# STATISTICAL PROPERTIES OF THE QUADRATURE COMPONENTS OF AN IONOSPHERIC SIGNAL

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When a quasi-monochromatic radio signal is reflected from the ionosphere, it acquires amplitude modulation R(t) and phase modulation Φ(t), described by the modulation vector (the modulation function) U

$$E(t) = U(t)\,e^{-i\omega_0 t} = R(t)\,e^{i\Phi(t)-i\omega_0 t} = [E_c(t) + iE_s(t)]\,e^{-i\omega_0 t}, \tag{1}$$

where $E_c(t)=R(t)\cos\Phi(t)$, $E_s(t)=R(t)\sin\Phi(t)$ are the Cartesian components (the quadrature components) in the Van der Pol plane rotating around the origin or coordinates with velocity $\omega_0$. The origin of the modulation is the fluctuating nature of the ionospheric plasma, and its variability in space and time.

To solve the converse problem, i.e., to determine the properties of the ionosphere itself from the behavior of R and Φ, methods involving the observation of the envelope R (the amplitude method) or the phase Φ (the phase method) are usually employed. Knowing the processes $E_c$, and $E_s$ one can uniquely determine both R and Φ

$$R(t) = \sqrt{E_c(t)^2 + E_s(t)^2} \text{ and } \Phi(t) = \operatorname{arctg}\frac{E_c(t)}{E_s(t)}. \tag{2}$$

In the theory of ionospheric investigations R and Φ are found using the components $E_c$ and $E_s$, connected by a Hilbert transformation [1]

$$E_s(t) = \frac{1}{\pi}\lim_{T\to\infty}\int_{-T}^{T}\frac{E_c(\tau)}{t-\tau}\,d\tau. \tag{3}$$

In this case $E_c$ and $E_s$ must possess the following properties. The correlation coefficients $\rho_c(\tau)$ and $\rho_s(\tau)$ of the processes $E_c$ and $E_s$ must be equal. The cross correlation coefficient $\rho_{cs}(\tau)$ must be an odd function, equal to zero when $\tau = 0$. The variances $\sigma_c^2$, $\sigma_s^2$ of the components $E_c$ and $E_s$ must be equal. The values of $E_c$ and $E_s$ must not exceed the envelope of R.

No complete experimental data exists at the moment on the properties of both quadrature components together, and on the correctness of the use of transformation (3) in the ionospheric case. This makes it of interest to design appropriate apparatus and to make an experimental study of the properties of the component $E_c$ and $E_s$.

An experimental investigation was made using the original ionospheric coherent reception apparatus, designed in the Physics Faculty, for recording $E_c(t)$ and $E_s(t)$ simultaneously. In Fig. 1a we show specimens of the recordings of $E_c$ and $E_s$. On the same record the envelope $R_e(T)$ is recorded, obtained by the usual radio-physical (power) method, unlike relation (2). More detailed information on the method employed can be found in [1-7].

We observed a single signal (2.5-7.5 mHz) reflected from the F2 layer, with vertical probing. Only 100 sessions of duration T = 3-4 min each were subjected to statistical processing and analysis. The value of T determined the sufficiency of the sample volume.

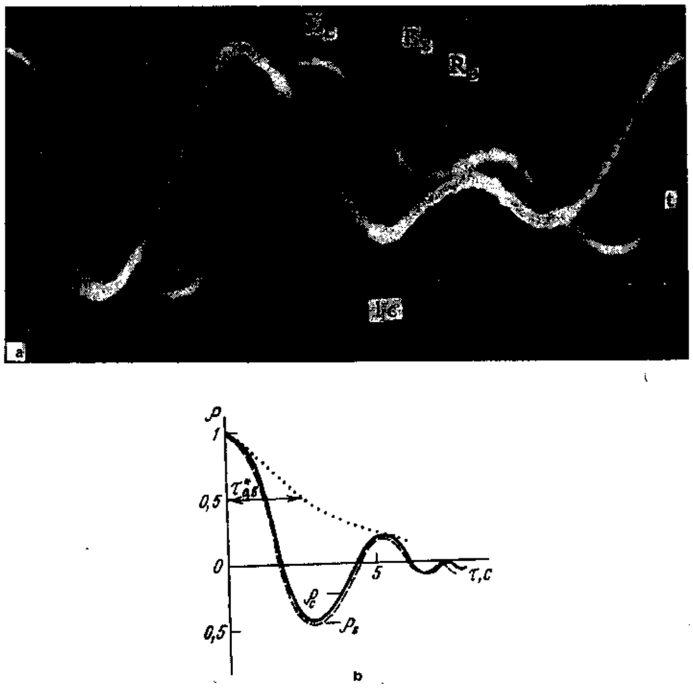


Fig. 1

Using a computer, we determined for all the sessions the autocorrelation coefficients ($\rho_c$, $\rho_s$) and the cross correlation coefficient ($\rho_{cs}$), and the variances $\sigma_c$, $\sigma_s$.

An analysis of the material obtained enabled us to draw the following conclusions regarding the properties of $E_c(t)$ and $E_s(t)$.

For all the sessions the coefficient $\rho_c(\tau)=\rho_s(\tau)$ was found with an accuracy exceeding the statistical error. The statistical error was found as described in [4, 5]. Figure 1b shows a typical example of the correlation coefficient of $E_c$ and $E_s$.

The calculated coefficients $\rho_{cs}(\tau)$ were odd, while the deviation $\rho_{cs}(0)$ from the expected zero value was within the limits of statistical error, i.e., at coincident instants of time $E_c(t)$ and $E_s(t)$ were uncorrelated.

In 95% of the cases the variances $\sigma_c^2$ and $\sigma_s^2$ agreed within the limits of statistical error. This confirms the uniformity of the phase distribution in the interval $[-\pi, \pi]$, in good agreement with the data obtained in our next paper that will be published in this journal. In this case the statistical error was calculated as described in [6]. Finally, the experimental check showed that the components $E_c$ and $E_s$ in not less than 95% of the cases possessed Hilbert-conjugate properties.

The analysis also showed that the envelope $R(t)$, reconstructed from $E_c$ and $E_s$, according to expression (2), and the envelope $R_e(t)$ obtained by the usual radio-physical method, agree with a high degree of accuracy (an error of $\leq 5\%$). The reconstruction of the envelope of R with respect to $E_c$ and $E_s$ can be used to construct a more noise-immune method of distinguishing the envelope.

This method of investigating an ionospheric signal from the quadrature components $E_c$ and $E_s$ possesses some advantages compared with the purely amplitude or phase method, because the records of $E_c(t)$ and $E_s(t)$ (6.1) contain complete information on the variations of both $R(t)$ and $\Phi(t)$.

It is not difficult to extract this information using a computer.

## REFERENCES


1. B. R. Levin, Theoretical Principles of Statistical Radio Engineering, Pt. 1 [in Russian], Moscow, 1969.
2. S. F. Mirkotan and I. A. Biryulin, Ionos. Issled., no. 9, p. 18, 1961.
3. S. F. Mirkotan, A. G. Vologdin and V. A. Smorodinov, Radiotekh. i Elektron., vol. 23, no. 3, 1978.
4. S. F. Mirkotan, Vest. Mosk. Un. Ser. fiz., no. 1, p. 151, 1956.
5. S. F. Mirkotan and A. G. Vologdin, Geomag. i. Aeronom., no. 4, p. 640, 1974.
6. N. Arlei and K. Bukh, Introduction to Probability Theory and Mathematical Statistics [in Russian], Moscow, 1951.
7. A. G. Vologdin, Candidate Dissertation, Moscow, 1973.


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