \leqslant 1$. These relations follow from energy and momentum conservation for reaction (1).

To find the angular distribution of the radiation, we integrate Eq. (6) over energy ω of the photon in the range $\varepsilon_0 \leqslant \omega \leqslant E$, where ε_0 is the minimum photon energy in units of the rest mass of the electron, which could be determined experimentally. We find

$$\begin{aligned}\frac{d\sigma_\gamma(\theta)}{d\Omega_\gamma} = \frac{\alpha G^2 m^2}{2\pi^3}\Big\{ &\frac{1}{\Delta^2}\Big[\frac{1}{2}\Big(\frac{1}{4}\gamma^3+\frac{1}{4}\gamma^2-3\Big)\ln(1+\Gamma(\theta)) + \\ &+2(1+\gamma)L\Big(\frac{2}{2+\gamma}\Big)-2(1+\gamma)L\Big(\frac{2\varepsilon_0\Delta}{2+\gamma}\Big)+\frac{91}{12}-\frac{1}{4}\gamma^2- \\ &-2((1+\gamma)\ln(1+2E)-3)\ln\varepsilon_0\Delta\Big]+\frac{1}{\Delta}\Big[\Big(\frac{3}{4}\gamma-\frac{3}{8}\gamma^2+2\varepsilon_0-\frac{1}{1+2E}\Big)\times \\ &\times\ln(1+\Gamma(\theta))-\frac{19}{2}\varepsilon_0-\frac{1}{4}\varepsilon_0\gamma-\frac{9}{4}-\frac{17}{6}E+\frac{3}{4}\gamma+\frac{1}{4}\varepsilon_0\gamma^2- \\ &-4\frac{E(1+E)}{1+2E}\ln\varepsilon_0\Delta\Big]+\frac{1}{4}(9+\gamma)\varepsilon_0^2+\frac{3}{2}\Big(E-\frac{1}{3}\Big)+ \\ &+\Big(\frac{\gamma}{4(1+2E)}-\frac{1}{2}\varepsilon_0^2\Big)\ln(1+\Gamma(\theta))+\frac{2E^2}{1+2E}\ln\varepsilon_0\Delta+\Big(3+4E-\frac{3}{4}\gamma\Big)\varepsilon_0+ \\ &+\varepsilon_0\Delta\Big[\frac{1}{2}-2E-\frac{3}{2}\varepsilon_0\Big(E+\frac{1}{2}\Big)-\frac{1}{3}\varepsilon_0^2\Big]+E\Delta^2\varepsilon_0^2\Big(\frac{1}{2}+\frac{1}{3}\varepsilon_0\Big)\Big\},\end{aligned} \tag{7}$$

where

$$\Gamma(\theta)=2E(1-\varepsilon_0\Delta),\quad L(x)=\int_0^x \frac{\ln(|1-y|)}{y}dy.$$

We easily see from Eq. (7) that the maximum radiation occurs as $\theta \to 0^\circ$; i.e., the radiation maximum corresponds to the forward emission of a photon, parallel to the momentum of the incident neutrinos. As high neutrino energies $(E \gg 1)$, the radiation is essentially concentrated in a narrow column having a vertex angle $\theta \sim \frac{2}{E}$. In this case Eq. (7) yields

$$\frac{d\sigma_\gamma(E\gg 1,\theta=0^0)}{d\Omega_\gamma} \cong \frac{\alpha G^2 m^2}{\pi^3}E^2\ln E\Big(\ln\frac{E}{\varepsilon_0}\Big). \tag{7'}$$

Integrating Eq. (6) over the angular variables $d\Omega_\gamma=\sin\theta\, d\theta\, d\varphi$ for the escape of the photon, we find the energy distribution of the bremsstrahlung for the case of $\omega \geqslant \frac{E}{1+2E}$:

$$\begin{aligned}\frac{d\sigma_\gamma}{d\omega}=\frac{\alpha G^2 m^2}{\pi^2}\Big\{&\frac{3}{2}\gamma-\frac{1}{2}\gamma^2+\frac{11}{2}-2L\Big(\frac{2}{2+\gamma}\Big)+2L\Big(\frac{2\omega}{1+2E}\Big)+ \\ &+\Big[\frac{1}{4}\gamma^2-\Big(1+\frac{\omega}{E}\Big)+\frac{1}{8}\frac{\gamma^2}{\omega^2}+\frac{E}{\omega}\Big(2+\gamma+\frac{3}{4}\gamma^2-\frac{3}{8}\gamma^3+\frac{2}{1+2E}\Big)\Big]\times \\ &\times\ln(1+2E-2\omega)+2\Big(3-\ln(1+2E)+2\frac{E(1+E)}{\omega(1+2E)}\Big)\ln\frac{E}{\omega}+ \\ &+\Big(\frac{1}{4}\gamma^2+\frac{1}{4}\gamma^3-\frac{5}{4}\gamma-\frac{53}{6}\Big)\frac{E}{\omega}+\Big(\frac{7}{2}+\frac{1}{12}\gamma\Big)\frac{\omega}{E}-\frac{1}{6}\frac{\omega^2}{E^2}+ \\ &+\Big(\frac{1}{4}\gamma^2-\frac{1}{4}\gamma^3-\frac{1}{3}\gamma\Big)\frac{E^2}{\omega^2}\Big\}.\end{aligned} \tag{8}$$

In the limiting case of high neutrino energies, $E \gg 1\ (\gamma \ll 1)$, we find from (8)

$$\begin{aligned}\frac{d\sigma_\gamma}{d\omega}=\frac{\alpha G^2 m^2}{\pi^2}\Big[&\frac{11}{2}+\frac{\pi^2}{3}+2L\Big(\frac{\omega}{E}\Big)+\Big(2\frac{E}{\omega}-\frac{\omega}{E}-1\Big)\ln 2(E-\omega)+ \\ &+2\Big(3-\ln 2E+\frac{E}{\omega}\Big)\ln\frac{E}{\omega}-\frac{53}{6}\frac{E}{\omega}+\frac{7}{2}\frac{\omega}{E}-\frac{1}{6}\frac{\omega^2}{E^2}\Big].\end{aligned} \tag{9}$$

Integrating (7) over the solid angle $d\Omega_\gamma$, and using $\varepsilon_0 \ll 1$, we find the total cross section for the emission of hard photons with energies $\omega \geqslant \varepsilon_0$ in reaction (1):

$$\begin{aligned}\sigma_\gamma=\Big(\frac{2\alpha G^2 m^2}{\pi^2}\Big)\frac{2E^2}{1+2E}\Big\{&\frac{175}{24}+\frac{1}{4}\gamma-\frac{3}{8}\gamma^2+(1+\gamma)L\Big(\frac{2}{2+\gamma}\Big)- \\ &-4\ln 2E+(1+\gamma)\Big[\frac{3}{2}\ln 2E-\frac{1}{2}\ln\Big(1+\frac{\gamma}{2}\Big)\Big]\ln(1+2E)-\end{aligned} \tag{10}$$

$$-2[(1+\gamma)\ln(1+2E)-2]\ln 2\varepsilon_0+\frac{1}{32}\gamma(16+20\gamma-3\gamma^2)\ln^2(1+2E)-$$

$$-\left(\frac{13}{6}+\frac{23}{6}\gamma+\frac{1}{8}\gamma^2-\frac{3}{8}\gamma^3\right)\ln(1+2E)\Big\}. \qquad (10)$$

We see that the cross section σ_γ contains a lower limit ε_0 for the energy of the emitted photon.

To eliminate energy ε_0 from cross section (10), we also consider the scattering of a neutrino by an electron ($\nu_e e\to\nu_e' e'$) with an account of the emission of soft photon ($\sigma_{\Delta E_\gamma}$), the radiation corrections (σ_r) due to the emission of a virtual photon by the electron, and the vacuum polarization.

The total cross section for neutrino-electron scattering, $\nu_e e\to\nu_e' e'$, is, with an account of radiation corrections of order α,

$$\sigma'=\sigma_0+\sigma_r+\sigma_{\Delta E_\gamma}=\left(\frac{2G^2m^2}{\pi}\right)\frac{2E^2}{1+2E}\Big\{1-\frac{\alpha}{\pi}\Big[(2(1+\gamma)\ln(1+2E)-4)\times$$

$$\times\ln\frac{m}{2\Delta E_\gamma}+\frac{4+\gamma}{6(2+\gamma)}\ln\frac{\Lambda^2}{m^2}-\frac{1}{6}\left(35+41\gamma+8\gamma^2+\gamma^3+\frac{1}{1+2E}\right)\times$$

$$\times\ln(1+2E)-\frac{1}{8}\gamma(4+\gamma^2+\gamma^3)\ln^2(1+2E)+\frac{15}{2}-\frac{1}{27}+\frac{11}{6}\gamma+$$

$$+\frac{5}{6}\gamma^2-\frac{43}{108}\frac{1}{1+2E}+\frac{3}{2}(1+\gamma)\ln^2(1+2E)+$$

$$+2(1+\gamma)\left(2\ln(1+\gamma)-3\ln\left(1+\frac{\gamma}{2}\right)\right)\Big\}\ln(1+2E)+$$

$$+2(1+\gamma)L\left(-\frac{1}{1+2E}\right)+(1+\gamma)L\left(\frac{1}{(1+2E)^2}\right)\Big]. \qquad (11)$$

Here ΔE_γ is the maximum energy of the soft photon, and Λ is the momentum-cutoff constant, which is taken to be $\Lambda\leqslant 100$ GeV for weak lepton interactions.

According to Eqs. (10) and (11), the total cross section σ for neutrino-electron scattering which takes into account all possible electromagnetic effects of order α, is

$$\sigma=\sigma'+\sigma_\gamma=\left(\frac{2G^2m^2}{\pi}\right)\frac{2E^2}{1+2E}\left[1-\frac{\alpha}{\pi}f_1(E,\Lambda)\right], \qquad (12)$$

where

$$f_1(E,\Lambda)=\frac{4+\gamma}{3(2+\gamma)}\ln\frac{\Lambda}{m}+\Big[-\frac{11}{3}-3\gamma-\frac{29}{24}\gamma^2-\frac{13}{24}\gamma^3-\frac{1}{6(1+2E)}-$$

$$-\frac{3}{2}(1+\gamma)\ln\left(1+\frac{\gamma}{2}\right)+4(1+\gamma)\ln\left(1+\frac{1}{1+2E}\right)\Big]\ln(1+2E)+$$

$$+4\ln 2E-\frac{1}{32}\gamma(32+20\gamma+4\gamma^2+\gamma^3)\ln^2(1+2E)+$$

$$+2(1+\gamma)L\left(-\frac{1}{1+2E}\right)+(1+\gamma)L\left(\frac{1}{(1+2E)^2}\right)-(1+\gamma)L\left(\frac{2}{2+\gamma}\right)+$$

$$+\frac{37}{216}+\frac{19}{12}\gamma+\frac{29}{24}\gamma^2-\frac{43}{108}\frac{1}{1+2E}. \qquad (12')$$

We see that cross section σ no longer contains $\varepsilon_0\left(=\frac{\Delta E_\gamma}{m}\right)$, but depends only on the cutoff prime Λ for weak interaction of leptons.

At high neutrino energies, $E\gg 1$ $(\gamma\ll 1)$, the function $f_1(E,\Lambda)$ has a simple form:

$$f_1(E,\Lambda)\simeq\frac{1}{3}\left(\ln 2E+\frac{\pi^2}{2}+\frac{37}{72}+\ln\frac{\Lambda^2}{m^2}\right). \qquad (13)$$

From Eqs. (12) and (13) we see that, in contrast with (11), the correction increases in proportion to ln IIE, and not in proportion to $\ln^2 2E$. At very high neutrino energies, the radiation correction is 2-3%, with Λ = 100 GeV. Equations (7) and (8) show that the bremsstrahlung cross section of $\nu_e e$ scattering increases rapidly with increasing energy of the incident neutrinos. In numerical estimates we will therefore use the maximum neutrino energy E_ν which can at present be used experimentally. Since, for solar neutrinos produced by the decay of ^8B we have $E_{\nu,\max}=14{,}06$ Mev, we choose $E_\nu=14$ Mev (E = 28) and carry some numerical estimates on the basis of Eq. (7) and (8). In Eq. (7) we assume that $\varepsilon_0=1$ $(k_{0,\min}=m)$. The results are shown in the accompanying table.

θ	0°	5°	10°	15°	20°	30°
$\frac{d\sigma_\gamma(\Theta)}{d\Omega_\gamma}$, cm²/sr	$1{,}16\cdot 10^{-44}$	$0{,}86\cdot 10^{-44}$	$4{,}2\cdot 10^{-45}$	$2{,}6\cdot 10^{-45}$	$5{,}7\cdot 10^{-46}$	$4{,}2\cdot 10^{-46}$

With $E_\nu = 14$ Mev and ω = 0.1E we find $d\sigma_\nu/d\omega = 2{,}4\cdot 10^{-47}$ cm^2 from (8). In the case of high-energy neutrinos on the other hand, with $E_\nu = 1$ GV (E = 2000) and $\theta = 0°$ the cross section is (for $\varepsilon_0 = 1$):

$$\frac{d\sigma_\gamma}{d\Omega_\gamma} \cong 0{,}72\cdot 10^{-39} \text{ cm}^2/\text{sr}.$$

REFERENCES

1. R. P. Feynman and M. Gell-Mann, Phys. Rev., 109, 193, 1958.
2. B. K. Kerimov and Yu. I. Romanov, ZhETF, 46, 1962, 47, 1123, 1964.
3. B. K. Kerimov and Yu. I. Romanov, Vestn. Mosk. un-ta, fiz., astron., No. 5, 1964.
4. B. K. Kerimov and Yu. I. Romanov, Izv. AN SSSR, ser. fizich., 27, 1122, 1965.
5. A. A. Sokolov, Yu. G. Pavlenko, Yu. Ivanov, and B. K. Kerimov, DAN SSSR, 157, 1096, 1964.
6. M. A. Markov, Neutrinos [in Russian], Nauka, Moscow, 1964.
7. T. D. Lee and A. Sirlin, Rev. Mod. Phys., 36, 666, 1964.
8. A. M. Perelomov, Yadernaya fizika, 1, 1045, 1965.
9. A. I. Akhiezer and V. B. Berestetskii, Quantum Electrodynamics [in Russian], Nauka, Moscow, 1969.

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