ACTIVE COMPENSATION OF NOISE IN THE MICHELSON INTERFEROMETER

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A general approach is developed to noise compensation in the Michelson interferometer. Compensating systems containing sensitive elements with distributed parameters are examined. A highly stable practical system is proposed.

Interferometric methods are widely used for the detection of very small lengths and displacements. They are successfully used to solve many scientific and technological problems such as calibration of detectors and sources of sound, determination of the amplitudes and resonance frequencies of mechanical elements, studies of ferroelectric phenomena at critical points, etc. Interferometric methods are distinguished by good spatial resolution and high sensitivity. Under the most favorable conditions, they can ensure measurement precision of about $10^{-6}\lambda$ [1, 2]. However, these advantages of the method are difficult to exploit fully in practice all interferometric systems are extremely sensitive to all kinds of external effects, the most important of which are acoustic noise and random effects in laser emission.

Consider the principle of the Michelson interferometer (Fig. 1). The single-mode laser beam is divided by the semitransparent mirror (2) into two beams. The first of these is directed onto the surface under investigation (3) and after reflection is received by the photodetector (5). The second reference beam is reflected by mirrors (4) and (2) and is also received by the photodetector.

Oscillations of the object x(t) and the effect of external microseismic noise produce a path difference $\psi(t) = x(t) + \xi(t)$ between the interferring beams. The quantity $\xi(t)$ is the path difference due to the external sources. It has been shown [3] that the photodetector output current is given by

$$I(t) = A\left[\sin\frac{4\pi}{\lambda}\psi(t) + \varkappa\right] + \eta(t), \tag{1}$$

where $\varkappa$ is a coefficient defining the working point on the photodetector; λ is the wavelength of the laser radiation; $\eta(t)$ is the photodetector noise. When the intensity of the interferring beams is high enough, $A \gg |\eta(t)|$, and the photodetector noise may be neglected. When the oscillations under investigation and the microseismic effects are such that

$$\frac{4\pi}{\lambda}|\psi(t)| \ll 1, \tag{2}$$

the expression given by (1) assumes the form

$$I(t) \simeq A\left(\frac{4\pi}{\lambda}\psi(t) + \varkappa\right). \tag{3}$$

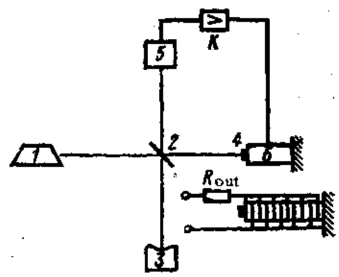

Fig. 1

Let us further suppose that the spectrum of the signal under investigation is quite different from the noise spectrum. The useful signal x(t) can then be readily isolated by the usual methods, using filters. The noise intensity is frequently appreciable and (2) is not valid. When this is so, the signal spectrum contains both the noise and useful signal frequencies as well as combination frequencies, and the spectral isolation of the useful signal cannot be carried out. Moreover, existing methods for compensating laser radiation noise [2], which are satisfactory at low microseismic noise intensity $\xi(t)$, are then ineffective. Passive noise compensation, which depend on direct suppression of $\xi(t)$, include mouting the interferometer on a massive base, special design of the components of the instrument, etc. The possibilities of such methods are restricted and, as a rule, they cannot be fully exploited in practice.

Negative feedback systems have been used to compensate the influence of noise [4]. The reference mirror (4) is mounted on a piezoceramic rod (6) to which a potential difference is applied. This varies its length in accordance with some law $y(t)$ and displaces the mirror (4). The path difference between the interferring beams is $\psi(t) = x(t) + \varepsilon(t)$, where $\varepsilon(t) = \xi(t) - y(t)$ is the compensation error signal. The function $y(t)$ must be such that $|\varepsilon(t)| << \lambda/4\pi$ so that, as shown above, the useful signal $x(t)$ can be isolated by spectral methods. This feedback arrangement can be used to increase the accuracy of interferometric measurements but it suffers from a number of shortcomings which restrict its possibilities. Thus, the piezoceramic block is a mechanical oscillatory system with distributed parameters, executing longitudinal oscillations. The amplitude-frequency characteristic of such a system has well-defined resonances at natural frequencies $\omega_1, \ldots, \omega_n$. The first few complex natural frequencies of the oscillatory system $\Omega_j = \mu_j + i\omega_j$, $j = 1, \ldots, N$ have small damping coefficients $|\mu_j| << \omega_j$. Closure of the feedback circuit may result in the self-excitation of the compensating system because the gain K in the feedback circuit in the region of the frequencies ω_j is greater than unity and the phase stability condition is violated. Let us consider possible ways of increasing the stability of compensating systems.

1. One way is to increase the resonance frequencies to high enough values to ensure that the stability condition is satisfied. This can be done by making the piezoceramic rod as short as possible. However, a reduction in the length of the rod is accompanied by an increase in the voltage necessary to ensure the required increase in the length, which in turn is determined by the amplitude of the microseismic noise. A high voltage across the rod is undesirable for technical reasons. One way of reducing the voltage and increasing the sensitivity of the system is to make a piezoblock in the form of a large number of individual plates (30 plates in [4]) connected in parallel. However, the natural frequencies then turn out to be relatively low, about 10 kHz.

2. The second method is to reduce the gain in the feedback circuit at the resonance frequencies until the stability condition is satisfied. However, this is accompanied by a reduction in the precision of compensation.

3. The third method is to increase the damping coefficient $|\mu_j|$ by passive methods, for example, by placing the rod in a viscous medium.

As a rule, these methods are only partially effective and do not solve the problem. Moreover, their application leads to a reduction in the static sensitivity.

In this paper, we propose an active method of increasing the stability. It is based on the utilization of the distributed properties of a sectionalized piezoceramic rod. Consider the structural compensation system illustrated in Fig. 2. We shall suppose that its operation is linear ($|\varepsilon(t)| << \lambda/4\pi$ and the characteristics of all branches are linear). The nonlinear block Φ can then be replaced by a perfect amplifier with gain K_1. The dynamics of the piezoceramic rod will be described by the transfer function $W_1(x, \xi, p)$ which we shall write in the form [5]:

$$W_1(x, \xi, p) = \sum_{j=1}^{\infty} \frac{\Phi_j(x)\Phi_j(\xi)}{(p - \mu_j)^2 + \omega_j^2}, \qquad (4)$$

where $\{\Phi_j(x)\}$ is a set of orthonormal eigenfunctions and $\Omega_j = \mu_j + i\omega_j$ are the complex frequencies of longitudinal oscillations of the rod. $W_2(p) = K$ represents the transfer function of the amplifier. The intermediate block $W_x(x, p)$ describes the transformation of the function $\varphi(x, t)$ (relative elongation of the rod at the point x) into the localized signal $y(t) = \varphi(l, t)$ ($x = l$ is the coordinate of the free end of the rod). By definition of the intermediate block $W_x(x, p)$, we have $y(p)=\int_0^l \varphi(x, p) W_x dx$, i.e., $W_x(x, p) = \delta(x - l)$.

The capacitance C_0 of the piezoceramic rod, which is assumed constant, and the output resistance R_{out} of the amplifier, form the elementary block $W_b(p)$ with transfer function $W_b(p) = 1/(T_0 p + 1)$, $T_0 = R_{out}C_0$. The intermediate block $W_c(\xi, p)$ represents the voltage distribution over the sectionalized rod. The transfer funtion of this block is equal to the ratio of the force acting on the rod to the applied voltage. We must now determine this force. Suppose that the potential difference across each element of the composite rod is u(p). The expression for the relative deformation in the case of a reversible longitudinal piezoelectric effect is [6]: $\frac{\partial \varphi(x, t)}{\partial x} = s\sigma + d_1 E(x, t)$, where E(x, t) is the electric field in the rod; d_1 is the piezoelectric modulus; σ is the stress; and s is the compliance. Hence the elastic force can be described by

$$\frac{\partial \sigma}{\partial x} = \frac{1}{s}\left(\frac{\partial^2 \varphi}{\partial x^2} - d_1 \frac{\partial E}{\partial x}\right).$$

It follows that the distribution of the force along the rod axis, due to the inverse piezoelectric effect, is given by

$$F(x, p) = -d\frac{\partial E(x, p)}{\partial x} = d\frac{\partial^2 V(x, p)}{\partial x^2},\ d = d_1/s. \tag{5}$$

It is assumed that the potential distribution V(x, p) along the rod axis is linear, i.e., $V(x, p) = u(p)x/l$, so that $E(x, p) = -u(p)/l$. Substituting the resulting expression in (5), we obtain

$$F(x, p) = [\delta(x) - \delta(x - l)]\, u(p)\, d/l. \tag{6}$$

Thus, in the above method of connection of the piezoceramic rod, the rod experiences two forces that are equal in magnitude and opposite in direction, and are applied at x = 0 and $x = l$. The expression for the intermediate block therefore becomes

$$W_c(\xi, p) = \frac{F(\xi, p)}{u(p)} = [\delta(\xi) - \delta(\xi - l)]\, d/l.$$

Having found all the transfer functions of the elementary blocks in the structural system, we can write down the transfer function W(p) of the closed compensation system [5] (the ratio of the output signal y(p) to the input signal $\xi(p)$):

$$W(p) = \frac{K_0 \int_0^l\int_0^l W_c(\xi, p) W_1(x, \xi, p) W_x(x, p) W_b(p)\, dx d\xi}{1 + K_0 \int_0^l\int_0^l W_c(\xi, p) W_1(x, \xi, p) W_x(x, p) W_b(p)\, dx d\xi}, \tag{7}$$

or

$$W(p) = \frac{K_0 [W_1(l, 0, p) - W_1(l, l, p)] W_b(p)}{1 + K_0 [W_1(l, 0, p) - W_1(l, l, p)] W_b(p)}. \tag{8}$$

For static stability, the coefficient $K_0 = K_1 K d/l$ must be negative, i.e., $K_0 < 0$. It is desirable to increase $|K_0|$ because this leads to a reduction in dynamic errors in the compensation system ($\varepsilon(p) \to 0$ for $|K_0| \to \infty$). The stability condition imposes a restriction on the magnitude of the gain, and we shall investigate this now. The complex frequencies $p_j = \delta_j + i\nu_j$ of the closed control system are given by the following characteristic equation:

$$K_0 W_b(p) \sum_{j=1}^{\infty} \frac{q_j}{(p - \mu_j)^2 + \omega_j^2} = -1,\ q_j = \Phi_j(l)[\Phi_j(0) - \Phi_j(l)]. \tag{9}$$

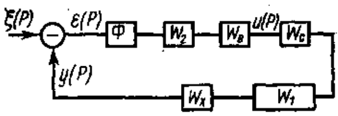

Fig. 2

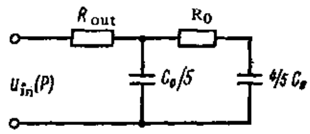

Fig. 3

Closure of the feedback circuit ensures the individual harmonics of the transient process become coupled by the controlling system. It is clear from physical considerations that if the natural frequencies ω_j are well spaced out, then the coupling of the individual harmonics in the closed system will be weak if, of course, the coefficient $|K_0|$ is small enough. The change in the complex frequencies will then be governed by the individual resonant terms corresponding to the different roots. It follows that to determine the complex frequency $p_n = \delta_n + i\nu_n$, we can neglect in (9) the summation over the j nonresonant terms with $j \neq n$. The characteristic equation then assumes the form

$$K_0 W_1(p) \frac{q_n}{(p-\mu_n)^2+\omega_n^2} = -1, \quad n = 1, \ldots, \infty.$$

This can readily be transformed to the following form:

$$\delta_j = \mu_j - \frac{q_j}{2\nu_j} \, Im \frac{K_0}{T_0 P_j + 1}, \qquad (10)$$

$$\nu_j^2 = \omega_j^2 + (\delta_j - \mu_j)^2 - q_j \operatorname{Re} \frac{K_0}{T_0 P_j + 1}.$$

The stability condition $\max\limits_j \delta_j < 0$ then yields the following critical value for the gain:

$$K_{crit} = \min_j |2\mu_j (1 + T_0^2 \nu_j^2)/T_0 q_j|. \qquad (11)$$

We shall confine our attention to the first N natural oscillation modes for which the damping coefficient is small, $|\mu_j| \ll \omega_j$, $j = 1, \ldots, N$, so that if we use (11) we find that (10) has the following approximate solution:

$$\delta_j \simeq \mu_j + K_0 T_0 q_j / 2(1 + T_0^2 \omega_j^2), \qquad (12)$$

$$\nu_j \simeq \omega_j, \; j = 1, \ldots, N.$$

Let $v_j = \partial\delta_j / \partial K_0$ represent the velocity of the j-th root on the complex plane of p. It is clear that when $v_j > 0$ the j-th root moves in the direction of increasing stability, and when $v_j < 0$, the stability of the root deteriorates with increasing $|K_0|$. It is readily shown from (12) that

$$v_j = T_0 q_j / 2(1 + T_0^2 \omega_j^2).$$

It is not difficult to see the reasons for the instability of the compensating system. In fact, by virtue of the boundary conditions $q_j < 0$ and $v_j < 0$, the stability of the system deteriorates for all j as $|K_0|$ increases. Let us consider the ways in which the operation of the compensation system can be improved. It follows from (12) that the sign of the velocity depends on the values of the eigenfunctions $\Phi_j(x)$ at the points at which the forces are applied. We shall use distributed forces and note that different distributions of forces can be obtained by applying different potential differences to the components of the sectionalized rod.

Suppose the piezoceramic rod of length $l = 1$ consists of N individual elements. We shall assume, for simplicity that they all have the same polarization and are insulated from each other. We apply a control voltage $\alpha_i u(p)$ to the element of number i. According to (6) this produces a pair of forces at the bounding points x_{i-1} and x_i, which are respectively equal to $u(p)\alpha_i \delta(x - x_{i-1}) d/\Delta_i$ and $-u(p)\alpha_i \delta(x - x_i) d/\Delta_i$, $\Delta_i = x_i - x_{i-1}$,

$i = 1, \ldots, N$. However, at the point x_{i-1} we also have the force due to the preceding element number $i - 1$, and this is equal to $\frac{d}{\Delta_{i-1}} u(p) \alpha_{i-1} \delta(x - x_i)$.

It follows that the resultant force at the point x_{i-1} is

$$F_{i-1} = du(p)\,\delta(x - x_{i-1})\left(\frac{\alpha_i}{\Delta_i} - \frac{\alpha_{i-1}}{\Delta_{i-1}}\right).$$

The total force acting on the rod is

$$F = du(p)\left[\frac{\alpha_1}{\Delta_1}\delta(x) - \frac{\alpha_N}{\Delta_N}\delta(x-1) + \sum_{i=2}^{N}\left(\frac{\alpha_i}{\Delta_i} - \frac{\alpha_{i-1}}{\Delta_{i-1}}\right)\right]\delta(x - x_{i-1}),$$

and, correspondingly, for the intermediate block $W_c(\xi, p)$ we have

$$W_c(\xi, p) = d\left[\frac{\alpha_1}{\Delta_1}\delta(\xi) - \frac{\alpha_N}{\Delta_N}\delta(\xi - 1) + \sum_{i=2}^{N}\delta(\xi - x_{i-1})\left(\frac{\alpha_i}{\Delta_i} - \frac{\alpha_{i-1}}{\Delta_{i-1}}\right)\right]. \quad (13)$$

As before, we assume that $W_b(p) = 1/(T_0 p + 1)$. By analogy with (12), we can readily show that the damping coefficients are

$$\delta_j = \mu_j + \frac{K_0 T_0 \Phi_j(1)}{2(1 + T_0^2\omega_j^2)}\left[\frac{\alpha_1}{\Delta_1}\Phi_j(0) - \frac{\alpha_N}{\Delta_N}\Phi_j(1) + \sum_{i=2}^{N}\left(\frac{\alpha_i}{\Delta_i} - \frac{\alpha_{i-1}}{\Delta_{i-1}}\right)\Phi_j(x_{i-1})\right]. \quad (14)$$

This formula incorporates all the previous cases. When $\alpha_i = 1$, $i = 1, \ldots, N$, (14) becomes identical with (4) for a composite rod connected as shown in Fig. 1a, and $\alpha_i = \begin{cases} 1, & i = 1, \ldots, N_1 \\ 0, & i = N_1 + 1, \ldots, N \end{cases}$ is equivalent to the connection of a free segment of length $\Delta = \sum_{i=N_1+1}^{N} \Delta_i$ to the free end of the rod. To increase the stability of a given number M of components of the transient process, it is clear from (14) that the coefficients α_i, $i = 1, \ldots, N$ must be chosen so that

$$\Phi_j(1)\left[\frac{\alpha_1}{\Delta_1}\Phi_j(0) - \frac{\alpha_N}{\Delta_N}\Phi_j(1) + \sum_{i=2}^{N}\left(\frac{\alpha_i}{\Delta_i} - \frac{\alpha_{i-1}}{\Delta_{i-1}}\right)\Phi_j(x_{i-1})\right] \geqslant 0. \quad (15)$$

When this is so, $v_j \geq 0$ for all $j = 1, \ldots, M$. On the other hand, to achieve the maximum amplitude of static displacement for a given applied voltage, it is necessary for the linear form $I = \sum_{i=1}^{N}\alpha_i$ to achieve its maximum possible value ($\alpha_i = 1$ corresponds to the maximum admissible voltage across a rod element). We thus arrive at the following linear programming problem: it is required to choose the quantities $-1 \leq \alpha_i \leq 1$ so that the following inequalities are satisfied:

$$\sum_{i=1}^{N} a_{ij}\alpha_i \geqslant 0, \; j = 1, \ldots, M, \quad (16)$$

and the linear form $I = \sum_{i=1}^{N}\alpha_i$ assumes its maximum possible values, where

$$a_{ji} = \Phi_j(1)\left[\frac{\Phi_j(x_{i-1})}{\Delta_{i-1}} - \frac{\Phi_j(x_i)}{\Delta_i}\right].$$

The case I = N corresponds to the absence of restrictions on parameters α_i. The linear form I characterizes the static sensitivity of the system to the control system, and when $I < 0$ the system is statically unstable.

In general, the increase of stability achieved by the above method is accompanied by a deterioration in the operation of the compensation system under static conditions because the displacement amplitude is reduced ($I < N$). This defect can, however, be largely removed. Thus, the control voltage can be applied to each element of the rod through a link with transfer function $\alpha_i(p)$ such that $\alpha_i(0) = 1$. The transfer functions can be synthesized so that for frequencies for which the natural oscillations must be damped $\mathrm{Im}\,\alpha_i(p) = \alpha_i$, where α_i are the coefficients obtained by solving the problem defined by (16). Thus, in the static case, we have I = N as before. The synthesis can be carried out, for example, as follows. For those elements of the rod for which $\alpha_i < 0$, the control voltage can be applied through a phase shifting chain producing a phase change of π outside the compensation region. It is then clear that all the coefficients $\alpha_i(p)$ will be positive for $p = 0$ and $\alpha_i(p) = \alpha_i$ in the region of resonance frequencies. If it is not necessary to damp out a large number of natural oscillations, the coefficients α_i can be found by analyzing (15) to obtain the velocities of the roots. Figure 3 illustrates the use of distributed control designed to damp out natural oscillations. It incorporates a sectionalized rod in a feedback loop. Calculations have shown that this arrangement can ensure a considerable increase in the stability of the compensation system. In fact, tests have shown that the stability could be improved by an order of magnitude.

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