DYNAMIC DAMPING DURING MEASUREMENTS OF SMALL FORCES

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Measurements of small forces by radiophysical methods based on the response of a mechanical oscillator are discussed. It is shown that a quasi-ideal transducer can be developed by using the dynamic damping of the transducer noise at a partial oscillator frequency. A calculation is given for an oscillator with a parametric capacitive transducer.

In experiments with test bodies [1], the external effect is detected through the response of a mechanical oscillator. The data is usually coupled to a transducer which transforms mechanical effects into an electrical signal. The most frequently employed are the piezoelectric and magnetostrictive effects, parametric capacitive and inductive transducers, etc. [2]. Experimental techniques have now been perfected so that a very low level of intrinsic Brownian noise in the test oscillator can be achieved by cooling down to $T_\mu \sim 10^{-1}$°K and by ensuring that the mechanical quality factor is high, e.g., $Q_\mu \sim 10^{10}$ [2]. Under these conditions, the sensitivity of the system consisting of the mechanical oscillator and the transducer is determined by electrical fluctuations in the transducer. The radical solution of the problem of perfect transformation with low intrinsic noise will probably be based on the principle of nondestructive quantum-mechanical measurements [3].

In this paper, we consider a quasi-ideal transducer based on the general properties of a coupled oscillatory system with two degrees of freedom. The coupling between the mechanical degree of freedom (test oscillator) and the electrical degree of freedom (transducer) is usually chosen on the basis of the matching condition, i.e., the condition that the maximum amount of mechanical energy should be transformed into electrical energy. The matching coupling λ_{opt} ensures maximum response at the output of the transducer and a fixed signal-to-noise ratio. This ratio can be increased by increasing the coupling. However, this is achieved at the expense of the absolute signal size: the signal amplitude falls by the same factor by which the sensitivity is increased. However, the development of low-noise parametric amplifiers with noise temperature 10^{-5}°K or less [4] means that a subsequent amplification of the weakened signal may be possible.

The system which we shall consider is illustrated in Fig. 1. The quantities X_1, M, K_1 refer to the test oscillator and X_2, m, K_2 to the transducer which, for the sake of simplicity, is represented by an equivalent mechanical system. The external effect is represented by the force F_s acting on the coordinate X_1; the observed response of the transducer is represented by the coordinate X_2. The fluctuation force is due to the transducer and acts on the coordinate X_2. It has a white spectrum, at least within the signal frequency band. Figure 1b shows two resonance curves: the amplitude of X_2 as a function of the frequency of the force applied to X_2 (solid curve) and the amplitude of X_2 as a function of the frequency of the force applied to X_1 (broken curve). The response to the force applied to X_2 is 0 at the partial frequency of the test oscillator $n_1^2 = K_1M^{-1}$ (damping is assumed small and $n_2^2 = (K_1 + K_2)/m >> n_1^2$). This is the well-known dynamic damping effect [5]: the effect of the force on X_2 is compensated by the reaction due to the degree of freedom X_1. At the same time, the X_2 response at this frequency is finite if the perturbing force is applied to the test oscillator X_1. However, the force acting on X_1 is the signal force and that acting on X_2 is the fluctuation force. It is

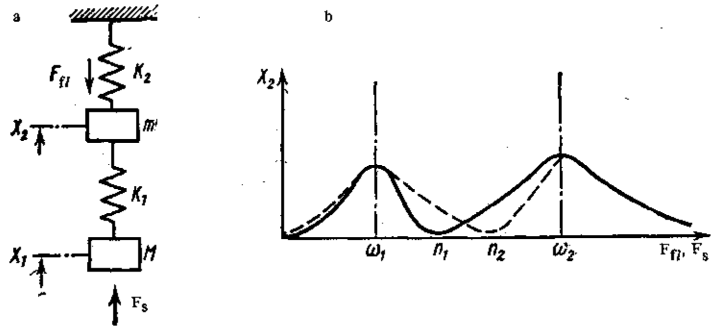

Fig. 1

therefore clear that when the signal force F_s has a spectrum occupying a narrow frequency band near the partial frequency n_1, the sensitivity is much greater than in the region of the maximum on the signal response curve. Figure 1b corresponds to a coupling coefficient $K_1(K_1 + K_2)^{-1}$ which is greater than the matching coupling. Further increase in the coupling leads to a still greater separation of the two humps, a reduction in the noise level within the band $\hat{\tau}^{-1}$, and an increase in the sensitivity with further reduction in the signal response. For a force F_s with a very narrow spectrum near n_1, the sensitivity can, at least theoretically, be as high as desired.

The possible substantial increase in the coupling coefficient is typical of parametric transducers, for example, the capacitive resonant transducer for which the coupling is proportional to the square of the excitation amplitude. In this case there is no simple analogy with the above example because of the considerable difference between the frequencies of the test oscillator (ω_μ) and that of the electrical circuit of the transducer (ω_e): $\omega_\mu \ll \omega_e$. However, it has been shown [6] that the signal η_{sd} produced after synchronous detection of the transducer output with a reference signal having the phase of the excitation in the circuit is the analog of X_2 in the preceding example. The signal-to-noise ratio at the output of such a synchronous detector will increase with increasing excitation voltage by a factor of (V/V_{opt}) where V_{opt} is the matching voltage. The signal would be reduced by an equal factor.

We now give a concrete example of a calculation of the signal-to-noise ratio for an oscillator with a capacitive transducer. In this system, the oscillations of the test mass M are used to vary the capacitance in an external electric circuit, and this modulates the voltage V applied to the circuit from an external source at a high frequency ω_{ex}. The output voltage of the transducer has the form $\eta(t) = (C_0 + a)\cos \omega_{ex}t + b \sin \omega_{ex}t$, $|a|$, $|b| \ll C_0$, where the slowly varying functions a, b are given by

$$\begin{aligned}&(p^2 + \omega_\mu^2)\xi - \omega_\mu^2 \lambda C_0 a \approx \omega_\mu^2 f_e(\tau) - \\ &- \left(p + \frac{\omega_e}{2Q_e}\right) a + \Delta b \approx \left(\frac{\omega_e}{2}\right) U_s,\end{aligned} \tag{1}$$

$$\left(\frac{\omega_e}{2}\right) C_0\xi + \Delta a + \left(p + \frac{\omega_e}{2Q_e}\right) b \approx \left(\frac{\omega_e}{2}\right) U_c.$$

In these expressions, we used the notation of [6]: $C_0 = V_{ex}/V_0$; V_0 is a normalizing constant; $\xi = X/d$, X is the displacement of the mechanical oscillator; d is the initial gap in the capacitance C of the circuit; $f_e = F_s/m\omega_\mu^2 d$ is the external signal force; $U_c U_s$ are white, independent, noise distributions with spectral intensity $N_e = 8\varkappa T_e/Q_e\omega_e C$; ω_μ, ω_e are the resonance frequencies of the oscillator and of the circuit, respectively; and $p = d/dt$. In contrast to [6], the above equations are written in simplified form, corresponding to a known "phase" $C(t) = C_0 \cos \omega_{ex}t$ of the excitation, i. e., $\varphi_e = 0$

(see [6]). The signal $F_s(t)$ is assumed to be a short train of length τ and carrier frequency $\omega_s \sim \omega_\mu$, $|\omega_s - \omega_\mu| = \chi << \omega_\mu$. We now confine our attention to the solution of (1) for the following two special cases that are of interest in practice: $\omega_\mu << (\omega_e/2Q_e)$ and $\Delta = \omega_\mu >> (\omega_e/2Q_e)$.

1. The slow component which contains information about the signal can be isolated from $\eta(t)$ in the following three ways: a) by compensating the excitation $C(t)$ and then using linear detection at the output, we obtain the variable $\eta_0 = (a^2 + b^2)^{1/2}$; b) by detecting without compensation of the excitation, we obtain

$$\eta_d = \{(C_0 + a)^2 + b^2\}^{1/2} \approx C_0 + a + b^2/2C_0;$$

and c) by using synchronous detection with reference signal $C_0 \mathrm{const}\,\omega_{ext} t$, we obtain the output $\eta_{sd} = C_0/2(C_0 + a)$. When $\chi = 0$, the signal-to-noise ratio after the matched filter for the variable $C_0 = [C_0]_{opt}$ reaches a maximum for some optimum value η_0. The variable η_{sd} is characterized by a continuous increase in μ with increasing C_0, and the physical reason for this is analogous to the dynamic damping of noise mentioned above.

Returning now to the variable η_d, we note that the signal is largely concentrated in $a(t)$ because $|a_s| \sim |b_s| >> |b_s/C_0||b_s|$. To estimate the fluctuation intensity, we confine our attention to the condition $V_k > V_{opt}(C_0 > [C_0])_{opt}$ for which the determinant of (1) within the band $(\omega_\mu \pm \hat{\tau}^{-1})$ is approximately given by Det $(j\omega) \approx -(\lambda/2)\omega_e\omega_\mu^2 C_0^2$ = const (see [6]). This can be used to determine quite readily the energy spectrum of the noise at the output of the amplitude detector because the noise components a_{fl} and b_{fl}^2 are uncorrelated. It turns out that the signal-to-noise ratio for the variable η_d when $V_k > V_{opt}$ is given by

$$\mu_d \approx 4\,\frac{(f_0\hat{\tau})^2}{\lambda^2 N_e}\left\{\pi^{-1}\left(\frac{Q_e}{4}\right)(N_e\hat{\tau}^{-1}) + 4\,(1/\omega_{ex}\tau_m)^2\,\frac{2}{(\lambda C_0)^2}\right\}^{-1}.$$

Analysis of this formula shows that there is a characteristic plateau which occurs when

$$(V_k)_{pl} \approx V_{opt}(\hat{\tau}/_m)\left\{\frac{m\omega_1^2 d^2}{\varkappa T_e}\,Q_e^{-1}\,\frac{\Delta}{\omega\mu}\right\}^{1/2}.$$

Hence, when $\tau_m = \hat{\tau}$, we have

$$(V_k)_m \approx \left\{\frac{m\omega_\mu^2 d^2}{\varkappa T_e Q_e}\,\frac{\Delta}{\omega_1}\right\}^{1/2} V_{opt}.$$

For typical values of the parameter, we find that $(V_k)_{pl}/V_{opt} \gg 1$. Fig. 2 presents the results of the numerical calculation of sensitivity for these three cases.

2. Optimum filtration $\eta(t)$ with the aid of a synchronous detector followed by a matched filter will ensure the following result for the signal-to-noise ratio:

$$\mu(\chi) = C_0(f_0^2\hat{\tau}/N_e)^{1/2}\left|\frac{\sin\chi\hat{\tau}/2}{\chi\hat{\tau}/2}\right|. \qquad (2)$$

At the same time, the transformation coefficient is given by

$$k(\chi) = af_0^{-1} \approx 0{,}25\,C_0 Q_e\omega_\mu\hat{\tau}\left|\frac{\{\sin[\omega_{ex}(1-\varkappa) - \omega_e]\,\hat{\tau}/2}{[\omega_\mu(1-\varkappa) - \omega_e]\,\hat{\tau}/2}\right|; \qquad (3)$$

$$\varkappa = \lambda C_0^2 Q_e/4.$$

It is convenient to introduce the generalized filtration quality parameter $\theta = \mu(\chi)k(\chi)$. By maximizing this parameter we ensure optimum operation, combining good sensitivity with high transformation coefficient. There are two interesting cases: a) signal frequency equal to the partial frequency of the oscillator $\omega_s = \omega_\mu$ and b) signal

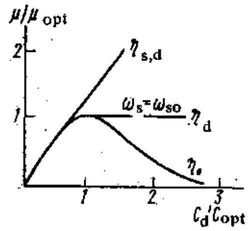

Fig. 2

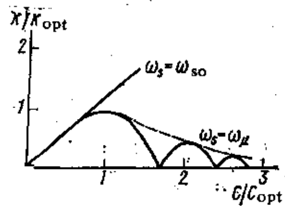

Fig. 3

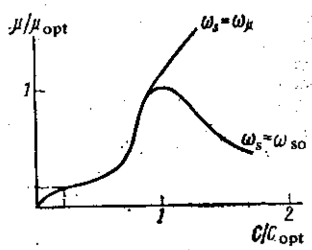

Fig. 4

frequency equal to the natural frequency ω_{s0} of the coupled system. In the first case, the signal-to-noise ratio increases without limit as the excitation increases and k decreases. In the second case, we have the opposite situation. In either case,

$$\theta \approx (f_0^2/N_e\, 1/\hat{\tau})^{1/2}\lambda^{-1} \cdot \sin\left[\frac{\lambda}{4}C_0^2 Q_e\omega_\mu\hat{\tau}\right] = \text{const}\sin\left[\frac{\lambda}{4}C_0^2 Q_e\omega_\mu\hat{\tau}\right]. \quad (4)$$

This parameter reaches a constant maximum value when the excitation is

$$[C_0^2]_{opt} \approx 2\pi^{-1}(\lambda Q_e\omega_\mu\hat{\tau})^{-1},$$

and this is equal to the optimum value of the excitation (see [1, 6]).

The dependence of the transformation coefficient on the size of the excitation is shown in Fig. 3. In practice, the matched filter for signals in the form of a short train can be replaced by a bandwidth of $2\hat{\tau}^{-1}$ around the carrier frequency ω_μ: for weak coupling $C_0 \ll [C_0]_{opt}$ and for strong coupling $C_0 \gg [C_0]_{opt}$, the band-pass filter is in practice little different from the optimum filter. Figure 4 shows the results of numerical calculations of the signal-to-noise ratio when the band-pass filter is employed.

3. The utilization of the damping method for achieving better sensitivity will, in practice, depend on the noise properties of the amplifier that follows the capacitive transducer. Equation (3) readily yields the expression for the signal power (in terms of the amplitude F_0 of the force) and the displacement $\Delta x = \frac{F_0\hat{\tau}}{2}(m\omega_\mu)^{-1}$):

$$P \approx 2\pi^{-1}\frac{(F_0\hat{\tau})^2}{m}\frac{\omega_e}{\omega_n}\left(\frac{V_{opt}}{V_k}\right)^2 \sim \frac{m(\Delta\omega_\mu)^2}{\hat{\tau}}\frac{\omega_e}{\omega_\mu}\left(\frac{V_{opt}}{V_k}\right)^2. \quad (5)$$

Comparison of (5) with the noise power $\varkappa T_{ex}(\Delta f)^{-1}$ enables us to formulate the conditions that have to be imposed on the noise temperature of the next stage. To demonstrate the possible increase in sensitivity due to the damping effect, let us consider the following example. Suppose the mechanical oscillator is such that $m \sim 10$ g, $\omega_\mu \sim 100$ and that the parameters of the transducer circuits are $\omega_e \sim 10^7$, $Q_e \sim 10^2$, $d \sim 10^{-2}$ cm, $C \approx 100$ pF. Such values are readily realized in practice. If the external force train has a duration of $\tau \sim 10$ sec (100 periods), we have $V_{opt} \simeq 3$V and we have a margin of two orders of magnitude before the breakdown voltage is reached.

When $V_k = V_{opt}$, we can detect a displacement

$$\Delta x = \left(\frac{\varkappa T}{m\omega_\mu\omega_e}\right)^{1/2} \sim 3\cdot 10^{-12} \text{ cm}.$$

From (5), we find that if we use the damping effect, the noise temperature of the amplifier might satisfy the condition $T_n \leq 10^{26}(\Delta x)^2(V_{opt}/V_k)^2$. Parametric amplifiers incorporating field effect transistors have noise temperatures of the order of a few degrees at the frequency of about 1 mHz [4]. An amplifier of this kind can therefore be used to record $(\Delta x)_{eff} \approx \Delta x$, $V_{opt}/V_k \sim 5\cdot 10^{-14}$ cm for $(V_{opt}/V_k) \sim 3\cdot 10^{-2}$. The limiting signal power is about 10^{-22} W. A sensitivity improvement of this kind can be achieved by increasing the excitation frequency ω_{ex}, but, because of the root dependence, the frequency would have to be increased by more than three orders of magnitude, i.e., $\omega_{ex} > 10^{10}$, and this is often technically less acceptable. Moreover, quantum-mechanical restrictions become significant as the excitation frequency increases [3].

4. The above method of reducing the transducer noise level through the damping effect is probably most effective in the case of a sufficiently long train when the spectral density of the signal is concentrated around the excitation frequency ω_{ex} and when the transducer noise is of nonthermal origin. We note that for our example of a parametric transducer, the damping effect should be seen in the suppression of the spectral intensity of the noise around the combination frequencies.

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