A MULTILAYER INTERFERENCE ABSORBER WITH A WORKING LAYER OF ARBITRARY THICKNESS

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The conditions for maximum energy absorption, and equations for the selectivity of a multilayer interference absorber with a working layer of arbitrary thickness are obtained. It is shown that to synthesize a multilayer interference absorber with a specified selectivity (absorption band) one must choose the multilayer dielectric mirrors which surround the working layer with the required dispersion of the phase of the reflection coefficient, or vary the thickness of the working layer.

It was suggested in [1, 2] that a multilayer interference absorber could be used as the sensitive element in wave-energy receivers (for example, in optical receivers). As shown in [2] in a multilayer interference absorber, using a thin layer of weakly absorbing material, which under normal circumstances is practically transparent to radiation, one can achieve practically complete absorption due to resonance effects. In [1, 2] a multilayer interference absorber was considered in which the absorbing working layer had an optical thickness (the product of the real part of the refractive index and the layer thickness) of $\lambda_0/2$, where λ_0 is the wavelength of the radiation used. In [2] equations were obtained which enable one to choose the number of layers of the absorber and the layer material giving an absorption of ~100%. As can be seen from a graph of the spectral dependence of the absorption coefficient given in [3], the multilayer interference absorber possesses pronounced selective properties: the absorption is high over a narrow spectral range close to the wavelength λ_0. Henceforth a multilayer interference absorber designed to receive radiation of one definite wavelength will be called a single-frequency absorber.

In this paper we consider the conditions for constructing a single-frequency multilayer interference absorber in which the absorbing working layer with complex refractive index $n + i\varkappa$ has an arbitrary thickness d. Equations will be obtained for the selectivity, i.e., the ratio of the wavelength of the received radiation λ_0 to the half-width of the absorption line $\Delta\lambda$ for the absorber.

Figure 1 shows a schematic diagram of a multilayer interference absorber with a central working layer of refractive index $n + i\varkappa$, surrounded by two layered media 1 and 2. The received radiation is a plane wave propagating from a nonuniform nonabsorbing medium of refractive index n_l in the direction of the z axis (the wave vector is normal to the surface of the absorber). When traveling through the absorber the wave propagates in a uniform nonabsorbing medium of refractive index n_r. We will use the following notation: t_1 and r_1 are the amplitude transmission and reflection coefficients for medium 1 in the direction of the z axis, and t_1' and r_1' are the same coefficients for the wave propagating in the opposite direction. The coefficients with subscripts 2 relate to the second medium. In the general case media 1 and 2 may be absorbing so that all the reflection and transmission coefficients calculated ignoring absorption will be denoted by the additional index (0), for example $t_1(0)$, $r_2(0)$, etc.

As in [1, 2], the imaginary part of the refractive index of the working layer $\varkappa$ is a small parameter, with respect to which the expansion will be carried out. The equations we require are Eqs. (3) of [2], which express the amplitude transmission coefficient t and reflection coefficient r of the whole absorber in terms of the coefficients of medium 1 and medium 2 and the parameters of the working layer:

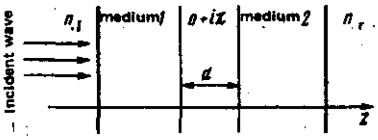

Fig. 1. Sketch of the multilayer interference absorber of wave energy.

$$\frac{1}{t}=\frac{1}{t_1\cdot t_2}\left[e^{-i\widehat{\alpha}}-e^{i\widehat{\alpha}}r_1'r_2+\frac{\varkappa}{n}\sin\alpha\,(r_2+r_1')+O(\varkappa)\right],$$

$$\frac{r}{t}=\frac{1}{t_1^{+}\cdot t_2}\left[r_2e^{i\widehat{\alpha}}-\frac{\varkappa}{n}\sin\alpha+O(\varkappa)\right]+$$

$$\frac{r_1}{t_1t_2}\left[e^{i\widehat{\alpha}}+r_2\frac{\varkappa}{n}\sin\alpha+O(\varkappa)\right], \tag{1}$$

where

$$t_1^{+}=t_1/(t_1t_1'-r_1r_1'),\ \widehat{\alpha}=\frac{2\pi}{\lambda}(n+i\varkappa),\quad d=\alpha+i\beta,$$

$\alpha=\frac{2\pi}{\lambda}nd;\ \beta=\frac{\varkappa}{n}\alpha$, and λ is the wavelength of the incident radiation.

We will assume that the amplitude coefficients of media 1 and 2 differ from those calculated ignoring absorption by a quantity of the order of $\varkappa$

$$\begin{aligned}
t_1&=t_{1(0)}[1-c_1(\varkappa)+O(\varkappa)],\\
t_1'&=t_{1(0)}[1-c_1'(\varkappa)+O(\varkappa)],\\
r_1&=r_{1(0)}[1-p_1(\varkappa)+O(\varkappa)],\\
r_1'&=r_{1(0)}'[1-p_1'(\varkappa)+O(\varkappa)],\\
t_2&=t_{2(0)}[1-c_2(\varkappa)+O(\varkappa)],\\
r_2&=r_{2(0)}[1-p_2(\varkappa)+O(\varkappa)].
\end{aligned} \tag{2}$$

Here $c_1(\varkappa),\ c_1'(\varkappa)$, etc., are quantities which have an order of smallness not less than $\varkappa$.

These assumptions hold, for example, if we take as media 1 and 2 quarter-wave mirrors, as considered in [3].

All the quantities in (2) are, of course, functions of the wavelength λ.

We will also introduce the following notation. Let φ_r and φ_t be the phase shifts on reflection and transmission ignoring absorption

$$t_{1(0)}=|t_{1(0)}|e^{i\varphi_t},\ r_{1(0)}=|r_{1(0)}|e^{i\varphi_{r_1}},\ r_{1(0)}'=|r_{1(0)}'|e^{i\varphi_{r_1}'},\ r_{2(0)}=|r_{2(0)}|e^{i\varphi_{r_2}}.$$

The capital letters T and R with corresponding indices will denote the energy transmission and reflection coefficients

$$T_1=\frac{n}{n_1}|t_1|^2,\ T_{1(0)}=\frac{n}{n_1}|t_{1(0)}|^2,\ T_2=\frac{n_r}{n}|t_2|^2,$$

$R_{1(0)}'=|r_{1(0)}'|^2$, etc.

For the power coefficients and phase shifts in nonabsorbing media we have the following relations (see, for example [4]):

$$T_{1(0)}=t_{1(0)}t_{1(0)}',\ R_{1(0)}'=R_{1(0)},\ \varphi_t'=\varphi_t;\ \varphi_2=2\varphi_t+\pi-\varphi_r. \tag{3}$$

We will assume that $T_{1(0)}=O(\varkappa)$ and $T_{2(0)}=O(\varkappa)$, i.e., that the power transmission coefficients of media 1 and 2 are of order of smallness $\varkappa$.

Then, using (3), we have apart from terms $O(\varkappa)$

$$t_1t_1'=T_{1(0)}e^{i2\varphi_t},\ r_1r_1'=R_{1(0)}e^{i(\varphi_{r_1}+\varphi_{r_1}')}\cdot[1-C(\varkappa)], \tag{4}$$

where $C(\varkappa)=p_1(\varkappa)+p_1'(\varkappa)$.

Hence,

$$\frac{t_1}{t_1^{+}}=t_1t_1'-r_1r_1'=e^{i2\varphi_{t_1}}[T_{1(0)}+R_{1(0)}-R_{1(0)}C(\varkappa)+O(\varkappa)]=$$

$$=e^{i2\varphi_{t_1}}[1-C(\varkappa)+O(\varkappa)]. \tag{5}$$

We will now transform the first equation of (1), using (2), the equations $e^{i\widehat{\alpha}} = e^{i\alpha}[1 - \beta + O(\varkappa)]$ and $e^{-i\widehat{\alpha}} = e^{i\alpha}[1 + \beta + O(\varkappa)]$ and the assumptions regarding the order of smallness of $T_{1(0)}$ and $T_{2(0)}$, in view of which $R_{1(0)} = 1 + O(\varkappa)$ and $R_{2(0)} = 1 + O(\varkappa)$:

$$\frac{1}{t} = \frac{1}{t_1 t_2}\Big\{e^{i\alpha} - e^{i\alpha} r'_{1(0)} r_{2(0)} [1 - p'_1(\varkappa)][1 - p_2(\varkappa)] + \beta (e^{-i\alpha} + e^{i\alpha} r'_{1(0)} r_{2(0)}) + \frac{\varkappa}{n} \sin\alpha (r_{2(0)} + r'_{1(0)}) + O(\varkappa)\Big\} =$$

$$= \frac{1}{t_1 t_2}\Big\{e^{-i\alpha} - \sqrt{R_{1(0)} R_{2(0)}}\, e^{i(\alpha + \varphi'_{r_1} + \varphi_{r_2})} + e^{i(\alpha + \varphi'_{r_1} + \varphi_{r_2})}[p'_1(\varkappa) + p_2(\varkappa)] + \beta [e^{-i\alpha} + e^{i(\alpha + \varphi'_{r_1} + \varphi_{r_2})}] + \frac{\varkappa}{n} \sin\alpha (e^{i\varphi_{r_2}} + e^{i\varphi'_{r_1}}) + O(\varkappa)\Big\}.$$

We will put

$$\Delta\varphi = 2\alpha + \varphi'_{r_1} + \varphi_{r_2} \pmod{2\pi} \tag{6}$$

and require that the phase shifts on reflection are the same and that $\Delta\varphi$ is a quantity of the order of $\varkappa$: $\Delta\varphi = O(\varkappa)$.

Then

$$e^{i(\alpha + \varphi'_{r_1} + \varphi_{r_2})} = e^{-i\alpha} e^{i\Delta\varphi} = e^{-i\alpha} + O(\varkappa),$$

$$e^{-i\alpha} - \sqrt{R_{1(0)} R_{2(0)}}\, e^{i(\alpha + \varphi'_{r_1} + \varphi_{r_2})} = e^{-i\alpha}[1 - \sqrt{R_{1(0)} R_{2(0)}} - i\Delta\varphi + O(\varkappa)] =$$
$$= e^{-i\alpha}[e^{i(\alpha + \varphi_{r_2})} + e^{i(\Delta\varphi - \varphi_{r_2} - \alpha)}] = e^{-i\alpha}\, 2\cos(\alpha + \varphi_{r_2}) + O(\varkappa)$$

and the expression for 1/t takes the form

$$\frac{1}{t} = \frac{e^{-i\alpha}}{t_1 t_2}\Big[\frac{1}{2}(T_{1(0)} + T_{2(0)}) - i\,\Delta\varphi + p'_1(\varkappa) + p_2(\varkappa) + 2\beta + 2\frac{\varkappa}{n} \sin\alpha \cos(\alpha + \varphi_{r_2}) + O(\varkappa)\Big], \tag{7}$$

where

$$a(\varkappa) = p'_1(\varkappa) + p_2(\varkappa) + 2\beta + 2\frac{\varkappa}{n} \sin\alpha \cos(\alpha + \varphi_{r_2}). \tag{8}$$

Carrying out similar transformations taking (3)-(5) into account we obtain for the second equation of (1)

$$\frac{r}{t} = \frac{e^{i(2\varphi_f + \alpha + \varphi_{r_2})}}{t_1 t_2}\Big[\frac{1}{2}(T_{1(0)} - T_{2(0)}) + i\Delta\varphi - a(\varkappa)\Big]. \tag{9}$$

We will now change from Eq. (7) for the amplitude transmission coefficient of the absorber to an expression for the power coefficient

$$T = \frac{n_{r_1}}{n_t} |t|^2:$$

$$\frac{1}{T} = \frac{1}{T_1 T_2}\Big\{\Big[\frac{1}{2}(T_{1(0)} + T_{2(0)}) + \operatorname{Re} a(\varkappa)\Big]^2 + [\Delta\varphi - \operatorname{Im} a(\varkappa)]^2 + O(\varkappa^3)\Big\}.$$

whence, since $T_1 T_2 = T_{1(0)} T_{2(0)} + O(\varkappa^2)$, we obtain the following expression for T apart from a function which is infinitely small compared with $\varkappa$:

$$T = \frac{4 T_{1(0)} T_{2(0)}}{[T_{1(0)} + T_{2(0)} + 2 \operatorname{Re} a(\varkappa)]^2 + 4[\Delta\varphi - \operatorname{Im} a(\varkappa)]^2} = \frac{4k}{[1 + k + x]^2 + 4y^2}, \tag{10}$$

where

$$k = \frac{T_{2(0)}}{T_{1(0)}};\quad x = \frac{2 \operatorname{Re} a(\varkappa)}{T_{1(0)}};\quad y = \frac{\Delta\varphi - \operatorname{Im} a(\varkappa)}{T_{1(0)}}. \tag{11}$$

It follows from (9), apart from a function which is infinitely small compared with $\varkappa$ that

$$\frac{R}{T}=\frac{1}{T_{1(0)}T_{2(0)}}\left\{\frac{1}{2}\left[T_{1(0)}-T_{2(0)}-\operatorname{Re}a(\varkappa)\right]^2+\left[\Delta\varphi-\operatorname{Im}a(\varkappa)\right]^2\right\},$$

whence R, using the notation introduced in (11), is given by

$$R=\frac{[1-k-x]^2+4y^2}{[1+k+x]^2+4y^2}. \tag{12}$$

Equations (10) and (12) give the following expression for the absorption A = 1 - R - T:

$$A=\frac{4x}{[1+k+x]^2+4y^2}. \tag{13}$$

Relations (10), (12) and (13) differ from those obtained in [2, 3] in the presence of the additional terms with $y=\frac{\Delta\varphi-\operatorname{Im}a(\varkappa)}{T_{1(0)}}$, which take into account the change in the phase shifts when the wavelength λ changes, and enable one to consider the problem of the shape and half-width of the absorber absorption line. In [2, 3] the expressions for R, T, and A were obtained for a fixed wavelength λ_0 with additional assumptions for which y = 0.

As follows from (13) maximum absorption $A_{max}=\frac{1}{1+k}$ is achieved when y = 0 and x = 1 + + k. The wavelength λ_0 for which y = 0 obviously corresponds to the center of the absorption band.

Note that the expression for x

$$x=\frac{2\operatorname{Re}a(\varkappa)}{T_{1(0)}}=2\,\frac{\operatorname{Re}[p_1'(\varkappa)+p_2(\varkappa)]+2\frac{\varkappa}{n}[\alpha+\sin\alpha\cos(\alpha+\varphi_{r_1})]}{T_{1(0)}} \tag{14}$$

also differs from the expressions obtained in [2, 3] in the assumption that the thickness of the working layer satisfies the condition nd = $\lambda_0/2$ where λ_0 is the working wavelength of the absorber, and enables absorbers with arbitrary working layer thicknesses to be considered.

It was shown in [1, 2] that it is best to use quarter-wave mirrors as media 1 and 2 surrounding the working layers (i.e., systems of alternating layers of refractive index n_1 and n_2, in which the optical thickness of each layer, that is, the product of its thickness and the refractive index, is equal to $\lambda_0/4$, where λ_0 is the working wavelength of the absorber). It is easy to show that in the region of the reflection plateau of such mirrirs the functions $p_1'(\varkappa)$ and $p_2(\varkappa)$ are real. Their form can be obtained using the method described in [3], where expressions are given for $p_1'(\varkappa)$ and $p_2(\varkappa)$ for one type of quarter-wave mirror.

Thus, for a broad class of systems $y=\frac{\Delta\varphi}{T_{1(0)}}$, and the first of the conditions for obtaining an absorption maximum at a given wavelength λ_0 - y = 0 takes the form $\Delta\varphi=0$, or from (6) and the expression for α

$$\frac{2\pi nd}{\lambda_0}+\varphi_{r_1}'(\lambda_0)+\varphi_{r_2}(\lambda_0)=O\,(\mathrm{mod}\;2\pi). \tag{15}$$

We will consider the expressions obtained for T, R, and A in a small neighborhood of λ_0 of the order of $\varkappa$, i.e., for wavelengths λ satisfying the condition $\delta\lambda=\lambda-\lambda_0=O(\varkappa)$. We will assume that in this case the relative variations in $T_{1(0)}$, $T_{2(0)}$, $p_1'(\varkappa)$ and $p_2(\varkappa)$ are also of the order of smallness $\varkappa$. This assumption is satisfied, for example, for all quarter-wave mirrors in the region of the reflection plateau. Hence, it can be assumed that $a(\varkappa)$ occurs in all the expressions with wavelength λ_0, since, as follows from (8), $a(\varkappa)=O(\varkappa)$.

It can also be assumed that $T_{1(0)}$ and $T_{2(0)}$ are taken at the wavelength λ_0. In view of this everywhere from now on in expressions (8) for $a(\varkappa)$ and (14) for x instead of α we will write $\alpha_0=\frac{2\pi}{\lambda_0}nd$. Then the second condition, written in a more convenient form for designing the absorber takes the form

$$T_{1(0)} = \frac{1}{1+k}\left\{2\left[p_1'(\varkappa) + p_2(\varkappa)\right] + 4\,\frac{\varkappa}{n}\left[\alpha_0 + \cos(\alpha_0 + \varphi_{r_2})\sin\alpha_0\right]\right\}, \tag{16}$$

where all the quantities are calculated at a wavelength λ_0.

This expression agrees with the expressions obtained in [1-3] for the cases considered in [1-3], and enables the required number of layers to be calculated and the materials of media 1 and 2 to be chosen (for more detail see [2, 3]).

We will now consider the problem of the selectivity of the absorption band of the filter.

As follows from the above comments, we can assume that the expression for x is independent of the wavelength λ. We then obtain from (13) that the half-width of the absorption line $\Delta\lambda=\lambda_1-\lambda_2$ can be found from the condition $2y(\lambda_1)=1+k+x$, or, when the condition for maximum absorption x = 1 + k is satisfied, from the condition $y(\lambda_1)=1+k$.

For $\operatorname{Im} a(\varkappa)=0$, $y=\frac{\Delta\varphi}{T_{1(0)}}$. From (6) and (15) we have

$$\Delta\varphi(\lambda) = \left(2\frac{d\alpha}{d\lambda} + \frac{d\varphi'_{r_1}}{d\lambda} + \frac{d\varphi_{r_2}}{d\lambda}\right)\Bigg|_{(\lambda=\lambda_0)}(\lambda-\lambda_0) + O(\lambda-\lambda_0) =$$

$$= \left(-2\frac{\alpha_0}{\lambda_0} + \frac{d\varphi'_{r_1}}{d\lambda}\Bigg|_{\lambda=\lambda_0} + \frac{d\varphi_{r_2}}{d\lambda}\Bigg|_{\lambda=\lambda_0}\right)(\lambda-\lambda_0) + O(\varkappa).$$

Taking (16) into account, we obtain the following expression for the half-width of the absorption line $\Delta\lambda$:

$$\operatorname{mod}\left(-\frac{2\alpha_0}{\lambda_0} + \frac{d\varphi'_{r_1}}{d\lambda}\Bigg|_{\lambda=\lambda_0} + \frac{d\varphi_{r_2}}{d\lambda}\Bigg|_{\lambda=\lambda_0}\right)\Delta\lambda =$$
$$= 2\left[p_1'(\varkappa) + p_2(\varkappa)\right] + 4\frac{\varkappa}{n}\left[\alpha_0 + \sin\alpha_0\cos(\alpha_0+\varphi_{r_2})\right]. \tag{17}$$

If media 1 and 2 are quarter-wave mirrors, the derivative of the phase shifts with respect to λ can be obtained in a similar manner to that used to obtain the expressions for $p_1'(\varkappa)$ and $p_2(\varkappa)$ in [3], i.e., we first find the derivative of the characteristic matrix with respect to λ, which for quarter-wave mirrors can be expressed in terms of a Chebyshev polynomial, and then the amplitude reflection coefficient is expressed in terms of the elements of this matrix and the derivative of the phase is calculated. For example, for the quarter-wave mirrors considered in [3], at the central wavelength of the mirror λ_0 we have

$$\frac{d\varphi'_{r_1}}{d\lambda} = \frac{d\varphi_{r_2}}{d\lambda} = -\frac{\pi}{\lambda_0}\,\frac{n_1 n_2}{n(n_1-n_2)}. \tag{18}$$

As an example of the use of Eq. (17) we will calculate the half-width of the absorption line of the multilayer interference absorber considered in [1], and compare the value obtained with the value obtained from a graph of A against λ for this absorber (see [1]), calculated on a computer using the accurate equations. For the absorber considered in [1] $\alpha_0=\pi$, $p_1'(\varkappa)=p_2(\varkappa)=0$, and Eq. (17), using (18), takes the form

$$\frac{\Delta\lambda}{\lambda_0}\left[1 + \frac{n_1 n_2}{n(n_1-n_2)}\right] = \frac{\varkappa}{n}. \tag{19}$$

Equation (19) gives the following value for the selectivity $\lambda_0/\Delta\lambda$ for the values of n_1, n_2, n, χ, λ_0 obtained in [1]:

$$\frac{\lambda_0}{\Delta\lambda} = 3{,}15\cdot 10^3.$$

From the graph of $A(\lambda)$ $\frac{\lambda_0}{\Delta\lambda}\cong 3\cdot 10^3$. Hence, the relations obtained enable the selectivity of the absorption band to be calculated fairly accurately.

Equation (17) can also be used to synthesize a multilayer interference absorber with a specified selectivity: to obtain the desired value of $\lambda_0/\Delta\lambda$, as is seen from (17), we can choose mirrors with different derivatives of the phase shifts on reflection, or vary the value of α_0, i.e., the thickness of the absorbing layer d $\left(\alpha_0 = \frac{2\pi}{\lambda_0} nd\right)$.

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