A UNIVERSAL METHOD FOR THE SYMBOLIC REPRESENTATION OF CASCADE CONNECTED TWO-PORT NETWORKS

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Standard matrix methods were shown in [1] to be unsuitable for the symbolic representation of cascade connected two-port networks (quadrupoles) when at least one of the branches has an infinite impedance. In this paper, we give a further development of the previously proposed representation of cascade connected two-port networks, and show that the method is valid for any values of the branch impedances, both finite and infinite.

1. A two-port network is completely defined by specifying the impedances Z_{10}, $Z_{11'}$, $Z_{10'}$, $Z_{01'}$, $Z_{00'}$, $Z_{1'0'}$ of the branches between its terminals, i.e., the input terminals 1 and 0 and the output terminals 1' and 0'. The row of impedances (Z_{10}, $Z_{11'}$, $Z_{10'}$, $Z_{01'}$, $Z_{00'}$, $Z_{1'0'}$) can therefore be regarded as the symbol for the two-port network (Fig. 1a). By specifying the numerical values of the impedances in this row, we obtain the symbol for the particular two-port network. Thus, the symbol for

$$Z_{10} = Z_{10'} = Z_{01'} = Z_{1'0'} = \infty\ \Omega,\ Z_{11'} = Z_1 \text{ and } Z_{00'} = Z_0$$

is (∞, Z_1, ∞, ∞, Z_0, ∞), and the symbol for

$$Z_{10} = Z_{11'} = Z_{00'} = Z_{1'0'} = \infty\ \Omega,\ Z_{10'} = Z_1 \text{ and } Z_{01'} = Z_0$$

is (∞, ∞, Z_1, Z_0, ∞, ∞). In the symbolic representation of networks, we shall always write ∞ instead of $\infty\ \Omega$, i.e., we shall not name the unit of impedance.

The above two symbols represent the two-port networks illustrated in Figs. 1b, c, respectively. However, as before [1], we shall represent these networks by the simpler symbols (Z_1, Z_0) and $\langle Z_1, Z_0\rangle$ defined by

$$(Z_1, Z_0) \underset{D}{=} (\infty, Z_1, \infty, \infty, Z_0, \infty),$$
$$\langle Z_1, Z_0\rangle \underset{D}{=} (\infty, \infty, Z_1, Z_0, \infty, \infty), \qquad \text{(D1)}$$

where the symbol $\underset{D}{=}$ means "equal by definition."

The definition of (Z_1, Z_0) and $\langle Z_1, Z_0\rangle$ as given by (D1) has the advantage as compared with previous definitions that there is no need to introduce the equations $(Z_1, Z_0) = (Z_3, Z_4)$ and $\langle Z_1, Z_0\rangle = \langle Z_3, Z_4\rangle$, as in [1], since now these equations are merely the consequences of the definition of equal rows, a concept that is very general and common in modern mathematics. Moreover, the equation

$$\langle\infty, \infty\rangle = (\infty, \infty) \qquad (1)$$

is now merely a consequence of the definition given by (D1) and not an equation in the sense discussed in [1].

It is clear from (D1) that $\langle Z_1, Z_2\rangle \neq (Z_1, Z_2)$ always except when $Z_1 = Z_2 = \infty\ \Omega$.

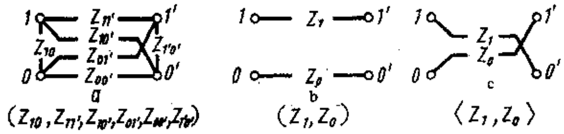

Fig. 1

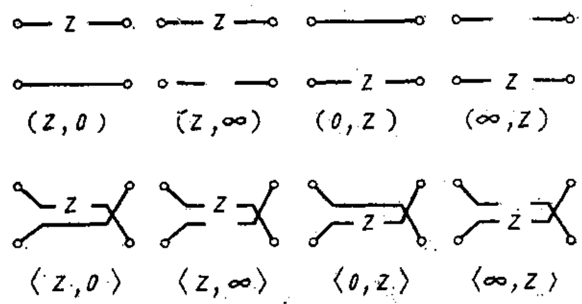

Fig. 2

The two-port networks illustrated in Fig. 1b,c will be referred to as direct and cross two-port networks, respectively, and the symbols (Z_1, Z_0) and $\langle Z_1, Z_0 \rangle$ will be referred to as the impedance symbols. Networks represented by the symbols (Z_1, Z_0) and $\langle Z_1, Z_0 \rangle$ will be called simply networks (Z_1, Z_0) and $\langle Z_1, Z_0 \rangle$, respectively. In general, whenever we discuss any of such circuits, a two-port network will be represented by the symbol $|Z_1, Z_0|$, which means either (Z_1, Z_0) or $\langle Z_1, Z_0 \rangle$.

2. A one-port network of impedance 0 or $\infty\Omega$ is called degenerate [1] and is represented by the lower case letter z. A network $|Z_1, Z_0|$ in which one of the branches is a degenerate one-port network will be called partially degenerate two-port network. There are, obviously, eight different types of partially degenerate networks:

$$(Z, 0), (Z, \infty), (0, Z), (\infty, Z), \langle Z, \infty \rangle, \langle Z, 0 \rangle, \langle \infty, Z \rangle, \langle 0, Z \rangle,$$

where Z is any one-port network (Fig. 2).

The network $|z_1, z_0|$ in which both branches are degenerate one-port networks z_1 and z_0 will be called a degenerate two-port network. In view of (1), there are only seven different types of degenerate network $|z_1, z_0|$:

$$(0, 0), (\infty, 0)\ (0, \infty), (\infty, \infty) = \langle \infty, \infty \rangle, \langle \infty, 0 \rangle, \langle 0, \infty \rangle, \langle 0, 0 \rangle$$

(Fig. 3). The symbols (∞, ∞) and $\langle \infty, \infty \rangle$ represent the same network, i.e., the empty two-port network [1], and we shall always represent them by the letter ω. Similarly, the symbols $(0, 0)$ and $\langle 0, 0 \rangle$ will henceforth be replaced by the more compact symbols 0 and $0^{\times}$.

Each variable assuming m $(m \geq 2)$ of the seven values

$$0, \langle \infty, 0 \rangle, (0, \infty), \omega, \langle \infty, 0 \rangle, \langle 0, \infty \rangle, 0^{\times},$$

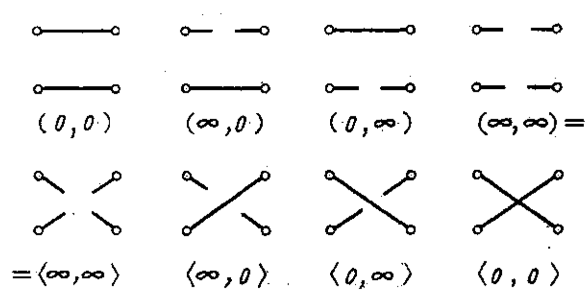

Fig. 3

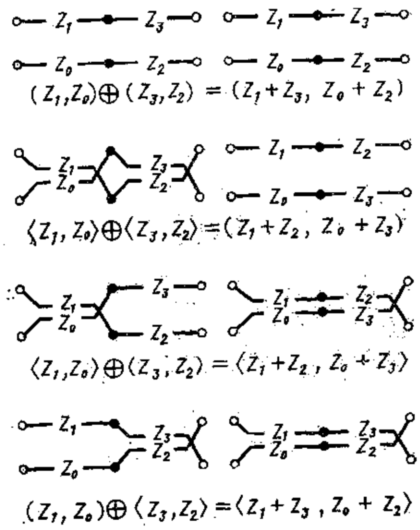

Fig. 4

can be looked upon as a symbol for an m-position two-port switch. For example, the variables k and τ with ranges $\{0, 0^\times\}$ and $\{0, \omega\}$ can be regarded, respectively, as symbols for a two-position commutator and a tumbler switch (or a two-port switch). In this interpretation of the variables k and τ, the equations $\kappa = 0$ and $\kappa = 0^\times$ signify that the signs of the voltages at the input and output of the commutator are, respectively, the same and opposite, whereas $\tau = 0$ and $\tau = \omega$ indicate, respectively, that the tumbler switch τ is closed and open.

The best known example of a three-position two-port switch is the three-position commutator. It can be represented by the variable k with the range of values $\{0, 0^\times, \omega\}$. We then find that $k = \omega$ indicates that the commutator k is in the neutral position in which it disconnects the two-port network of which it is a part.

The symbols $|z, Z_0|$ and $|Z_1, z|$, where z is the degenerate variable, i.e., variable with range of values $\{0, \infty\}$, can be used to denote partially degenerate two-port networks, one branch of which contains a circuit interrupting device such as an ordinary off-switch, a telegraphic key, a relay contact, or in general any one-port network consisting of relay contacts or one-port switches.

It is important to note that an impedance which is not in general degenerate may turn out to be degenerate at certain frequencies of the voltage applied to a one-port circuit having this impedance. For example, $Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = \infty$ Ω for $\omega = 0$ rad/sec, although in general this impedance is not degenerate. Similarly, the impedance $Z = (j\omega C + 1/j\omega L)^{-1}$ of a one-port circuit formed by a solenoid of inductance L connected in parallel with the capacitor of capacitance C has an infinite impedance when $\omega = \omega_0 = 1/\sqrt{LC}$.

The degeneracy of the impedance that sets in only at certain frequencies of the voltage applied to the one-port network will be called the frequency degeneracy of the impedance in contrast to the previously considered degeneracy associated with the particular values of R, C, and L which we shall refer to as parametric degeneracy. In general, a two-port network can be partially degenerate due to both the parametric and the frequency degeneracy of one of its branches. However, whatever the reason for the degeneracy of the branches of a two-port network, the network itself can always be represented by the symbol $|Z_1, Z_0|$, i.e., by one of the two impedance symbols (Z_1, Z_0) or $\langle Z_1, Z_0\rangle$. Matrix representations of circuits which we represent by the symbols (Z, ∞), (∞, Z), $\langle\infty, Z\rangle$, and $\langle Z, \infty\rangle$, was shown in [1] to be impossible.

3. The cascade connection of the networks $|Z_1, Z_0|$ and $|Z_3, Z_2|$ will be represented by the symbol $|Z_1, Z_0| \oplus |Z_3, Z_2|$, defined by

$$(Z_1, Z_0) \oplus (Z_3, Z_2) \underset{D}{=} (Z_1 + Z_3,\ Z_0 + Z_2), \tag{D2a}$$

$$\langle Z_1, Z_0\rangle \oplus \langle Z_3, Z_2\rangle \underset{D}{=} (Z_1 + Z_2,\ Z_0 + Z_3), \tag{D2b}$$

$$\langle Z_1, Z_0\rangle \oplus (Z_3, Z_2) \underset{D}{=} \langle Z_1 + Z_2,\ Z_0 + Z_3\rangle, \tag{D2c}$$

$$(Z_1, Z_0) \oplus \langle Z_3, Z_2\rangle \underset{D}{=} \langle Z_1 + Z_3,\ Z_0 + Z_2\rangle \tag{D2d}$$

(see Fig. 4). These formulas differ from those given in [1] by equations (15), (18), (19), and (20) by the fact that instead of the sign +, which was used to denote the cascade connection of two-port networks we now use the special symbol «⊕».

The use of this special symbol enables us to replace the phrase "cascade connection of two-port networks" by the shorter expression "operation $\oplus$."

To abbreviate our text still further, we shall agree to replace the symbols $|Z_1, Z_0|$, $|Z_3, Z_2|$, ..., $|Z_{2i+1}, Z_{2i}|$ by the letter ζ with the subscripts defined by

$$\zeta_i \underset{D}{=} |Z_{2i+1}, Z_{2i}| \tag{D3}$$

for any i = 0, 1, ... In other words, ζ_i represents a variable that can assume only two values, namely (Z_{2i+1}, Z_{2i}) or $\langle Z_{2i+1}, Z_{2i}\rangle$. Subscript zero will frequently be omitted, i.e., instead of ζ_0 we shall frequently use simply the letter ζ without a subscript. Since the variables $\zeta, \zeta_1, \zeta_2, \ldots$ are used here to represent only two-port networks, they will be referred to simply as the networks $\zeta, \zeta_1, \zeta_2, \ldots$

It is clear from (D2) that the operation $\oplus$, is not commutative, i.e., in general $\zeta_1 \oplus \zeta_2 \neq \zeta_2 \oplus \zeta_1$. It is, however, associative, i.e.,

$$(\zeta \oplus \zeta_1) \oplus \zeta_2 = \zeta \oplus (\zeta_1 \oplus \zeta_2) \tag{2}$$

is valid for any values of ζ, ζ_1, and ζ_2.

Since this equation is valid for $\zeta = (Z_1, Z_0)$, $\zeta_1 = (Z_3, Z_2)$, $\zeta_2 = (Z_5, Z_4)$, it follows from (D2a) that addition is associative. Let us verify that (2) is valid for $\zeta = \langle Z_1, Z_0\rangle$, $\zeta_1 = \langle Z_3, Z_2\rangle$, $\zeta_2 = \langle Z_5, Z_4\rangle$.

In fact, using (D2b), (D2d), the associative law of addition, (D2c) and (D2b) in succession, we obtain the following chain of equations

$$\begin{aligned} (\langle Z_1, Z_0\rangle \oplus \langle Z_3, Z_2\rangle) \oplus \langle Z_5, Z_4\rangle &= (Z_1 + Z_2,\ Z_0 + Z_3) \oplus \langle Z_5, Z_4\rangle = \\ = \langle (Z_1, Z_2) + Z_5,\ (Z_0 + Z_3) + Z_4\rangle &= \langle Z_1 + (Z_2 + Z_5),\ Z_0 + (Z_3 + Z_4)\rangle = \\ = \langle Z_1, Z_0\rangle \oplus (Z_3 + Z_4,\ Z_2 + Z_5) &= \langle Z_1, Z_0\rangle \oplus (\langle Z_3, Z_2\rangle \oplus \langle Z_5, Z_4\rangle), \end{aligned}$$

from which (2) follows for the above values of ζ, ζ_1, and ζ_2. Similarly, we can verify that (2) is a consequence of (D2) and the associative property of addition for other values of the variables ζ, ζ_1, and ζ_2.

Thus, the set of all networks ζ is a noncommutative semigroup under the operation $\oplus$ The neutral element ("zero") of this semigroup is the direct connector 0 since

$$\zeta \oplus 0 = \zeta = 0 \oplus \zeta \tag{3}$$

are valid for any network ζ.

This set is not a group because it is not always possible to associate with a network ζ an opposite network, i.e., the network $-\zeta$, satisfying

$$-\zeta \oplus \zeta = 0 = \zeta \oplus -\zeta. \tag{4}$$

In particular, the network $-\omega$ that is opposite to the empty network ω does not exist. This is clear from

$$\omega \oplus \zeta = \omega = \zeta \oplus \omega, \tag{5}$$

and it follows from (D2) and from the definition of ω that (5) is valid for any network ζ.

By virtue of the associative law (2), the cascade connection of networks ζ_0, ζ_1, ζ_2, ..., ζ_n can be represented by the expression $\zeta_0 \oplus \zeta_1 \oplus \zeta_2 \oplus \ldots \oplus \zeta_n$ which does not contain parentheses. This expression will be called the impedance symbol of the cascade network in which the i-th cascade is the network ζ_{i-1}, i = 1, 2, ... Since the operation $\oplus$ is not commutative, we cannot write down a general expression for the terms involved in this operation because an interchange of any two terms results in the symbol for another cascade connection which is not equivalent to the original connection.

The impedance symbol $\zeta_0 \oplus \zeta_1 \oplus \zeta_2 \oplus \cdots \oplus \zeta_n$, can be used to represent the cascade connection of any networks ζ_0, ζ_1, ζ_2, ..., ζ_n but the cascade connection of networks ζ_0, ζ_1, ζ_2, ..., ζ_n cannot be represented by the product $A_{\zeta_0} \cdot A_{\zeta_1} \cdot A_{\zeta_2} \cdot \ldots \cdot A_{\zeta_n}$ of its principal matrices A_{ζ_0}, A_{ζ_1}, $A_{\zeta_2}, \ldots$, A_{ζ_n}, whenever at least one of these networks is described by an impedance symbol of the form $|\infty, Z|$ or $|Z, \infty|$, where Z is any impedance. The impedance symbol $\zeta_0 \oplus \zeta_1 \oplus \zeta_2 \oplus \ldots \oplus \zeta_n$, is thus a universal representation of the cascade connection of two-port networks ζ_0, ζ_1, ζ_2, ..., ζ_n, whereas the matrix representation $A_{\zeta_0} \cdot A_{\zeta_1} \cdot A_{\zeta_2} \cdot \ldots \cdot A_{\zeta_n}$ does not have this property.

4. The operation of crossing of the output leads of the network ζ will be called cross-inversion of this network and will be indicated by the symbol $\zeta^{\times}$. It is clear from Fig. 5a that this operation can be represented by

$$\zeta^{\times} \underset{D}{=} \zeta \oplus 0^{\times}. \qquad \text{(D4)}$$

Substituting for ζ in this expression, we obtain

$$(Z_1, Z_0)^{\times} = \langle Z_1, Z_0 \rangle, \qquad \text{(6a)}$$

$$\langle Z_1, Z_0 \rangle^{\times} = (Z_1, Z_0) \qquad \text{(6b)}$$

(Fig. 5b,c) which shows that cross-inversion transforms each direct network into a cross network and vice versa. Only the empty network ω is unaffected by this:

$$\omega^{\times} = \omega, \qquad (7)$$

which is an obvious consequence of (1).

It is clear from (6) and (7) that cross-inversion $\zeta^{\times}$ is an involute transformation of network ζ:

$$(\zeta^{\times})^{\times} = \zeta. \qquad (8)$$

Since the operation $\oplus$ is associative, we have

$$\zeta \oplus \zeta_1^{\times} = (\zeta \oplus \zeta_1)^{\times}. \qquad (9)$$

In fact,

$$\zeta \oplus \zeta_1^{\times} = \zeta \oplus (\zeta_1 \oplus 0^{\times}) = (\zeta \oplus \zeta_1) \oplus 0^{\times} = (\zeta \oplus \zeta_1)^{\times}.$$

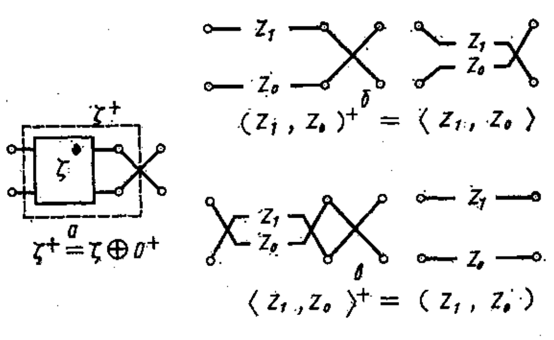

Fig. 5

Equation (9) can readily be generalized to any number of stages:

$$(\zeta_0 \oplus \zeta_1 \oplus \zeta_2 \oplus \dots \oplus \zeta_n)^{\times} = \zeta_0 \oplus \zeta_1 \oplus \zeta_2 \oplus \dots \oplus \zeta_n. \tag{10}$$

The physical meaning of this equation is quite clear: crossing of output leads of any cascade network is equivalent to the crossing of the output leads of the last cascade.

We note that the crossing of the input leads of a network ζ is not in general the cross-inversion $\zeta^{\times}$. In fact, it follows from (D2) that

$$0^{\times} \oplus (Z_1, Z_0) = \langle Z_0, Z_1 \rangle, \tag{11a}$$

$$0^{\times} \oplus \langle Z_1, Z_0 \rangle = (Z_0, Z_1), \tag{11b}$$

and, if we use (6), we can combined these into the single equation

$$0^{\times} \oplus | Z_1, Z_0 | = | Z_0, Z_1 |^{\times}. \tag{11}$$

It is clear from this that the crossing of the input leads of a network produces an interchange of the branches of the network and at the same time its cross-inversion.

Hence it follows in particular that in general $\kappa \oplus \zeta \neq \zeta \oplus \kappa$, i.e., it is not immaterial whether the commutator κ is connected to the input or the output of the network. In fact, in positions 0, the commutator κ changes only the sign of the voltage at the output of the network $\zeta \oplus \kappa$, whereas in the network $\kappa \oplus \zeta$ it will, in addition, interchange the branches of the network ζ.

If we apply in succession the formulas given by (D4), (9), (11), and (8), we obtain the following chain of equations:

$$0^{\times} \oplus | Z_1, Z_0 | \oplus 0^{\times} = (0^{\times} \oplus | Z_1, Z_0 |)^{\times} = 0^{\times} \oplus | Z_1, Z_0 |^{\times} = | Z_0, Z_1 |, \tag{12}$$

from which it follows that

$$0^{\times} \oplus | Z_1, Z_0 | \oplus 0^{\times} = | Z_0, Z_1 |, \tag{13}$$

and this has a simple physical interpretation.

Table 1

1

P	$\sim P$
F	T
T	F
U	U

1′

k	$k^{\times}$
0	$0^{\times}$
$0^{\times}$	0
ω	ω

Table 2

2

$P\ Q$	$\sim(P \supset \subset Q)$
$F\ F$	F
$T\ F$	T
$F\ T$	T
$T\ T$	F
$F\ U$	U
$T\ U$	U
$U\ F$	U
$U\ T$	U
$U\ U$	U

2′

$k_1 k_2$	$k_1 \oplus k_2$
$0\ 0$	0
$0^{\times}\ 0$	$0^{\times}$
$0\ 0^{\times}$	$0^{\times}$
$0^{\times}\ 0^{\times}$	0
$0\ \omega$	ω
$0^{\times}\ \omega$	ω
$\omega\ 0$	ω
$\omega\ 0^{\times}$	ω
$\omega\ \omega$	ω

5. A network ζ whose branch impedance are finite will be called a finite network. It will be even or odd, depending on whether it contains an even or an odd number of crossings $0^{\times}$.

From (D4), (3), and (8) we have

$$0^{\times} \oplus 0^{\times} = 0. \tag{14}$$

If we now use (13) either on its own or in conjunction with (14) we can replaced any even network ζ by an equivalent direct network, i.e., a network that does not contain a single crossing. Hence it follows that any odd network ζ is equivalent to a network containing only one crossing. It then follows from (11) and (12) that any odd network ζ can be transformed to an equivalent network that is the cross-inversion of a direct network.

The set of all finite two-port networks containing no breaks can therefore be divided into two classes, namely, even and odd networks, similarly to the way that the set of all integers can be divided into the two classes of even and odd numbers. The simplest examples of even and odd networks ζ are, respectively, the direct connector 0 and the crossed connector 0^X. These are the analogs of zero and 1 as representatives of even and odd numbers, respectively.

The equations

$$0 \oplus 0 = 0 = 0^\times \oplus 0^\times,\ 0 \oplus 0^\times = 0^\times = 0^\times \oplus 0 \quad (15)$$

are analogous to the congruences modulo 2

$$0+0 \equiv 0 \equiv 1+1,\ \ 0+1 \equiv 1 \equiv 1+0 \quad (16)$$

or the equations

$$0 \oplus 0 = 0 = 1 \oplus 1,\ \ 0 \oplus 1 = 1 = 1 \oplus 0 \quad (17)$$

for the numbers 0 and 1.

It is clear from (15) and (17) that the cascade connection of the networks 0 and $0^\times$ is an isomorphism of the cyclic composition of the numbers 0 and 1 and, consequently, the cascade connection $\kappa \oplus \kappa_1$ of two-position commutators κ and κ_1 will simulate the operation of this cyclic composition $m \oplus n$ of binary numbers m and n, i.e., the numbers $m, n \in \{0, 1\}$.

If the values 0 and 1 are in one-to-one correspondence with F (false) and T (true) of propositions p and q of classical two-point logic, then as shown in [2], the cyclic composition $m \oplus n$ of numbers m and n is an isomorphism of the operation $p \not\equiv q$, the negation of the equivalence $p \equiv q$ of propositions p and q, and the operation $n \oplus 1$ is an isomorphism of the negation ~p of proposition p. Consequently, if the networks 0 and $0^\times$ are placed in one-to-one correspondence with the numbers 0 and 1, then the operations $p \not\equiv q$ and ~p in classical propositional calculus can be simulated, respectively, by the cascade connections $\kappa \oplus \kappa_1^\times$ and $\kappa \oplus \kappa^\times$ of the two-position commutators κ and κ_1. In view of (D4), the negation operation ~p is simulated by the commutator k^X, which is a cross-inversion of commutator κ, i.e., a commutator that operates in antiphase with $k^\times$, which simulates proposition p.

We note that, since the commutator κ is a symmetric two-port network, and for any symmetric network $\zeta = |Z, Z|$ we have

$$0^\times \oplus \zeta = \zeta^\times, \quad (18)$$

the commutator $k^\times$ that is the cross-inversion of k, can be obtained by crossing its inputs or outputs.

The equivalence $p \equiv q$ of propositions p and q can be simulated by the cross-inversion $(k \oplus k_1)^\times$ of the network $k \oplus k_1$, i.e., either $k \oplus k_1 \oplus 0^\times$, or $0^\times \oplus k \oplus k_1$.

When the correspondence between 0 and 1 on the one hand and the F and T of propositions p and q is opposite to that adopted in [2], then propositions p, ~p, $p \equiv q$, and $p \not\equiv q$ can be simulated by the commutators $k^\times, k, k \oplus k_1$ and $(k \oplus k_1)^\times$, respectively.

The cascade connection of two-position commutators can therefore be used either to perform certain arithmetic operations in the binary system, or to perform certain logical operations in ordinary (two-point) propositional calculus.

6. The cascade connection of three-position commutators can be used, in precisely the same way, to perform certain logical operations in three-value logic and the analogous arithmetic operations. In fact, if we compare Table 1, which defines the negation ~P of a proposition P, with Table 1', which gives the corresponding information on the commutator $k^\times$, we see that the commutator $k^\times$ simulates negation ~P of proposition P if F (false), T (true), and U (uncertain or empty) relating to proposition P are taken to correspond to the positions 0, $0^\times$, and ω of the commutator k. It is only when this correspondence prevails between the status of propositions P and Q on the one hand and the positions of the three-position commutators k_1 and k_2 on the other hand, that the cascade connection $k_1 \oplus k_2$ will simulate the operation $\sim(P \supset\subset Q)$ involving the negation of weak or internal equivalents $P \supset\subset Q$ [3] of propositions P and Q (this will be obvious from a comparison of

Tables 2 and 2'). On the other hand, the operation $P \supset\subset Q$ can obviously be simulated by cross-inversion $(k_1 \oplus k_2)^{\times}$ of the cascade connection $k_1 \oplus k_2$ of the three-position commutators k_1 and k_2, i.e., simply by the crossing of the output leads of $k_1 \oplus k_2$.

When the correspondence between the status of propositions P and Q and the positions of the three-position commutators k_1 and k_2 is opposite to that adopted above, i.e., when F, T, and U of P and Q correspond to positions $0^{\times}$, 0 and ω of the commutators k_1 and k_2, then the circuit $k_1 \oplus k_2$ simulates the weak equivalents $P \supset\subset Q$, and the circuit $(k_1 \oplus k_2)^{\times}$ simulates the negation $\sim(P \supset\subset Q)$ of the operation $P \supset\subset Q$. When Q = P, the situation reduces to one of the basic laws of traditional logic, namely, the law of identity $P \supset\subset P$, or, more precisely, its generalization to the two-valued calculus of D. A. Bochvar. The model of $P \supset\subset P$ is the cascade connection $k \oplus k$ of two identical three-position commutators. From (3), (5), and (14), we then have

$$k \oplus k = \tau, \tag{19}$$

which shows that the operation $P \supset\subset P$ is also simulated by the tumbler switch τ. Hence it follows directly that

$$(k \oplus k)^{\times} = \tau^{\times}, \tag{20}$$

and this shows that negation $\sim(P \supset\subset P)$ of $P \supset\subset P$ can be simulated by the commutator network $(k \oplus k)^{\times}$ or, more simply, by the tumbler switch $\tau^{\times}$.

It is important to note that, since there are no definite principal matrices for three-position commutators and tumbler switches, the conventional matrix approach is insufficient for the symbolic representation of cascade-connected networks simulating the operations ~P and $P \supset\subset Q$ in the three-valued propositional calculus. The symbolic method described above, on the other hand, is valid when the variables P and Q are the propositions of both two-valued and three-valued logic.

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