MECHANICAL OSCILLATOR IN A LIGHT FLUX

V. B. Braginskii, V. A. Kuznetsov, and V. N. Rudenko

Vestnik Moskovskogo Universiteta. Fizika, Vol. 26, No. 2, pp. 192-197, 1971

UDC 535.214

An experimental study has been made of light-induced stiffness and radiometric oscillatory instability.

A high sensitivity can be achieved in the measurement of small forces (or small moments of forces) in macroscopic experiments if the dissipator coupling between the object under study and the laboratory can be reduced appreciably [1]. In high-Q mechanical oscillators in vacuum, effects arise which strongly affect the dynamic characteristics. Below we discuss two of these effects.

§1. Light-Induced Stiffness

A nonuniform light flux introduces a differential stiffness into a linear mechanical oscillator:

$$K_{\mathrm{LF}} \simeq \left(\frac{1+R}{c}\right)\frac{\partial N(r)}{\partial r},$$

where R is the reflection coefficient of the oscillator material, N(r) is the intensity of the light flux, and c is the speed of light. An additional differential stiffness due to radiation pressure can be produced in a uniform flux also, but with a torsional oscillator. A change in the oscillation period of an oscillator caused by this effect is easily detectable under laboratory conditions.

Figure 1 shows a diagram (top view) of the setup used to measure the light-induced stiffness. A torsional oscillator is made from a lightweight quartz dumbbell (12 cm long and 100 μ in diameter), to whose ends two blades (1 cm × 1 cm) of aluminum foil are attached. This dumbbell is suspended on a tungsten filament (6 μ in diameter and 30 cm long) in an evacuated volume. The vacuum can range from $2\cdot10^{-4}$ to $1\cdot10^{-7}$ torr. Clearly, a uniform light flux along direction *aa'* will produce a negative light-induced stiffness, or one along direction bb' will produce a positive stiffness.

Figure 2 shows the oscillation period τ_0 of the pendulum as a function of the pressure for various light flux intensities. It is assumed here that the light flux is directed along bb'; i.e., a positive stiffness is induced in the pendulum. Curve 1 corresponds to a vanishing flux, while curves 2-5 correspond to 0.2, 0.5, 1.3, and 1.7 W/cm^2, respectively. We see from Fig. 2 that in the absence of a light flux the oscillation period does not depend on the pressure up to $2\cdot10^{-4}$ torr, having a value of about $\tau_0\simeq380$. As the intensity of the light flux increases, the oscillation period contracts to $|\tau|_{LF}\simeq60$ (at an intensity of $W\simeq1.7$ W/cm^2). This result means, in particular, that the maximum light-induced stiffness under our conditions is higher by a factor of $[\tau_0/(\tau_0)_{\mathrm{LF}}]^2\simeq40$ then the torsional stiffness of the filament. The oscillation period is essentially independent of thepressure, except at the right-hand edge of Fig. 2 (at pressures above 10^{-5} torr, where radiometric forces become important).

§2. Relaxation Time of an Oscillator with Light-Induced Stiffness

During the measurement of the relaxation time of this torsional pendulum it was discovered that this time depends strongly on the light flux and pressure, so this dependence can be related to radiometric forces. The temperature difference between the elongated and shaded sides of the pendulum blade causes an excess (radiometric) pressure to be exerted by the gas molecules on the blade; this pressure has the same sign as the radiation pressure. In the experimental setup of Fig. 1, radiometric forces also cause restoring moments, i.e., a radiometric stiffness, since the total light flux incident on a blade depends on the angular position of the pendulum. The radiometric pressure is comparable in magnitude to the radiation pressure only at pressures below 10^{-4} torr (for a blade thickness of $\leqslant10^{-2}$ cm), so the contribution of the radiometric stiffness to the oscillation period is small essentially everywhere in this pressure range. Significantly, however, the radiometric stiffness arises in the oscillatory system after a delay, governed by the speed of the processes involved in establishing the temperatures of the pendulum blades. This delay should lead to a change in the damping factor for the pendulum oscillations. We turn first to a simple phenomenological model for this phenomenon. The equation for the torsional oscillations of a pendulum having an additional retraded stiffness is

$$\ddot{\varphi}(t) + 2\delta_0\dot{\varphi}(t) + \omega_0^2\varphi(t) + \theta^2\varphi(t-\tau) = 0. \tag{1}$$

where $\varphi(t)$ is the angular position; δ_0 is the intrinsic damping factor, i.e., shows the absence of a radiometric effect; ω_0 is the natural frequency, determined was shown above, by the light-induced stiffness; and the last term in Eq. (1) describes the effect of the retarded radiometric stiffness f_τ: $\theta^2 = f_\tau / I \sim P$, where I is the moment of inertia of the pendulum, and τ is the delay time.

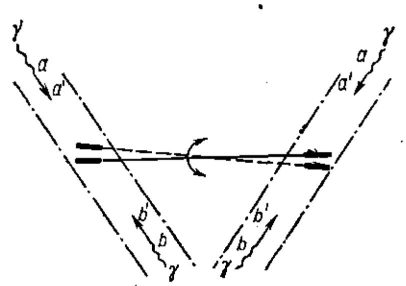

Fig. 1

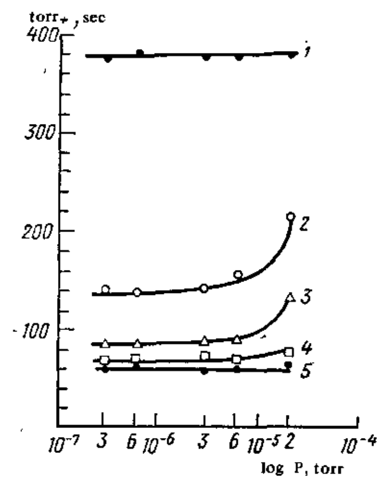

Fig. 2

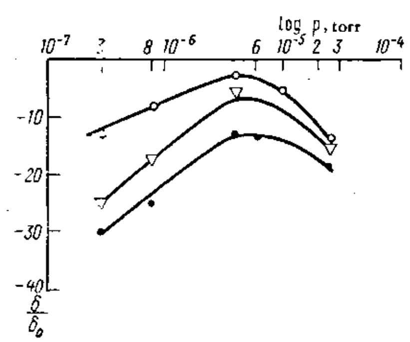

Fig. 3

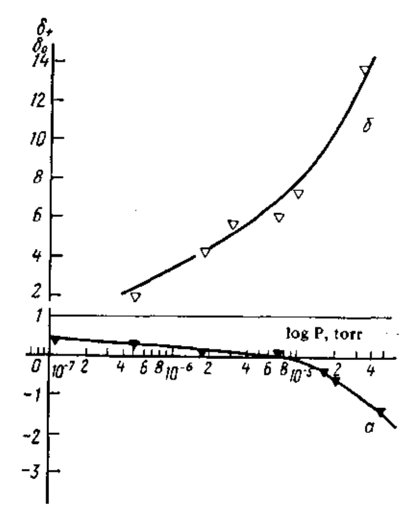

Fig. 4

Substitution of the solution $\varphi(t) = e^{(-\delta + i\omega)t}$ into this equation yields a system of transcendental equations for δ and ω_1—the new damping factor and frequency which take into account the radiometric forces:

$$\delta = \delta_0 - \theta^2 e^{\delta\tau}\frac{\sin\omega\tau}{2\omega},$$
$$\omega^2 = \omega_0^2 - \delta^2 + \theta^2 e^{\delta\tau}\left(\cos\omega\tau - \frac{\delta}{\omega}\sin\omega\tau\right). \tag{2}$$

When we impose the auxiliary conditions $\delta_0^2 \ll \omega_0^2;\ \theta^2 \ll \omega_0^2;\ \delta\tau \ll 1$, which correspond quite well to the experimental conditions at pressures below $5\cdot 10^{-4}$ torr, System (2), sympathize to

$$\begin{cases} \dfrac{\delta}{\delta_0} \simeq 1 - \dfrac{\theta^2}{2\omega_0\delta_0} \sin\omega_0\tau \\ \omega^2 \simeq \omega_0^2 \end{cases} \tag{3}$$

For the zone near the pressure $p \sim 5\cdot 10^{-4}$ torr, we have $\theta^2 \lesssim \omega_0^2$, and we should take into account the frequency correction in the second term of Eq. (3):

$$\omega^2 \simeq \omega_0^2 + \theta^2 \cos\omega_0\tau. \tag{4}$$

Equations (3) and (4) describe qualitatively the effect of the retarded radiometric stiffness on the temporal characteristics of the oscillatory system. As (3) and (4) show, the signs of the corrections to the frequency and damping factor depend on the ratio of the natural oscillation period to the delay. The correction to frequency ω_0 is small in magnitude and it is essentially vanishing for a sufficiently low pressure; on the other hand, the correction to the damping factor increases in importance with decreasing value δ_0, and for a system having a long term constant it may be important down to extremely low pressures. Qualitatively, the $\frac{\delta}{\delta_0} = f(p)$ dependence for a constant light intensity should be described by straight lines [see (3)] converging to 1 as $p \to 0$; the slope should be governed by the value of τ.

We turn now to the experimental results. The curves in Fig. 2 were obtained for a pendulum having blades of aluminum foil 100 μ thick. The slight increase in the oscillation period on curves 2-5 at pressures 10^{-4}–10^{-5} torr, results from the radiometric corrections to the natural frequency according to (4).

Figure 3 shows the pressure dependence of the ratio δ/δ_0 for the same pendulum and for three different light intensities: 0.2, 0.5, and 1.7 W/cm^2. The value of δ turns out to be negative everywhere in this pressure range; i.e., the radiometric forces convert the pendulum from the damping mode to the buildup mode.

It follows from Eq. (3) that this mode can be established under the condition $\sin\omega_0\tau > 0$. The curves display extrema (instead of the expected linearity) because of the decrease in δ_0 with increasing pressure. The damping caused by the viscosity of the suspension filament is negligible, corresponding to $\delta_{mech} \simeq 10^{-9}$ 1/sec; actual absorbed δ_0 value is governed only by friction with the residual gas and has a value in the range $(10^{-3}$–$10^{-5})$ sec^{-1} in the pressure range studied. In subsequent experiments we determined δ_0 by means of a system of permanent magnets arranged near the pendulum blades; the damping of the pendulum, due to the eddy-current loss in the blades, was held at the level $\delta_0 \simeq 10^{-3}$ sec^{-1}. The(δ/δ_0 = f(p) dependence obtained under these conditions from curve a in Fig. 4) corresponds to the quantitative picture which follows from (3): as the pressure is reduced, the ratio (δ/δ_0) tends toward 1; the curve is nonlinear only because of the logarithmic scale with use for the pressure axis. It follows from (3) when under the condition $\sin\omega_0\tau > 0$ a reduction of the pressure should result in a change in the sign of δ. We do in fact observe this change, from a buildup mode to a damping mode (with this pendulum model, the damping vanishes near $p \simeq 9\cdot 10^{-6}$). At this point the radiometric energy input compensates for the natural damping of the pendulum. If we have $\sin\omega_0\tau < 0$, [see (3)], we should find a damping mode. This latter condition can be satisfied by a suitable choice of ratio between the period and the delay. Experimentally, we achieved this mode over a pendulum having a blade ~ 15 μ thick and having a period of ~ 32 sec (at a power of $w \simeq 1.5$ W). The(δ/δ_0 = f(p) dependence for this pendulum is shown by curve b in Fig. 4.

Using the experimental data (Figs. 2-4), we estimated the delay τ on the basis of (3). For a pendulum having blades ~ 100 μ thick, we find $\tau \simeq (1-3)\cdot 10^2$ sec (we assumed in the calculation that the retardation was due to only the thermal properties of the pendulum and the surrounding medium and that τ was proportional to the blade thickness). This delay is consistent with a theoretical estimate based on the solution of the corresponding heat-conduction problem.

The most dangerous of the parasitic effects in this experimental setup is outgassing from the heated pendulum blade. To suppress this effect, we heated the blades in vacuum before the experiments until a constant pressure was achieved. It should also be noted that this effect should make only a constant contribution to δ_0, essentially independent of the pressure in the pressure range studied.

On the whole, the experimental results are consistent with the assumption that the radiometric effect is responsible for the change in the relaxation time of an oscillator with light-induced stiffness.

§3. Radiometric Instability in a Fabry-Perot Resonator

It was shown in [2]that when an optical indicator, in particular, a Fabry-Perot resonator, is used to measure small displacements of a test object (attached rigidly to one of the resonator mirrors), there is an inverse dynamic effect of the indication system on the test object. As a result, the test object may become unstable, if the working point is on the right-hand slope of the resonant curve of the resonator and if the intrinsic friction of the test object is sufficiently low. This effect severely limits the use of a Fabry-Perot resonator for indicating small displacements, but, fortunately, the effect does not occur on the left-hand slope of the resonant curve.

Under actual laboratory conditions, however, a resonator is in a residual-gas medium. We would therefore expect that a test object (or mirror) could become unstable as a result of external radiometric forces, by a mechanism analogous to that responsible for the effects discussed in the preceding section.

During mechanical oscillations of a mirror there is a change in the intensity of the electromagnetic radiation within the resonator, and there is a corresponding change in the temperature at the mirror surfaces, but only after a delay governed by the thermal inertia. In this case, the radiometric pressure creates a retarding stiffness and thus changes the damping factor of the mechanical oscillatory system. The correction $\Delta\delta$ to the damping factor is given by (3), in which ω_0 must be understood as the frequency ω_M of the mechanical oscillations of the mirror, and where we have $\theta^2 = K_\tau/m$, where K_τ is the radiometric stiffness and m is the mirror mass. A simple calculation yields

$$\Delta\delta \simeq (\delta - \delta_0) \simeq \mp P \frac{\pi W_0 h}{T_0 \lambda \varkappa N} \frac{\sin \omega_M \tau}{\omega_M}. \tag{5}$$

where P is the residual gas pressure, W_0 is the power supplied to the resonator, λ is the radiation wavelength, h is the mirror thickness, $\varkappa$ is the thermal conductivity, and T_0 is the temperature of the surrounding medium. The plus sign in (5) corresponds to tuning on the right-hand slope of the resonant curve, and the minus sign corresponds to the left-hand slope. There is an instability if the correction to the damping factor is negative and is greater in magnitude than the intrinsic damping factor $\delta_0 = \omega_M/2Q_M$, where Q_M is the mechanical Q of the mirror. As (5) shows, this situation can be realized on either the left-hand or right-hand slope of the resonant curve, depending on the sign of $\sin \omega_M \tau$, which changes, with fixed ω_M, as a function of the mirror thickness (and thus of the delay τ). If $\Delta\delta$ is negative, the critical pressure, i.e., that above which the system becomes unstable, is found from the condition $\Delta\delta = \delta_0$:

$$P_{cr} \simeq \left| \frac{T_0 \lambda \varkappa m}{\pi w_0 h} \frac{\omega^2_{mech}}{2 Q_M \sin \omega_M \tau} \right|. \tag{6}$$

For example, with $m = 10$ g, $\omega_M = 6$ rad/sec, $h = 1$ cm, $\lambda = 6 \cdot 10^{-5}$ cm, $w_0 = 3000$ mW, $Q_{mech} = 10^5$, $T_0 = 300°$K, and $\varkappa = 5 \cdot 10^{+4}$ ergs/sec·cm·deg (for glass), expression (6) yields $p_{cr} \sim 10^{-10}$ torr (for $\sin \omega_M \tau \simeq 1$); i.e., the pressure requirement may be extremely stringent if the frequency of the mechanical oscillations is quite low.

In conclusion the authors thank M. N. Kuklachev for assistance in the experiments.

REFERENCES

1. V. B. Braginskii, Vestn. Mosk. un-ta, fiz., astron., no. 2, 65, 1965.
2. V. B. Braginskii, and A. B. Manukin, ZhETF, 52, 1967.

1 June 1970

Department of Physics of Oscillations