CAUSALITY AND REGULARIZATION

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Within the framework of perturbation theory it is shown that the integral condition of causality is satisfied for a regularized scattering matrix.

The causality principle plays a fundamental role in the quantum theory of fields. Several different formulations of this condition are known. One of the more fruitful of these has been the formulation proposed by N. N. Bogolyubov [1]. It is very important to establish to what extent the principle of causality can be made consistent with the other aspects of the theory. The most crucial point here is the agreement of this principle with the regularization procedure. In this paper we examine a generalized Bogolyubov causality condition within the framework of perturbation theory for the case of the neutral scalar field, centering our attention on the regularization procedure. It should be noted that the limitation to the neutral scalar field in this case is not essential; it was motivated by a desire to avoid awkwardness of notation.

Let H_F be a Fock space of asymptotic states. We will examine in H_F the family of unitary operators of the form

$$S(g, \xi, h) = T \exp \left\{ i \int dx \left[g(x) \mathscr{L}_\xi(x) + h(x) \varphi(x) \right] \right\}, \tag{1}$$

where $\mathscr{L}_\xi(x)$ is the seed Lagrangian of the interaction for the displaced field $\varphi(x)+\xi(x)$, and $g(x)$, $\xi(x)$, $h(x)$ are numerical functions. An expression of the type $T\{\mathscr{L}_\xi(x)\mathscr{L}_{\xi'}(x')\varphi(x'')\}$ will be understood to be a usual chronological product, the coefficient functions of which are renormalized by the R-operation of Bogolyubov and Parasyuk [2].

If the functions $g(x)$, $\xi(x)$, $h(x)$ belong to the Schwartz space $\mathscr{S}(R_4)$ of basis functions, then it is well known that an expression of the type (1) specified in H_F a well-defined unitary operator, leaving aside questions regarding the convergence of the perturbation theory expansion.

Following [3], we will adopt as the causality condition the relation

$$S(f_1 + f_2 + f_3) = S(f_1 + f_2) S^+(f_2) S(f_2 + f_3), \tag{2}$$

if $supp f_1 \gtrsim supp f_3$, where $f=(g, \xi, h)$, $supp f = supp g \cup supp \xi \cup supp h$ and the sign $\gtrsim$ denotes later or space-like.

It is seen that for $\xi_i = h_i = f_2 = 0$ condition (2) reverts to the usual integral causality condition of Bogolyubov. The need for condition (2) can arise in the examination of models which depart from the framework of perturbation theory. Thus, in [4], condition (2) figures as one of the fundamental assumptions of the S-matrix approach, permitting the formulation of the axioms of the algebraic approach of Haag and Araka. The connection of this condition with other formulations was examined in [3].

The necessary regularization procedure is conveniently taken into account by including it in the construction of the space of states and operators of the field. To this end we employ the construction proposed in [5]. We introduce the space of quasi-states $H = \bigoplus_n H_n$, where

$$H_n = \{\bigoplus_k f_{nk}(p_1, \ldots, p_n; \mu_1, \ldots, \mu_k; N)\},$$

f_{nk} are smooth functions of their arguments, $p_1, \ldots, p_n$ are four-momenta, and $\mu_1, \ldots, \mu_n$ and

N are parameters by means of which regularization is introduced into the theory (analogous to cut-off parameters). In the final results we will take the limit $\mu_i \to 0$, $N \to \infty$ (lifting the cut-off). The functions f_{nk} are symmetric in the arguments p_i and μ_j separately, and as functions of p_i they belong to the space of multipliers $O_M(V_+^n)$ in the Schwartz space $\mathcal{S}(R_4)$. Here V_+ is the future light cone. The vector $|f\rangle$ will be called regular if it is a finite sum over n and k of functions f_{nk} which as functions of p_i belong to $\mathcal{S}(V_+^n)$ and have $\mathcal{S}(V_+^n)$ limits with respect to $\mu_1, \ldots, \mu_k$, and N. The subspace H^R of regular state vectors will be provided with a scalar product

$$\langle h | f\rangle = \sum_n \sum_k \sum_r \lim_{N\to\infty} \lim_{\mu_1\to 0} \ldots \lim_{\mu_k\to 0} P(\mu) \int dp_1 \ldots dp_n \times$$
$$\times \tilde{D}^-(p_1; \mu_1; N) \ldots \tilde{D}^-(p_n; \mu_n; N)\, h^*_{nk}(p_1, \ldots, p_n;\ \mu_{n+1}, \ldots$$
$$\ldots, \mu_{n+k}; N)\, f_{nr}(p_1, \ldots, p_n;\ \mu_{n+k+1}, \ldots, \mu_{n+k+r}; N). \quad (3)$$

Here $P(\mu)$ is the symmetrizer with respect to all μ_j, and

$$\tilde{D}^-(p; \mu; N) = \sum_{j=0}^{M(N)} C_j(\mu; N)\, \tilde{D}^-(p; m^2 M_j(\mu; N)).$$

Here $\tilde{D}^-(p; m^2M_j(\mu; N))$ is the Fourier image of the usual scalar field packet with a square mass of $m^2M_j(\mu; N)$. C_j and M_j are finite for finite μ and N ($M_j \to \infty$ for $\mu \to 0\ \forall j$),

$$\sum_{j=0}^{M(N)} C_j(\mu; N)\, M_j^{\alpha}(\mu; N) \ln^{\beta} M_j(\mu; N) = 0 \quad (4)$$

for all α and β in the interval $0 \leqslant \alpha, \beta \leqslant N$.

Condition (4) ensures the regularization of the $D^-(x; \mu; N)$, and also of the chronological packets $D^c(x; \mu; N)$ connected with them by the usual relation. For finite μ and N these packets will be continuously differentiable functions of x.

The scalar product (3) is positive semidefinite, and hence defines in H^R a seminorm, with respect to which H^R factors and assumes the structure of the usual Fock space H_F. Namely, every vector $f \in H^R$ gives rise to a vector $f^0 \in H_F$

$$f_n = \bigoplus_k f_{nk}(p_1, \ldots, p_n;\ \mu_1, \ldots, \mu_k; N) \to f_n^0 = \sum_k \lim_{N\to\infty}$$
$$\lim_{\mu_1\to 0} \ldots \lim_{\mu_k\to 0} f_{nk}(p_1, \ldots, p_n;\ \mu_1, \ldots, \mu_k; N). \quad (5)$$

We introduce into the space H the creation and annihilation operators. Let $\chi(p) \in O_M(V_+)$; then by definition

$$\varphi^+\{\chi\} f_{nk}(p_1, \ldots, p_n;\ \mu_1, \ldots, \mu_k; N) = \frac{1}{\sqrt{n+1}} \sum_{i=1}^{n+1} \chi(p_i) \times$$
$$\times f_{nk}(p_1, \ldots, \hat{p}_i, \ldots, p_{n+1};\ \mu_1, \ldots, \mu_k; N);$$
$$\varphi^-\{\chi\} f_{nk}(p_1, \ldots, p_n;\ \mu_1, \ldots, \mu_k; N) = \int dp\, \chi(p) \frac{\sqrt{n}}{k+1} \times$$
$$\times \sum_{j=1}^{k+1} \tilde{D}^-(p; \mu_j; N) f_{nk}(p_1, \ldots, p_{n-1}, p;\ \mu_1, \ldots, \hat{\mu}_j, \ldots, \mu_{k+1}; N).$$

Here the symbol ($\wedge$) denotes the absence of an argument. In the case where $\chi(p) \in \mathcal{S}(V_+)$, the operators $\varphi^{\pm}\{\chi\}$, in accordance with Eq. (5), generate in H_F the usual creation and annihilation operators $\varphi_0^{\pm}\{\chi\}$ for the free field.

Unlike in the space H_F, in H, if it is assumed that $\chi(p) = (2\pi)^{-3/2} \exp\{\pm ipx\}$, the creation and annihilation operators are correctly defined at a point. Accordingly, the usual formula for the T-product through the Θ function uniquely determines the product of the field operators, while the formulas

$$\hat{S}(q)=\sum_{n=0}^{\infty}\frac{i^n}{n!}\int dx_1\ldots dx_n q(x_1)\ldots q(x_n)\,T\{\mathcal{L}(x_1)\ldots\mathcal{L}(x_n)\},\tag{6}$$

$$\hat{S}^+(q)=\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int dx_1\ldots dx_n q(x_1)\ldots q(x_n)\,T^+\{\mathcal{L}(x_1)\ldots\mathcal{L}(x_n)\}\tag{7}$$

within the framework of perturbation theory properly define the operators in H for a wide class of functions q(x). In Eq. (7) the symbol T^+ denotes the antichronological product.

As it was shown in [5] that in the case $q(x)=g(x)\in\mathcal{S}(R_4)$ Eq. (6) defines an operator $\hat{S}(g)$ which is regular in H, i.e., an operator which leaves invariant the subspace H^R, while the operator $\hat{S}(g)$ in the Fock space H_F generated by S(g) after factorization of H^R coincides with the usual S-matrix, regularized according to Bogolyubov and Parasyuk [2].

The corresponding results of [5] consist of the following. After reduction to the normal form $\hat{S}(g)$ reduces to the sum of terms of the form

$$R((\mu);N)=\int dk_1\ldots dk_r dp_1\ldots dp_{\mathcal{S}}\,\varphi^+(k_1)\ldots\varphi^+(k_r)\times\\ \times\varphi^-(p_1)\ldots\varphi^-(p_{\mathcal{S}})\,F(k_1,\ldots,k_r;\,p_1,\ldots,p_{\mathcal{S}};\,(\mu)_I;\,N),\tag{8}$$

where F is the Feynman amplitude, regularized by means of the parameters μ and N, which corresponds to some diagram G, i.e., the Fourier image of the product of chronological packets $D^c(x_l-y_l;\,\mu_l;\,N)$ and basis functions $g(x_j)$.

In Eq. (8) the symbol (μ) denotes the whole set of parameters μ_i corresponding to the external lines of the diagram (implicitly contained in the operators $\varphi^{\pm}$ and the internal. The latter set is denoted by $(\mu)_I$. It was established in [5] that $F(k_1,\ldots,p_{\mathcal{S}};\,(\mu)_I;\,N)\in\mathcal{S}(V_+^{r+\mathcal{S}})$, whereby all the calculations can be made independent of $(\mu)_I$ and N. Furthermore,

$$\lim_{N\to\infty}\lim_{(\mu)\to0}F(k_1,\ldots,p_{\mathcal{S}};\,(\mu)_I;\,N)=F^0(k_1,\ldots,p_{\mathcal{S}}),$$

where F^0 is the usual Feynman amplitude, regularized by use of the R operation. Therefore, If we operate with R((μ);N) on some regular vector $f_{nk}(p_1,\ldots,p_n;\,\mu_1,\ldots,\mu_k;\,N)$, and take the limit over all μ and N, then in the obtained expression

$$\lim_{N\to\infty}\lim_{(\mu)\to0}\int dk_1\ldots dk_r dp_1\ldots dp_s\,\varphi^+(k_1)\ldots\varphi^+(k_r)\times\\ \times F(k_1,\ldots,p_s;\,(\mu)_I;\,N)\prod_{l=1}^{\mathcal{S}\leqslant n}\tilde{D}^-(p_l;\,\mu_l;\,N)\,f_{nk}(p_1,\ldots,p_n;\,\mu_1,\ldots,\mu_k;\,N),$$

all limits can be taken inside the integrals. One then obtains an expression which coincides completely with that which arises under the action of the S matrix, regularized according to Bogolyubov and Parasyuk [2], on the vector f_n^0 of the Fock space H_F.

An analogous assertion is also valid for $\hat{S}^+(g)$. We will hereinafter be dealing with products of several operators $\hat{S}(g_i)$ acting on vectors from H^R. It is easily shown that in this case the operator generated in H_F coincides with the corresponding product $S(g_i)$. Actually, we will operate, for example, with $\hat{S}(g_1)$ and $\hat{S}(g_2)$, where $g_1, g_2\in\mathcal{S}(R_4)$, on some vector $f_{nk}\in H^R$. After putting $\hat{S}(g_1)\hat{S}(g_2)$ into normal form and taking the limit with respect to all parameters μ_j and N, we arrive at a sum of expressions of the form

$$\lim_{N\to\infty}\lim_{(\mu)\to0}\int dk_1\ldots dk_r dk_1'\ldots dk_t' dp_1\ldots dp_{\mathcal{S}} dp_{\mathcal{S}+1}\ldots dp_n\times\\ \times\varphi^+(k_1)\ldots\varphi^+(k_r)\,F_1(k_1,\ldots,k_r,\,p_1,\ldots,p_{\mathcal{S}};\,(\mu)_I;\,N)\times\\ \times\prod_{l=1}^{L}[\tilde{D}^-(p_l;\,\mu_l;\,N)\,\delta(p_l-k_l')]\,\varphi^+(k_{L+1}')\ldots\varphi^+(k_t')\,F_2(k_1',\ldots\\ \ldots,k_t',\,p_{\mathcal{S}+1},\ldots,p_n)\prod_{l=L+1}^{n}\tilde{D}^-(p_l;\,\mu_l;\,N)\,f_{nk}(p_1,\ldots,p_n;\,(\mu);\,N).$$

Again using the properties of F_{G_1} and F_{G_2}, we can take all the limits under the integral signs, after which we arrive at the expression which arises under the action of $S(g_1)S(g_2)$ on vectors of the Fock space H_F. An analogous argument is valid for the multiplication of a large number of operators $\hat{S}(g_i)$, even if some of them are replaced by $\hat{S}^+(g)$.

Since in the space H the operator function $H\,T\{\mathcal{L}(x_1)\ldots\mathcal{L}(x_n)\}$ is continuous, bounded, and integrable, equations (6) and (7) are also correct for $q(x)=g^+(x)=\theta(x^0)g(x)$,

$$q(x)=g^-(x)=\theta(-x^0)g(x), \text{ where } g(x)\in\mathcal{S}(R_4).$$

In this case the integrals in Eqs. (6) and (7) can be understood to be improper Lebesgue integrals [6] over a measurable set with a characteristic function $\theta(x_1^0)\ldots\theta(x_n^0)$ from the summable function

$$g(x_1)\ldots g(x_n)\,T\{\mathcal{L}(x_1)\ldots\mathcal{L}(x_n)\}.$$

The operator $\hat{S}(g^-)$ and $\hat{S}(G^+)$ are "half" S matrices. We see that in the space H they can be defined completely properly. This is a big advantage of the space H over the usual Fock space. In distinction with the operator $\hat{S}(g)$, the operators $\hat{S}(g^\pm)$ do not leave an invariant subspace of H^R and therefore do not generate any operators in H_F. This is a reflection of the well-known fact that, due to the presence of surface divergences in Fock space, "half" S matrices do not exist.

We will now prove a number of assertions.

Assertion 1. In H it is true that:

$$\hat{S}(g^+)\hat{S}^+(g^+)=1, \tag{9}$$

$$\hat{S}(g^+)\,\hat{S}(g^-)=\hat{S}(g),\ \hat{S}^+(g^-)\,\hat{S}^+(g^+)=\hat{S}^+(g). \tag{10}$$

Proof. For proof of Eq. (9) it is sufficient to show that in H

$$\int\limits_{E_0} dx_1\ldots dx_n\; g(x_1)\ldots g(x_n)\sum_{m=0}^{n}\frac{(-1)^m}{m!\,(n-m)!}P(x_i)\times \tag{11}$$

$$\times\, T\{\mathcal{L}(x_1)\ldots\mathcal{L}(x_m)\}\,T^+\{\mathcal{L}(x_{m+1})\ldots\mathcal{L}(x_n)\}=0,$$

where the integration is over a measurable region E_0 with characteristic function $\theta(x_1^0)\ldots\theta(x_n^0)$. It is easy to show by induction that the expression under the integral sign in Eq. (11) is equal to zero at every point. Consequently, the Lebesgue integral of this function over the region E_0 is also zero.

For proof of Eq. (10) we will turn to Eqs. (6) and (7), written for $q(x) = g(x)$. In this case the integrals in them can be understood as improper Lebesgue integrals over all of the space R^{4n}. Because all the integrands are continuous summable functions, one can apply the theorem of Foubini on the reduction of a double integral to an interated one. Dividing the space R^{4n} into four measurable sets, corresponding to the expansion

$$1=\theta(x^0)\,\theta(y^0)+\theta(x^0)\,\theta(-y^0)+\theta(-x^0)\,\theta(y^0)+\theta(-x^0)\,\theta(-y^0),$$

we can in a trivial way reduce the left-hand sides of Eq. (19) to the right-hand sides.

Assertion 2. If $supp g_1 \gtrsim supp g_3$, then in H the following equation is valid:

$$\hat{S}(g_1+g_2+g_3)=\hat{S}(g_1+g_2)\,\hat{S}^+(g_2)\,\hat{S}(g_2+g_3). \tag{12}$$

Proof. We will assume that the plane $x^0 = 0$ divides the carriers g_1 and g_3. This limitation is not essential, since if $supp g_1 \gtrsim supp g_3$, then there exists a space-like hypersurface ∃ which divides $supp g_1$ and $supp g_3$. Then it is possible to define this hypersurface by the equation $x^0 = \eta(x)$, where η is a continuous function. In addition, in Eqs. (6) and (7) for $q(x) = g^\pm(x)$ we will replace the arguments $(x^0 - 0)$ of all the θ functions by $(x^0 - \eta(\mathbf{x}))$. It is seen that all our reasoning goes as before, since $\theta(x^0-\eta(\mathbf{x}))$ is a measurable function (η is a continuous function). We make the substitution $g_2(x) = g_2^+(x) + g_2^-(x)$, and then obtain by use of Assertion 1

$$\hat{S}(g_1+g_2)\hat{S}^+(g_2)\hat{S}(g_2+g_3) = \hat{S}(g_1+g_2^+)\hat{S}(g_2^-)\hat{S}^+(g_2^-)\hat{S}^+(g_2^+)\times$$
$$\times\hat{S}(g_2^+)\hat{S}(g_2^-+g_3) = \hat{S}(g_1+g_2^+)\hat{S}(g_2^-+g_3) = \hat{S}(g_1+g_2+g_3).$$

We now examine in H the operator $\hat{S}(g,\xi)$, which is obtained from $\hat{S}(q)$ by substituting $\mathscr{L}(x)$ for $\mathscr{L}_\xi(x)$, where $\xi(x)\in\mathscr{S}(R_4)$. Just as are the operators $\hat{S}(g)$ and $\hat{S}(g^\pm)$, the operators $\hat{S}(g,\xi)$, $\hat{S}(g^\pm,\xi)$ are well defined in H. Assertion 1 obviously remains in force if $\mathscr{L}(x)$ is replaced by $\mathscr{L}_\xi(x)$, and therefore,

$$\hat{S}(g,\xi) = \hat{S}(g^+,\xi)\hat{S}(g^-,\xi). \tag{13}$$

Because $\xi(x)$ is multiplied either by $\theta(x^0)$ or $\theta(-x^0)$, in the right-hand side of Eq. (13), $\hat{S}(g^\pm,\xi) = \hat{S}(g^\pm,\xi^\pm)$, and therefore,

$$\hat{S}(g,\xi) = \hat{S}(g^+,\xi^+)\hat{S}(g^-,\xi^-).$$

Reasoning by analogy with the proof of Eq. (12), we arrive at

$$\hat{S}(g_1+g_2+g_3,\ \xi_1+\xi_2+\xi_3) = \hat{S}(g_1+g_2,\ \xi_1+\xi_2)\times$$
$$\times\hat{S}^+(g_2,\ \xi_2)\hat{S}(g_2+g_3,\ \xi_2+\xi_3), \tag{14}$$

if $\operatorname{supp} g_1 \cup \operatorname{supp}\xi_1 \geqslant \operatorname{supp} g_3 \cup \operatorname{supp}\xi_3$. It is easy to also show that in H the relation

$$\hat{S}(g,\ \xi_1+\xi_2+\xi_3) = \hat{S}(g,\ \xi_1+\xi_2)\hat{S}^+(g,\ \xi_2)\hat{S}(g,\ \xi_2+\xi_3) \tag{15}$$

is satisfied if $\operatorname{supp}\xi_1 \geqslant \operatorname{supp}\xi_3$.

We will now examine operators in H of the following form:

$$\hat{S}(g,\xi,h) = T\exp\left\{i\int dx\,[g(x)\mathscr{L}_\xi(x) + h(x)\varphi(x)]\right\}. \tag{16}$$

It is convenient for analysis to introduce the regularized retarded and advanced functions $D^r(x;\mu;N)$ and $D^a(x;\mu;N)$, which are connected with $D^\pm(x;\mu;N)$ and $D^c(x;\mu;N)$ by the usual relations. Using the properties of the carriers D^r and D^a one can readily show that the relation

$$\hat{S}(g,\xi,h) = \hat{S}(g,\ \xi+h^a,\ 0)\hat{S}(0,0,h) = \hat{S}(0,0,h)\hat{S}(g,\ \xi+h^r,\ 0), \tag{17}$$

is satisfied, where

$$h^{a,r}(x) = \int dy\,D^{a,r}(x-y;\mu;N)\,h(y).$$

We note that $h^a(x)$ and $h^r(x)$ do not belong to $\mathscr{S}(R_4)$ although they are infinitely differentiable and polynomially bounded functions. Because in determining $\hat{S}(g,\xi+h^{a,r})$ by Eq. (16) each of the functions $h^a(x)$ and $h^r(x)$ are multiplied by $g(x)\in\mathscr{S}(R_4)$, these operators are well defined on H.

Assertion 3. The following is valid in H:

$$\hat{S}(g,\xi,h) = \hat{S}(g^+,\xi^+,h^+)\hat{S}(g^-,\xi^-,h^-). \tag{18}$$

Proof. We note that Eq. (17) remains in force if $(g,\ \xi,\ h)$ is replaced by $(g^+,\ \xi^+,\ h^+)$ or by $(g^-,\ \xi^-,\ h^-)$; therefore, the right-hand side of Eq. (18) is equal to

$$\hat{S}(0,0,h^+)\hat{S}(g^+,\ \xi^++h^{+r},\ 0)\hat{S}(g^-,\ \xi^-+h^{-a},\ 0)\hat{S}(0,0,h^-). \tag{19}$$

Since $h^{+r}(x)=0$ for for $h^{-a}(x)=0$ for $x^0>0$ and

$$\operatorname{supp} g^+ \cup \operatorname{supp}\xi^+ \cup \operatorname{supp} h^{+r} \geqslant \operatorname{supp} g^- \cup \operatorname{supp}\xi^- \cup \operatorname{supp} h^{-a},$$

it follows from Eq. (14) that the right-hand side of Eq. (18) is equal to

$$\hat{S}(0, 0, h^{+})\, \hat{S}(g^{+} + g^{-}, \xi^{+} + \xi^{-} + h^{+\prime} + h^{-a}, 0)\, \hat{S}(0, 0, h^{-}),$$

from which by virtue of Eq. (17), we obtain Eq. (18).

Assertion 4. If $\mathrm{supp}\, g_1 \cup \mathrm{supp}\, \xi_1 \cup \mathrm{supp}\, h_1 \geqslant \mathrm{supp}\, g_3 \cup \mathrm{supp}\, \xi_3 \cup \mathrm{supp}\, h_3$, then the following is true in H:

$$\hat{S}(g_1 + g_2 + g_3, \xi_1 + \xi_2 + \xi_3, h_1 + h_2 + h_3) = \hat{S}(g_1 + g_2, \xi_1 + \xi_2, h_1 + h_2) \times \times \hat{S}^{+}(g_2, \xi_2, h_2)\, \hat{S}(g_2 + g_3, \xi_2 + \xi_3, h_2 + h_3). \quad (20)$$

Proof. We will write $g_2(x) = g_2^{+}(x) + g_2^{-}(x)$, and write $\xi(x)$ and $h(x)$ analogously. Then Eq. (20) is obtained from Eq. (18) by arguments similar to those in the proof of Eq. (14).

Assertions 2 and 4 and Eqs. (14) and (15) were obtained for operators $\hat{S}$ on H. However, as we have already said, in the case that $f_i \in \mathscr{S}(R_4)$, the operators $\hat{S}(f_1)$ generate operators $S(f_1)$ on the Fock space H_F whose algebraic properties are the same as those of $\hat{S}$. Therefore, assertions 2 and 4 and Eqs. (14) and (15) are valid also for the operators $S(f_1)$. We have thus completed the proof of the integral condition of causality (2).

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