DEBYE SCREENING AND ION CURRENT ONTO A PROBE IN THE CASE OF ARBITRARY ELECTRON ENERGY DISTRIBUTION FUNCTION

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We obtain an expression for the electron Debye radius and derive formulas for calculating the ion current onto a probe for an arbitrary isotropic electron distribution function. An algorithm is given for extracting the second derivative of the electron current when measuring the electron distribution function by the method of Dreyvesteyn.

Knowledge of the ion current onto a probe is needed for two reasons. First, situations are frequently encountered (VHF and UHF discharges, dense plasma) where it is impossible to determine the electron concentration from the size of the electron current under the influence of a space potential. In this case the concentration of electrons is determined from the value of the ion current at large negative probe potentials. Secondly, in measuring the electron energy distribution function (EEDF) one must evaluate the effect of the interfering ion current. Therefore, determining the ion current onto a probe for an arbitrary isotropic EEDF is of practical interest. This is especially important in the collisionless limiting regime, because in the orbital motion case the ion current does not depend on the energy distribution of the electrons, while in the collision regime the Dreyvesteyn formula is not applicable.

The theory of the current onto a probe for a Maxwellian electron distribution was developed by Yu. M. Kagan and V. I. Perel [1]. I. A. Vasil'ev showed in [2,3] that for the limited class of functions described by the expression

$$f_0(v) = C_k \exp\left\{-\left(\frac{mv^2}{2\alpha}\right)^k\right\}. \quad (1)$$

both the screening of the plasma and the size of the ion current onto a probe can depend significantly on the form of the EEDF.

The analysis of [1-3] showed that the method applied there can be extended to the case of arbitrary isotropic distribution function.

For determination of the ion current one needs to know the distribution of the electron concentration and the perturbing electric field. If the isotropic part of the EEDF in the unperturbed plasma is $f_0\left(\frac{mv^2}{2}\right)$, then in the absence of collisions the distribution function in the perturbed region will have the form [1]

$$f(\mathbf{r}, \mathbf{v}) = n_0 f_0\left(\frac{mv^2}{2} + eV(\mathbf{r})\right). \quad (2)$$

Here n_0 is the concentration of electrons in the unperturbed plasma, $\mathbf{r}$ and $\mathbf{v}$ are the coordinate and velocity of the electron, $v = |\mathbf{v}|$, e and m are the charge and mass of the electron, and V(r) is the perturbing field potential. The distribution of the electron concentration in the perturbed region is found by integrating Eq. (2) over the velocity space

$$F(eV) \equiv \frac{n_e(eV)}{n_0} = \quad (3)$$

$$= \frac{\int_0^\infty f_0\left(\frac{mv^2}{2} + eV\right) 4\pi v^2\, dv}{\int_0^\infty f_0\left(\frac{mv^2}{2}\right) 4\pi v^2\, dv} = \frac{\int_{eV}^\infty \sqrt{x - eV}\, f(x)\, dx}{\int_0^\infty \sqrt{x}\, f(x)\, dx}. \qquad (3)$$

Eq. (3) is the natural generalization of the Boltzmann distribution in a potential field for an arbitrary electron energy distribution.

Using Eq. (3) one can examine the problem of the electron Debye radius, which, as is well known, reduces to the problem of solving Poisson's equation for the self-consistent field. In analogy with [2], we will limit consideration to the case of stationary ions and examine the limit of small potentials. Equation (3) then takes the form

$$\frac{n_e(eV)}{n_0} = 1 + V \frac{\lim_{V\to 0} \int_0^\infty \frac{\partial f_0}{\partial V}\left(\frac{mv^2}{2} + eV\right) v^2\, dv}{\int_0^\infty f_0\left(\frac{mv^2}{2}\right) v^2\, dv}. \qquad (4)$$

Here we have used

$$f_0\left(\frac{mv^2}{2} + eV\right) \simeq f_0\left(\frac{mv^2}{2}\right) + V \lim_{V\to 0} \frac{\partial f_0}{\partial V}\Bigg|_{\frac{mv^2}{2} + eV}. \qquad (5)$$

Substituting Eq. (4) into Poisson's equation and performing the integration by parts, we obtain for the electron Debye radius

$$h_e^2 = \frac{m}{4\pi n_0 e^2} \frac{\int_0^\infty f_0\left(\frac{mv^2}{2}\right) v^2\, dv}{\int_0^\infty f_0\left(\frac{mv^2}{2}\right) dv} = \frac{kT_h}{4\pi n_0 e^2}, \qquad (6)$$

where

$$T_h = \frac{m}{k} \langle v^{-2}\rangle^{-1} = \frac{m}{k} v_h^2. \qquad (7)$$

Since the screening time is determined by the electron Langmuir frequency

$$\omega_{0e}^2 = \frac{4\pi n_0 e^2}{m},$$

then

$$v_h = h_e \omega_{0e} = \sqrt{\frac{kT_h}{m}}$$

has the physical sense of the rate at which the screening occurs. In the particular case of a Maxwellian distribution

$$v_h = v_{T_e} = \sqrt{\frac{kT_e}{m}}.$$

The quantity T_h can conditionally be called the effective screening temperature. In the general case h_e is determined not only by the mean energy of the electrons, as was assumed in several papers (see, e.g., [9]) on the basis of the expression for h_e for a Maxwellian distribution, but also depends on the parameters characterizing the given distribution function. Nevertheless, it follows from Eqs. (6) and (7) that for finding h_e it is sufficient to know one of the moments of the distribution function, namely, $\langle v^{-2}\rangle^{-1}$. Therefore, for calculation of h_e it is not necessary to know the analytic form of the distribution function.

Equation (6) emphasizes the role of the slow electrons in screening. We will illustrate this for one case of practical importance. Let the EEDF be given by the expression

$$f_0\left(\frac{mv^2}{2}\right)=\frac{n_1}{n_0}C_1e^{-\frac{mv^2}{2kT_1}}+\frac{n_2}{n_0}C_2e^{-\frac{m(v-v_0)^2}{2kT_2}}, \tag{8}$$

where $n_0 = n_1 + n_2$, and C_1 and C_2 normalize the terms in n_1/n_0 and n_2/n_0, respectively. Using Eq. (6) we find

$$h_e^2=\frac{T_1T_2}{4\pi e^2(n_1T_2+n_2T_1\varphi(x_0))}, \tag{9}$$

where $\varphi(x_0)$ is a function of the argument $x_0=\sqrt{\frac{m}{2kT_1}}v_0$, limited to the region $0<\varphi(x_0)\leqslant 1$. For $v_0\to\infty$ one sees that $x_0\to\infty$ and $\varphi(x_0)\to 0$. In this case, with the noticeable increase in the mean energy, it follows from Eq. (9) that h_e tends to a value which corresponds to a Maxwellian distribution with temperature T_1 and electron concentration n_1. In the case $v_0=x_0=0$ one has $\varphi(x_0)=1$, and the expression for h_e differs only slightly from the expression for the Maxwellian distribution with temperature T_1 (the introduction of the functions (8) is sensible if the inequality $n_1\gg n_2$) is satisfied). Thus, the high-energy part of the EEDF is weakly dependent on h_e.

We will examine the question of the size of the ion current on a probe. The method of calculation in which the region of perturbation of the plasma by the probe is partitioned into a region of quasi-neutrality and a region of space charge was developed in [1-3] for distribution functions of the form (1). This scheme can easily be generalized to the case of an arbitrary isotropic distribution function, although this was not mentioned in [103]. We will develop the method of calculation as it applies to a cylindrical probe. The results for a spherical probe can be obtained in an analogous way, using [1-3].

We will write the expression for the ion current i_p per unit length of probe in a form similar to that used in [1,3]:

$$i_p'=2\pi en_0r_p\sqrt{\frac{2kT_h}{M}}\alpha'. \tag{10}$$

Here r_p is the radius of the ionic layer (the layer in which the concentration of electrons can be neglected), M is the mass of an ion, and α' is a dimensionless coefficient determined by the dependence of the ion current on the form of the EEDF and the energy of the ions in the plasma. It is given by the formula

$$\alpha'^2=\alpha'^2_{\max}=\frac{\eta_0+\gamma}{\pi^2\left[1+\frac{1}{4\pi^2\left(\frac{dF}{d\eta}\right)^2(\eta+\gamma)}\right]_{\eta=\eta_0}}, \tag{11}$$

where $\eta=\frac{eV}{kT_h}$, $\gamma=\frac{\varepsilon_0}{kT_h}\ll 1$, ε_0 is the energy of the ions, and η_0 is the solution of the equation [3]:

$$4\pi^2\left(\frac{dF}{d\eta'}\right)^3(\eta+\gamma)^2+2\frac{d^2F}{d\eta^2}(\eta+\gamma)+3\frac{dF}{d\eta}=0. \tag{12}$$

Equation (10) can have an alternative form. For the case of a Maxwellian electron distribution function, T_e was used in [1]. In [3] for a distribution function of the form (1), the mean energy or α was used as an energy parameter. Naturally, the coefficient α' in this case will be different. For an arbitrary form of the distribution function it is convenient to choose an energy parameter such that even strong variations of the EEDF are accompanied by insignificant changes in the coefficient α'. The effective screening temperature is such a parameter.

The connection between i_p and r_p, on the other hand, can be described by the 3/2 power law:

$$i_p=\frac{2\sqrt{2}}{9}\sqrt{\frac{e}{M}}\frac{V_p^{3/2}}{a\beta^2(r_p/a)}, \tag{13}$$

where a is the radius of the probe $V_p \approx V$ is the potential drop in the layer. Equations (10)-(13) permit complete determination of the ion current for known a, n_0, EEDF, and

given V. They differ from the formulas for the Maxwellian distribution [1] only in the quantities T_h and α', which depend on the actual form of the EEDF.

In the case of orbital motion when the condition $r_p/a > \frac{\sqrt{\eta}}{\pi\alpha'}$ (see [4]) is satisfied, the ion current does not depend on the form of the EEDF:

$$i_p = 2en_0 a\sqrt{\frac{2eV}{M}}. \qquad (14)$$

We will use the proposed theory for calculation of α' in the case that the EEDF has the form (1) and (8). We will compare the cases in which one fixes as an energy parameter of the EEDF the quantity α, $\left\langle\frac{mv^2}{2}\right\rangle$, or T_h (Figs. 1 and 2). For distribution functions which are depleted of high-energy electrons (k = 2-3 in Fig. 1), the change of α' is minimal if either T_h or $\left\langle\frac{mv^2}{2}\right\rangle$ is fixed. In Fig. 2 we show two cases for an EEDF of the form (8). The first case corresponds ($v_0 = 0$) to the sum of two exponentials with different temperatures $\left(\frac{T_2}{T_1}=2\right)$. In this case the distribution function f_0 is rich in fast electrons, but is monotonic, and α' changes insignigicantly with the growth of the fast-electron concentration. The second case $\left(\frac{mv_0^2}{2kT_1}=10,\ \frac{T_2}{T_1}=2\right)$ corresponds to an EEDF f_0 having a maximum in the high-energy region. In this case α' changes more slowly if one takes T_h as the energy parameter. Thus, large changes in the EEDF only weakly affect α' and, hence, the size of the ion current. This means that the error introduced by the difference of i'' and i_e'' should also affect α' only slightly.

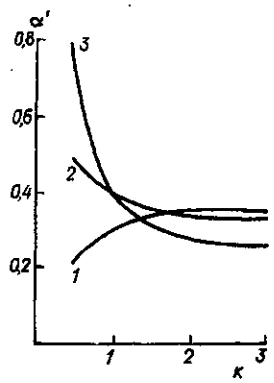

Fig. 1. The dependence of α' on k for $\gamma = 0$ for an EEDF of the form (1): curve 1 is for $\left\langle\frac{mv^2}{2}\right\rangle$ = const; curve 2 is for T_h = = const; and curve 3 is for α = const.

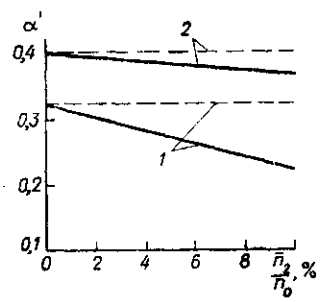

Fig. 2. The dependence of α' on n_2/n_0 for $\gamma = 0$ for an EEDF of the form (8): Curve 1 is for $\left\langle\frac{mv^2}{2}\right\rangle$ = const, curve 2 is for T_h = = const, $\left(\frac{mv_0^2}{2kT_1}=10,\ v_0=0\right)$.

The solution of a straightforward problem permits examination of the question of separating out the second derivative of the electron current i_e'' from the second derivative of the total current onto the probe. In [5], a method of finding i_p'' in the case of a Maxwellian EEDF, based on the use of Eqs. (10) and (13) and the Dreyvesteyn formula, is described. Since the form of the EEDF affects only the quantities T_h and α', one can apply the method of [5] in the case of an arbitrary distribution function as well. By analogy with [5] we obtain

$$\frac{i_e''}{i_p''} = \frac{3}{2\sqrt{\pi}}\sqrt{\frac{M}{m}}\left(\frac{a}{h_e}\right)^2\Phi(x)\sqrt{\eta}\,f_0(\eta). \qquad (15)$$

The radius of the ionic layer $x = r_p/a$ is determined from the equation

$$\varphi(x) = \frac{4}{9\alpha'}\left(\frac{h_e}{a}\right)^2\eta^{3/2}, \qquad (16)$$

where $\varphi(x) = x\beta^2(x)$.

The algorithm for calculating the distribution function reduces to the following sequence of steps. The probe curve and the second derivative of the current onto the probe are plotted. Considering the zeroth approximation that $i'' = i_e'' \sim f_0(eV)$, one finds T_h and α'. Then from the ion current one finds n_0 and calculates h_e. One then determines i_p'' and the

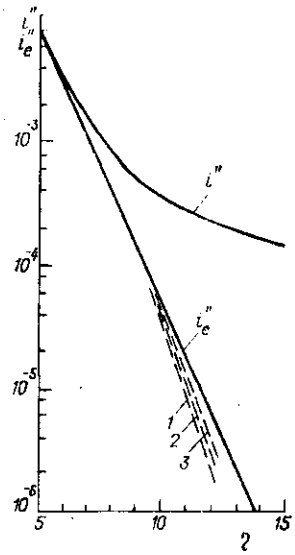

Fig. 3. Reduction of the Maxwellian distribution function. Curves 1-3 are the values of i''_e after the first, second, and third iteration, respectively.

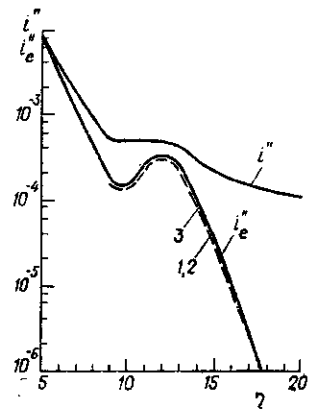

Fig. 4. Reduction of the EEDF of the form (8). The notation is the same as in Fig. 1.

first approximation for f_0: $f_0 : f_0 \sim i''_e \simeq i'' - i''_p$. The calculation is then repeated. The results of the calculation for two characteristic functions are shown in Figs. 3 and 4. Figure 3 is for the Maxwellian distribution, while Fig. 4 is for a distribution of the form (8) with $\frac{mv_0^2}{2kT_1} = 12$, $\frac{T_2}{T_1} = 0.5$, $\frac{n_2}{n_0} = 0.003$, where $n_0 = n_1 + n_2$, and C_1 and C_2 are normalization constants, with the value of i''_e corresponding to the chosen form of the distribution function, we recalculated the value of the second derivative of the total current onto the probe i_e. Considering i" as the initial measured distribution function, we applied to it the algorithm described above. The numbers 1, 2, and 3 on the curves in Figs. 3 and 4 indicate the values of i''_e after the first, second, and third iterations, respectively. As is seen in the figures, the method is rapidly convergent because of the weak dependence of T_h and α' on the form of f_0 in the high-energy region. In all cases the measurement band of the EEDF is broadened at least by an order of magnitude.

In [6] a method of determining the EEDF in the high-energy region based on graphical determination of the second derivative of the ion current is proposed. There it is assumed that $i_p \sim V^n$, where n changes from 0.5 to 1.5 depending on h_e, a, and the mean-free-path λ_e [7]. Plotting the function i"(V) to large inhibitory potentials, the authors assumed that for these potentials $i'' \simeq i''_p$, extrapolated the curve $i''_p(V)$ to the region of small potentials, and, measuring it from the curve i", determined i''_e.

It is clear that this method is not applicable for slowly falling-off EEDF's.

The calculation of the effect of i''_p performed in [8] is in fact based on the use of a function $i_p(V)$ of the form $i_p \sim \left(1 + 2\frac{h_e}{a} \cdot V^{1/2}\right)$. This formula describes the experimental data better, but for small h_e/a is insensitive to the power of V. It is obviously more correct to take a function of the form $i_p \sim (1 + CV^n)$, where n and C depend on h_e and a. We remark that according to our calculation such a form is closest of all to the results of [1]. Here $C \sim h_e/a$, and n increases from 0.58 to 0.8 when h_e/a decreases from 1 to 10^{-2}. At the same time, from a pure power-law dependence there should be a decrease in n when h_e/a is decreased. The dependence of n on α' (i.e., on the form of the EEDF) is exceedingly weak.

Thus, the changes in the high-energy part of the EEDF caused by not taking the ion current into account have a weak effect on the value of the second derivative of the ion current. Therefore, the region of appreciable effect of i_p'' can be estimated by the measured function i"(V). If the function i"(V) is plotted to potentials where $i_p'' \gg i_e''$, then it is easier to use the method of [6]; if $i_p'' \ll i_e''$, then it is necessary to solve the opposite problem, as was done in this paper.

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