INVESTIGATION OF THE PROBABILITY PROPERTIES OF THE PHASE OF AN IONOSPHERIC SIGNAL

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When studying the turbulent structure of an ionospheric plasma using radio-probing methods, one starts by assuming a certain mechanism for the scattering of radio waves by nonuniformities. This mechanism determines the fluctuation properties of the parameters of the reflected signals. The signal phase is one such parameter. The phase variations due to large-scale ionospheric nonuniformities have been investigated in detail [1]. There are very few publications in which the fast phase variations are investigated and in these the action of the large-scale nonuniformities were neutralized by the use of certain filtering operations.

In this paper we present some fundamental results of an investigation of the probability properties of the total phase $\Phi(t)$ of an ionospheric signal referred to the interval $[-\pi, \pi]$. Unlike [2] no filtering operations were applied to $\Phi(t)$, i.e., we investigated the behavior of $\Phi(t)$ as a result of the combined action of different factors which is emphasized by the term "total."

The ionospheric signal is a narrow-band random process modulated in amplitude $R(t)$ and phase $\Phi(t)$

$$E(t) = E_0(t) + E_p(t) = A_0 \cos(\omega_0 t - \varphi_0) + r(t) \cos[\omega_0 t - \varphi(t)] = R(t) \cos[\omega_0 t - \Phi(t)], \quad (1)$$

where $E_0(t)$, $E_p(t)$ are the specular and scattered components, $E_p(t)$ is a normal random process with $\overline{E_p(t)} = 0$ and $\sigma^2 = \overline{E_p^2}$.

The one-dimensional distributions $W(\Phi)$ and $W(z)$, $z = \cos\Phi$ for the case when $A_0 =$ = const, and $\varphi_0 = \text{const}$ (the Rice model) are given in [3] (expressions (8.59), (8.87)). In the Rice model a characteristic feature for $W(\Phi)$ is the nonuniformity, and for $W(z)$ the characteristic feature is the asymmetry.

For an actual ionosphere the condition $\varphi_0 = \text{const}$ breaks down; for observation intervals of length $T \sim 3$-4 min

$$\varphi_0(t) = \Omega t + \Psi(t), \quad (2)$$

where $\Omega \approx$ const, and $\Psi(t)$ is a stationary random process with a uniform distribution. Case (2) is called the μ-model [4]. Usually for the above-mentioned intervals T the phase $\varphi(t)$ and $\Phi(t)$ is also the sum of the term $\Omega \cdot t$ and a certain random process, i.e., a displacement of the frequency spectrum of the signal by Ω occurs in the ionosphere [5] with a mean value $\Omega/2\pi = 0.2$-0.3 Hz. Expressions (3)-(5) also hold in this special case of the μ-model (see below). The appearance of the term Ωt is due to dynamic large-scale ionospheric nonuniformities and of the ionospheric layer as a whole [5]. For observation intervals T considerably greater than 3-4 min, $\Omega = \Omega(t)$ is a random function of time with a quasi-period of ~20-30 min. Taking relation (2) into account and assuming the components $E_0(t)$ and $E_p(t)$ to be independent, we can obtain (using the procedure described in [3]) the joint distribution $W(R, \Phi, \varphi_0)$. Integrating it over φ_0, and then over R and Φ one can obtain the one-dimensional distribution for the μ-model

$$W(\Phi) = \frac{1}{2\pi}, \quad |\Phi| < \pi, \quad (3)$$

$$W(z) = \frac{1}{\pi\sqrt{1-z^2}}, \quad |z| < 1, \tag{4}$$

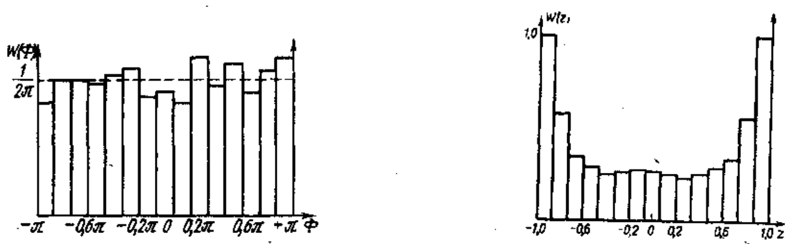
$$W(R) = \frac{R^2}{\sigma^2} e^{-\frac{R^2+A_0^2}{2\sigma^2}} I_0\left(\frac{A_0 R}{\sigma^2}\right), \tag{5}$$

where I_0 is the Bessel function.

Distributions (3) and (4) differ considerably from the corresponding expressions of the Rice model, which provides the possibility of an experimental check of the model. The distributions of R are the same in both models.

The distributions $W(\Phi)$ and $W(z)$ were investigated using special ionospheric coherent-reception equipment, which simultaneously recorded both quadrature components $E_c(t) = R(t)\cos\Phi(t)$ and $E_s(t) = R(t)\sin\Phi(t)$ of the signal (1). The method of recording E_c and E_s and of reconstructing $\Phi(t)$ from them is described in the article on p. 81 of this issue. The advantage of the method is the possibility of obtaining practically continuous records of the phase $\Phi(t)$ with an accuracy to within 5°.

We observed a signal reflected once from the F2 layer in the range 2.7-6.8 MHz (vertical incidence). About 100 sessions of observations of 3-4 min each were subjected to a statistical analysis. From the records of $E_c(t)$ and $E_s(t)$ we determined the distributions $W(\Phi)$ and $W(z)$, the central moments, and the correlation coefficients for R, E_c, E_z and z. Under our experimental conditions $\Phi(t)$ is a nonstationary random process: $\Phi(t) = \Omega t + \Phi^*(t)$ ($\Phi^*(t)$ is a stationary process), and, consequently, the distribution $W_T(\Phi, t)$ of the times that the phase lies in the range $[-\pi, \pi]$ depends, in principle, on the time t. However, in this case $W_T(\Phi, t)$ and, as shown in [6], the dependence of W_T on t can be neglected, which also agrees with expression (3). Below we assume $W_T = W(\Phi)$.



In the figure, where we show typical examples of histograms of $W(\Phi)$ and $W(z)$, the uniformity of $W(\Phi)$ and the symmetry of $W(z)$ can be seen. A quantitative statistical check of the correctness of the hypothesis of the uniformity of the distribution $W(\Phi)$ and the symmetry of $W(z)$, i.e., the correctness of expressions (3) and (4) were made using the Kolmogorov criterion. It was found that for 100% of the sessions the probability of the correctness of the hypothesis of the symmetry of $W(z)$ was $p = 0.75$; for 90% of the sessions $p = 0.85$, and for 70%, $p = 1.00$ (the level of significance of the criterion $\alpha = 1 - p$). A similar situation was established when checking the uniformity of $W(\Phi)$. In essence, the presence of a regular number Ωt (nonstationarity in $\Phi(t)$ and the condition $T \gg \frac{2\pi}{\Omega}$ lead to uniformity of the distribution $\Phi(t)$.

Finally these results provide a basis for the high reliability of the distribution laws (3) and (4), corresponding to the μ-model of the reflected signal. To solve problems of ionospheric propagation when it is necessary to take into account the form of the laws $W(\Phi)$, it is best to use expression (3) and (4) (instead of the usually employed $W(\Phi)$ law of the Rice model), as corresponding to the actual position. The results obtained may also be of interest when solving optimal reception problems.

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