SPIN EFFECTS USING THE MOTION OF ELECTRONS IN UNDULATOR TUBES

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An analysis is given of the influence of quantum-mechanical properties of radiation on the spin states of particles moving in an arbitrary undulator tube. The resulting expression for the spin-flip transition probability as a function of the particle energy and the anomalous magnetic moment is examined.

One of the manifestations of the quantum-mechanical properties of radiation is the self-polarization of electrons and positrons resulting from prolonged motion in a magnetic field. This new mechanism of polarization, whose existence was predicted by Sokolov and Ternov [1], is due to the dependence of the transition probability on the orientation of the electron spin in the initial state. Even before the experimental demonstration of the effect [2], many authors noted the advantages of this polarization mechanism as compared with the then existing method [3]. In the present paper, we investigate the influence of quantum-mechanical properties of radiation on the spin states of electrons moving in undulator tubes. There has been increased interest in undulator tubes in view of the possibility of producing polarized electromagnetic radiation in a broad spectral band [4-5].

The matrix element for the emission of a photon of frequency ω in the direction $\mathbf{n}$ with polarization $\mathbf{e}$ by a charged Dirac particle with anomalous magnetic moment will be written in the form

$$M=\frac{e\sqrt{4\pi\hbar c^{3}}}{c\hbar\sqrt{2\omega}}\int e_{\mu}^{*}\,j^{\mu}e^{ikx}\,d^{4}x, \qquad (1)$$

$$j^{\mu}=\bar{\psi}\left(\gamma^{\mu}+\frac{\mu'}{e}\,\sigma^{\mu\nu}k_{\nu}\right)\psi.$$

where ψ is the solution of the Dirac equation

$$\left[\gamma^{\mu}\left(i\hbar\partial_{\mu}-\frac{e}{c}\,A_{\mu}\right)-mc\right]\psi=0. \qquad (2)$$

It is well known that the quantum-mechanical effects are due to the quantization of the motion itself and to the effect of radiation on the motion of the electron in external fields. We shall neglect the former effects. The quantum-mechanical aspect of the problem will then involve only the spin states because the motion can be described by a wave packet $|i\rangle$ traveling over a classical path. We shall therefore seek the solution of (2) in the form

$$\Psi=u(p)\,e^{-\frac{i\varepsilon}{\hbar}t}|i\rangle,\quad u(p)=\sqrt{\frac{\varepsilon+mc^{2}}{2\varepsilon}}\begin{pmatrix}\varphi\\ \frac{c(\sigma\mathbf{p})}{\varepsilon+mc^{2}}\,\varphi\end{pmatrix}. \qquad (3)$$

Using the explicit form of the spinor u(p) given by (3), we obtain

$$-e_{\mu}^{*}\,j^{\mu}=\langle\varphi_{f}|A+i\sigma\mathbf{B}|\varphi_{i}\rangle\,e^{i\omega t-i\mathbf{kr}}, \qquad (4)$$

$$A=N\mathbf{e}^{*}\left\{\left(1-\frac{\hbar a\omega}{2mc^{2}}\right)\frac{c\mathbf{p}}{\varepsilon+mc^{2}}+\left(1+\frac{\hbar a\omega}{2mc^{2}}\right)\frac{c\mathbf{p}'}{\varepsilon'+mc^{2}}+\right.$$
$$\left.+\frac{\hbar a\omega\,[\mathbf{n}\,[\mathbf{pp}']]}{2m\cdot(\varepsilon+mc^{2})(\varepsilon'+mc^{2})}\right\}, \qquad (5)$$

$$B = N\left\{\left(1-\frac{\hbar a\omega}{2mc^2}\right)\frac{[\mathbf{e}^*\mathbf{p}]\,c}{\varepsilon+mc^2}-\left(1+\frac{\hbar a\omega}{2mc^2}\right)\frac{[\mathbf{e}^*\mathbf{p}']}{\varepsilon'+mc^2}+\right.$$

$$+\frac{\hbar a\omega}{2mc^2}[\mathbf{e}^*\mathbf{n}]-\frac{\hbar a\omega}{2m}\frac{1}{(\varepsilon+mc^2)(\varepsilon'+mc^2)}(\mathbf{p}([\mathbf{e}^*\mathbf{n}]\cdot\mathbf{p}')+\mathbf{p}'([\mathbf{e}^*\mathbf{n}]\cdot\mathbf{p})- \quad (6)$$

$$\left.-(\mathbf{p}\mathbf{p}')[\mathbf{e}^*\mathbf{n}])\right\},$$

$$N=\sqrt{\frac{(\varepsilon'+mc^2)(\varepsilon+mc^2)}{4\varepsilon\varepsilon'}},\quad \mu'=\frac{e\hbar}{2mc}a.$$

The quantity a is related to the Lande factor by the formula $a = (g/2) - 1$. For an electron $a \simeq \alpha/2\pi$ and for a proton $a = -1.793$.

Expanding (5) and (6) in powers of $\hbar\omega/\varepsilon$ up to terms $0\left(\frac{\omega^2\hbar^2}{\varepsilon mc^2}\right)$ we obtain

$$A = e_i^* a_i,\quad a_i = \frac{cp_i}{\varepsilon}\left(1+\frac{\hbar\omega}{2\varepsilon}\right),$$

$$B_i = -\frac{\omega\hbar}{2mc^2}e_k^*(\varepsilon_{ikl}b_l + c_{ij}\varepsilon_{jkl}n_l), \quad (7)$$

$$b_i = \alpha_1\beta_i - \alpha_2 n_i,\quad C_{ij} = \alpha_3\beta_i\beta_j, \quad (8)$$

$$\alpha_1 = a+\frac{mc^2}{\varepsilon+mc^2},\quad \alpha_2 = a+\frac{mc^2}{\varepsilon},\quad \alpha_3 = \frac{a\varepsilon}{\varepsilon+mc^2}.$$

We now use (1), (4), (7), and (8) to determine the probability of a transition with the emission of a photon, summed over the final states:

$$dw = \frac{e^2c^2}{\hbar\omega}\sum_{f,\gamma}|\langle\varphi_f|A_\omega+i\sigma\mathbf{B}_\omega|\varphi_i\rangle|^2\frac{d^3k}{(2\pi)^3}= \quad (9)$$

$$=\frac{e^2c^2}{\hbar\omega}\sum_{\gamma}Sp\left[\left(\frac{1+\sigma\zeta_i}{2}\right)(A_\omega^*-i\sigma\mathbf{B}_\omega^*)\left(\frac{1+\sigma\zeta_f}{2}\right)(A_\omega+i\sigma\mathbf{B}_\omega)\right]\frac{d^3k}{(2\pi)^3},$$

$$(A_\omega,\ \mathbf{B}_\omega) = \int_{-\infty}^{\infty}(A(t),\ \mathbf{B}(t))\,e^{i(\varepsilon/\varepsilon')kx(t)}\,dt. \quad (10)$$

For transitions with spin flip, we substitute $\zeta_i = -\zeta_f = \zeta$, so that after evaluating the space in (9), we obtain

$$dw = \frac{e^2c^2}{\hbar\omega}\sum_{\gamma}\{|\mathbf{B}_\omega|^2-|\zeta\mathbf{B}_\omega|^2-i\zeta[\mathbf{B}_\omega,\ \mathbf{B}_\omega^*]\}\frac{d^3k}{(2\pi)^3}. \quad (11)$$

Summing over the photon polarizations, we now obtain

$$dw = \frac{e^2c^2}{\hbar\omega}\{|\mathbf{b}|^2(1-(\zeta\mathbf{n})^2)+2(\zeta\mathbf{n})\,\mathrm{Re}\,(\mathbf{nb})(\zeta\mathbf{b}^*)-i(\zeta-\mathbf{n}(\zeta\mathbf{n}))[\mathbf{b},\ \mathbf{b}^*]+$$

$$+2\,\mathrm{Re}\,(c_{il}(\mathbf{b}^*\mathbf{n})-c_{il}n_ib_l^*)+|c_{ij}|^2-|c_{ij}n_j|^2-. \quad (12)$$

$$[-2\,\mathrm{Re}\,[c_{ij}\zeta_l\zeta_j(\mathbf{nb}^*)-c_{ij}\zeta_ib_j^*(\zeta\mathbf{n})]-|c_{ij}\zeta_j|^2+|c_{ij}\zeta_in_j|^2+2\,\mathrm{Im}\,(\zeta_k\varepsilon_{klj}n_lc_{jl}b_l^*)-$$

$$-i\zeta_s\varepsilon_{slj}(c_{lk}c_{jk}^*-c_{lk}n_kc_{jl}^*n_l)\}\frac{d^3k}{(2\pi)^2}.$$

High-energy particles radiate mainly into a small angle $\sim 1/\gamma = mc^2/\varepsilon$ around the direction of motion, and the characteristics of the emitted radiation depend on the ratio between the deflection ψ of the particles in the field and the angle $1/\gamma$ [6, 7]. We shall confine our attention to the situation commonly encountered in practice in which the radiation is generated on a large number of periodic-structure elements ($\psi\gamma \ll 1$) and the dipole approximation is valid, i.e.,

$$\gamma\frac{\Omega r_\perp}{c}\ll 1,$$

where $L = vT = (2\pi/\Omega)v$ is the period of the tube; $v = |\mathbf{v}|$ is the velocity component in the direction of the tube axis; and $\gamma^{-2} = 1 - (v^2/c^2)$. When this is so, it is convenient to use the following expressions:

$$\boldsymbol{\beta}_\omega = \frac{i}{\omega\Delta}\left(\dot{\boldsymbol{\beta}}_\omega + \frac{\mathbf{v}(\mathbf{n}\dot{\boldsymbol{\beta}}_\omega)}{c\Delta}\right),$$
$$\mathbf{n}_\omega = \frac{i\mathbf{n}}{\omega}\left(\frac{\mathbf{n}\dot{\boldsymbol{\beta}}_\omega}{\Delta^2}\right);\quad \Delta = 1 - (\mathbf{n}\mathbf{v})/c, \tag{13}$$
$$(\beta_i\beta_j)_\omega = \frac{i}{\omega c\Delta}(\dot{\beta}_{\omega i}v_j + v_i\dot{\beta}_{\omega j}) + \frac{iv_iv_j(\mathbf{n}\dot{\boldsymbol{\beta}}_\omega)}{\omega c^2\Delta^2}.$$

For an arbitrary, infinitely long, undulating tube,

$$\dot{\boldsymbol{\beta}}_\omega = \frac{\varepsilon'}{\varepsilon}\sum_{s=-\infty}^{+\infty}\dot{\boldsymbol{\beta}}_s 2\pi\delta(\omega\Delta - s\Omega);\quad \dot{\boldsymbol{\beta}}_s = \frac{1}{T}\int_0^T\dot{\boldsymbol{\beta}}(t)e^{is\Omega t}\,dt. \tag{14}$$

Substituting the explicit form of b_i and C_{ij} in the approximation given by (13) in (12), and integrating with respect to d^3k, we obtain the following expression for the spin-flip transition probability per unit time:

$$\begin{aligned} w_\zeta = \sum_{s=1}^{\infty}\frac{se^2\hbar\Omega\gamma^6|\dot{\boldsymbol{\beta}}_s|^2}{15m^2c^6}\Big\{&\alpha_1^2\left(6\gamma^2 - 4 + \frac{3}{\gamma^2}\right) - \alpha_1\alpha_2(12\gamma^2 - 7) + \\ &+ 3\alpha_2^2(2\gamma^2 - 1) + \frac{5\alpha_3^2v^2}{c^2\gamma^2} + 5\alpha_3\frac{v^2}{c^2}(\alpha_2 - \alpha_1) + \\ &+ \frac{v^2}{c^2}\zeta_1^2[\alpha_\perp^2(-3\gamma^2 + 7) - 3\gamma^2\alpha_2^2 + 6\gamma^2\alpha_1\alpha_2 + \alpha_3(\alpha_2 - 15\alpha_1) + \\ &+ \alpha_3^2\left(5 + 2\frac{v^2}{c^2}\right)\Big] + \frac{|\zeta\dot{\boldsymbol{\beta}}_s|^2}{|\dot{\boldsymbol{\beta}}_s|^2}\Big[\alpha_2^2 + \alpha_1^2\left(5 - \frac{v^2}{c^2}\right) - 5\alpha_1\alpha_2 + \\ &+ 5\alpha_3\frac{v^2}{c^2}(\alpha_1 - \alpha_2) - \frac{5\alpha_3^2v^2}{c^2\gamma^2}\Big] + \\ &+ 5\zeta_\parallel\frac{\mathrm{Im}[\dot{\boldsymbol{\beta}}_s,\ \dot{\boldsymbol{\beta}}_s^*]_\parallel}{|\dot{\boldsymbol{\beta}}_s|^2}\left(\alpha_1(\alpha_1 - \alpha_2) + \frac{\alpha_3^2v^2}{c^2\gamma^2}\right)\Big\}. \end{aligned} \tag{15}$$

It follows from this expression that the total probability is a quadratic function of the transverse polarization and a linear function of the longitudinal polarization. By taking the vector ζ to be parallel to the velocity v, and summing (15) over two independent components, we obtain the following expression for the spin relaxation time τ:

$$\frac{1}{\tau} = \frac{1}{\tau_0}\left[1 + \frac{a(6\gamma^2 + 7\gamma + 1)}{\gamma(6\gamma + 1)} + \frac{2a^2(4\gamma^2 + 3\gamma - 1)}{\gamma(6\gamma + 1)}\right],$$
$$\frac{1}{\tau_0} = \sum_{s=1}^{\infty}\frac{2se^2\hbar\Omega\gamma^5(6\gamma + 1)}{15m^2c^5(\gamma + 1)^2}|\dot{\boldsymbol{\beta}}_s|^2. \tag{16}$$

The asymptotic degree of polarization is

$$\mathbf{P} = \frac{5\left[\frac{a^2v^2}{c_2}\gamma^2 - \gamma a(\gamma + 1) - \gamma\right]\sum_{s=1}^{\infty}S\,\mathrm{Im}[\dot{\boldsymbol{\beta}}_s,\ \dot{\boldsymbol{\beta}}_s^*]}{\left[6\gamma^2 + \gamma + a(6\gamma^2 + 7\gamma + 1) + 2a^2(4\gamma^2 + 3\gamma - 1)\sum_{s=1}^{\infty}\right]S|\dot{\boldsymbol{\beta}}_s|^2}. \tag{17}$$

To estimate the self-polarization effect, we consider the motion of particles in a spiral undulating tube with magnetic field given by [8]:

$$\mathbf{H} = \left(H \sin \frac{2\pi}{L} z, \quad -H \cos \frac{2\pi}{L} z, \; 0\right).$$

With the initial conditions $\mathbf{r}(0) = 0$, $\mathbf{v}(0) = \left(0, \frac{L\Omega_0}{2\pi}, v\right)$ the acceleration of the electron is

$$\mathbf{w} = (w_0 \cos \Omega t, \; w_0 \sin \Omega t, \; 0), \tag{18}$$

$$\Omega = \frac{2\pi}{L} v, \quad \Omega_0 = \frac{eH}{mc\gamma}, \quad w_0 = v\Omega_0.$$

Substituting (18) in (14), and using (16), we find that for electrons (a << 1, $a\gamma$ >> 1):

$$\frac{1}{\tau} = \frac{2e^2 \hbar \Omega w_0^2 \gamma^4}{5m^2c^7}(1 + a) \tag{19}$$

The above formulas are valid for undulating tubes of finite length l provided

$$l > v\tau_0 = \frac{5m^2c^7}{2e^2\hbar\Omega\Omega_0^2 v\gamma^4}.$$

This condition can be realized in modern undulating systems for $\gamma \sim 10^7$.

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