ELECTRON-POSITRON PAIR PRODUCTION BY MASSIVE PHOTON IN MAGNETIC FIELD

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We examine the production of $e^+ e^-$ pairs by a massive photon and the annihilation of $e^+ e^-$ pairs with the formation of a massive photon in an ultrastrong magnetic field $\mathcal{H} \gtrsim B_0 = 4.41 \cdot 10^{13}$G. We obtain the conditions under which the external magnetic field begins to exert a substantial effect on these processes.

It is known that various processes such as the production of positronium [1], the creation of leptons and hadrons in the scattering of leptons on nuclei [2], and the decay of the ρ^0 meson [3] or the annihilation of charmonium into an electron-positron pair [4] take place through the formation of an intermediate massive photon. On the other hand, there have been indications in the literature of the possible existence in an ultrastrong magnetic field $\mathcal{H} > B_0 = m^2/e_0 = 4.41 \cdot 10^{13}$G* of a special resonance state with photon quantum numbers and a nonzero mass [5]. This conclusion was reached on the basis of analysis of the polarization operator of the electromagnetic field for $\mathcal{H} > B_0$ [6].

In this regard it is of interest to examine the decay of the massive photon with the creation of an electron-positron pair, and also the inverse process - the annihilation of a pair with the creation of a massive photon - in an intense external magnetic field.

1. **Production of the $e^+ e^-$ pairs.** We will choose the simpliest vector version of the interaction of the field of the electrons ψ with the vector field B, having the Lagrangian

$$L_{\rm int} = e\psi^+\gamma^0\gamma_\mu\psi B^\mu, \tag{1}$$

where e is the effective coupling constant. In the case of the resonance, which was predicted in Ref. [6], $e = e_0$ (e_0 is the electron charge), while in the case of the ρ^0 meson $e = e_2^0/2\gamma_\rho$ ($\gamma_\rho^2 \approx 0.5$ [7]).

The matrix element of the decay $B \to e^+ e^-$ with allowance for Eq. (1) can be written as

$$M_{fi} = \sqrt{4\pi}\, e\,(j^\mu e_\mu)/\sqrt{2q^0},$$
$$j^\mu = \int \psi^+\alpha^\mu e^{i\mathbf{qr}}\psi' d^3x, \quad (\mu = 0, 1, 2, 3),$$

*In a system of units with $c = \hbar = 1$.

where e_μ and **q** are the polarization vector and momentum of the B particle ($q^2 = q^{02} - \mathbf{q}^2 = M^2$ where M is the mass of the B particle). The electron ψ and positron ψ' states are described by the positive- and negative-frequency parts, respectively, of the solution of the Dirac equation in a constant homogeneous magnetic field $\vec{\mathcal{H}} \| oz$ [8]:

$$\psi = e^{-iEt} e^{ip_z z} f_{n,s,\zeta}(x, y), \quad \psi' = e^{iE't - ip'_z z} f_{n',s',\zeta'}(x, y).$$

Here E and E' are the electron and positron energy, respectively, $E = m(1 + 2n\mathcal{H}/B_0 + p_z^2/m^2)^{1/2}$; n, s, p_z, ζ and n', s', p'_z, ζ' are quantum numbers:

$$n = 0, 1, 2, \ldots, \quad s = 0, 1, 2, \ldots, \quad -\infty < p_z < \infty, \quad \zeta = \pm 1,$$

which are related by the following characteristics of the electron state in the negative field [8]:

$$R = 2[(n + 1/2)g]^{1/2}/m, \quad a = 2(sg)^{1/2}/m,$$

R is the radius of the quasiclassical orbit, a is the distance from the center of the orbit to the z axis, p_z is the projection of the momentum on the z axis; ζ is the spin number, which indicates one of the two possible polarization states of the electron: $\zeta = 1$ indicates the spin along the field $\mathcal{H}$; $\zeta = -1$, against the field. The parameter g, which gives the relative size of the magnetic field, is equal to

$$g = B_0/2\mathcal{H}.$$

The probability of pair production is evaluated by the formula

$$w = (\sqrt{4\pi}e)^2 \frac{1}{2q^0} \sum 2\pi\delta(q^0 - E - E') \, | j_\mu e^\mu |^2,$$

where the summation is over the electron and positron quantum numbers.

Averaging over the polarization of particle B, we obtain

$$\overline{| j_\mu e^\mu |^2}^e = -\frac{1}{3}(j^{\mu +} j_\mu),$$

where we have for $(j^{\mu+} j_\mu)$, after summing over the electron and positron spine [8],

$$-\sum_{\zeta\zeta'} j^{\mu+} j_\mu = \Big[\frac{m^2}{EE'}(I^2_{n-1,\,n'-1} + I^2_{nn'}) + 2\,\frac{2e_0\mathcal{H}\sqrt{nn'}}{EE'} I_{n-1,\,n'-1} I_{nn'} + \left(1 - \frac{p_z p'_z}{EE'} + \frac{m^2}{EE'}\right)(I^2_{n,n'-1} + I^2_{n-1,n'})\Big]\, \delta_{p_z + p'_z, q_z} I^2_{ss'}. \qquad (2)$$

Here the Laguerre function $I_{nn'}$, and $J_{ss'}$, defined by the equation

$$I_{nn'}(x) = \sqrt{\frac{n'!}{n!}}\, e^{-x/2} x^{\frac{n-n'}{2}} L_{n'}^{n-n'}(x),$$

are functions of the argument

$$x = q_\perp^2/2e_0\mathcal{H}$$

($L_n^{n-n'}(x)$ is a Laguerre polynomial, $q_\perp$ is the component of the B-particle momentum q orthogonal to the field $\vec{\mathcal{H}}$, and q_z is the longitudinal component, which for simplicity we will set equal to zero).

Taking into account the equations

$$\sum_{s'=0}^{\infty} I_{ss'}^2(x) = 1, \quad \sum_{s=0}^{s_{\max}} 1 = s_{\max} = e_0\mathcal{H}L^2/2\pi$$

and integrating over $p_z = p_z'$ with the aid of the delta function

$$\delta(q^0 - E - E') = \frac{\delta(p_z - p_z^+) + \delta(p_z - p_z^-)}{|\partial(E+E')/\partial p_z|},$$

where

$$\left|\frac{\partial(E+E')}{\partial p_z}\right| = \frac{\Delta_{nn'}}{\frac{1}{2} - 2\frac{e^2\mathcal{H}^2}{q_0^4}(n-n')^2},$$

$$\Delta_{nn'} = [1 + (2e_0\mathcal{H}/q_0^2)^2(n-n')^2 - (4e_0\mathcal{H}/q_0^2)(n+n') - 4m^2/q_0^2]^{1/2},$$

we obtain the probability of pair production by an unpolarized B particle:

$$w = \sum_{nn'} w_{nn'} = \frac{4}{3}e^2 q_0\left(\frac{e_0\mathcal{H}}{q_0^2}\right)\sum_{nn'}\Delta_{nn'}^{-1}\left[\left(\frac{m}{q_0}\right)^2(I_{nn'}^2 + I_{n-1,n'-1}^2) + \right.$$

$$\left. + \frac{4e_0\mathcal{H}}{q_0^2}\sqrt{nn'}\,I_{nn'}I_{n-1,n'-1} + \frac{1}{2}\left(1 - \frac{2e_0\mathcal{H}}{q_0^2}(n+n')\right)(I_{n-1,n'}^2 + I_{n,n-1}^2)\right].$$

In this formula the summation is over values of n and n' for which

$$\sqrt{m^2 + 2e_0\mathcal{H}n} + \sqrt{m^2 + 2e_0\mathcal{H}n'} \leqslant q^0,$$

and here $\Delta_{nn'}^2 \geqslant 0$.

As an example, we will examine the case of a particle at rest: $q = 0$, $q^0 = M$. Then

$$I_{nn'}(x) = \delta_{nn'}, \quad I_{ss'}(x) = \delta_{ss'}, \quad I_{n,n'-1}(x) = \delta_{n,n'-1}\ ,$$

etc. As can be seen from Eq. (3), only levels with $n = n'$ and $n = n' \pm 1$ contribute to the probability. We will write separately the probability of creation of a pair in states $n = n'$:

$$w_{nn} = \frac{4}{3}e^2\left(\frac{e_0\mathcal{H}}{M^2}\right)\frac{m^2 + 2e_0\mathcal{H}n}{\left(\frac{M^2}{4} - 2e_0\mathcal{H}n - m^2\right)^{1/2}}, \qquad (4)$$

for which here $0 < n \leqslant [(M^2/4 - m^2)/2e_0\mathcal{H}]$. If

$$\left(\frac{M^2}{4} - m^2\right)\Big/2e_0\mathcal{H} < 1, \qquad (5)$$

then only the single value n = 0 is possible; for this case we obtain ($n = n' = 0$, $\zeta = -\zeta' = -1$)

$$w_{00} = \frac{2}{3}e^2\left(\frac{e_0\mathcal{H}}{M^2}\right)\frac{m^2}{[M^2/4 - m^2]^{1/2}}. \qquad (6)$$

We will now write the probability of creation of a pair in states $n = n' - 1$:

$$w_{n,n+1} = \frac{2}{3}e^2\left(\frac{e_0\mathcal{H}}{M}\right)\frac{\left(1 - \frac{2e\mathcal{H}}{M^2}(2n+1)\right)}{\left[\left(1 - \frac{2e\mathcal{H}}{M^2}\right)^2 - \frac{4}{M^2}(2e_0\mathcal{H}n + m^2)\right]^{1/2}}, \qquad (7)$$

where

$$0 \leqslant n < \frac{1}{2e_0\mathcal{H}}\left[\frac{1}{M^2}(M^2/2-e\mathcal{H})^2-m^2\right]. \tag{8}$$

For $n' = n - 1$ we obtain analogously the probability $w_{n,n-1}$, which is given by Eq. (7) with the replacement $n \to n'$.*

If the right-hand side of inequality (8) is less than unity,

$$\frac{1}{2e\mathcal{H}}\left(\frac{1}{M^2}\left(\frac{M^2}{2}-e_0\mathcal{H}\right)^2-m^2\right)<1, \tag{9}$$

then probability (7) is nonzero only for $n = 0$:

$$w_{0,1}=\frac{2}{3}e^2\left(\frac{e_0\mathcal{H}}{M}\right)\frac{(1-2e_0\mathcal{H}/M^2)}{\left[\left(1-\frac{2e\mathcal{H}}{M^2}\right)^2-4\frac{m^2}{M^2}\right]^{1/2}}.$$

Finally, if

$$\sqrt{2e_0\mathcal{H}+m^2}+m>M, \tag{10}$$

then in this case it is generally impossible to create an electron and positron at different levels $n = n' \pm 1$.

Explicitly, we obtain from Eqs. (5), (9), and (10) the following threshold values for the magnetic field:

1). $$e_0\mathcal{H}>\frac{1}{2}\left(\frac{M^2}{4}-m^2\right).$$

The $e^+ e^-$ pair is created in states $n=n'=0,\ \zeta=-\zeta'=-1$ and cannot be created in states $n=n'=1, 2, \dots$.

2). $$e_0\mathcal{H}>\frac{3}{2}M^2-M\sqrt{2M^2+m^2}.$$

The components of the pair are formed in states $n = 0$, $n' = 1$ or $n = 1$, $n' = 0$; the states $n = n' + 1 = 2, 3, \dots$ or $n = n' - 1 = 1, 2, 3, \dots$ are forbidden.

3). $$e_0\mathcal{H}>M\left(\frac{M}{2}-m\right).$$

The pair production is in general impossible in states $n = n' \pm 1$.

2. **Pair annihilation.** The annihilation probability is determined by the general formula

$$dw=(\sqrt{4\pi}\,e)^2\,2\pi\delta(q^0-E-E')\,|j_\mu e^\mu|^2\frac{d^3q}{2q^0(2\pi)^3}. \tag{11}$$

We now average over the quantum numbers of the orbital centers and over the $e^+ e^-$ polarizations and then integrate over the momentum of particle B:

$$w=\frac{1}{4}\sum_{\zeta\zeta'}\frac{1}{s^2_{\max}}\sum_{ss'}\int dw.$$

*Expressions (4) and (7) for the probability of pair production correspond to the values obtained in Ref. [6] for the imaginary part of the polarization operator of the photon in a magnetic field. Formula (6) for w_{00} was obtained earlier by Loskutov and Skobelev [9].

The integral over q_z in Eq. (11) is replaced by a sum which is evaluated with allowance for the delta symbol $\delta_{p_z+p'_z, q_z}$, in the matrix element, where it is assumed that $p_z = p'_z = 0$. The integrals over q_x and q_y are evaluated with the aid of the delta functions:

$$\frac{1}{(2\pi)^2}\int dq_x dq_y \delta(q^0 - E - E') = \frac{1}{2\pi}\int q_\perp dq_\perp \delta(\sqrt{q_\perp^2 + M^2} - E - E').$$

Thus,

$$w = (2\pi)^2 e^2 \frac{1}{e_0 \mathscr{H} L^3}\frac{1}{4}\sum_{\zeta\zeta'}(\mathbf{j}^+\mathbf{j} - j^{0+}j^0), \tag{12}$$

where $\sum_{\zeta\zeta'}(\mathbf{j}^+\mathbf{j} - j^{0+}j^0)$ is determined by formula (2) of the preceding section.

The physically meaningful quantity here is the lifetime of an electron (positron) in a medium of positrons (electrons) of density $\rho = N/L^3$. Multiplying Eq. (12) by the number of positrons N, we obtain the inverse lifetime of the electron with regard to the creation of a massive photon of energy $q^0 \simeq E \gg m$:

$$\tau^{-1} = \rho\frac{(2\pi)^2 e^2}{e_0 \mathscr{H}}\frac{1}{4}\sum_{\zeta\zeta'}(\mathbf{j}^+\mathbf{j} - j^{0+}j^0). \tag{13}$$

It is of interest to find τ^{-1} for the collision of a relativistic electron having $E \gg m$ with a nonrelativistic positron having $E' \gtrsim m$ in a field $\mathscr{H} \sim B_0 = m^2/e_0$. For this case the argument of the Laguerre functions $I_{nn'}$, can be written approximately in the form

$$x \simeq n + 2\sqrt{n(n'+g)} - M^2/2e_0\mathscr{H}, \ (g = B_0/2\mathscr{H} \sim 1).$$

This value is close to the transition point of the function $I_{nn'}(x)$ for $n \gg 1$, $n' \ll n$:

$$x_0 = n + 2\sqrt{n(n'+1/2)}.$$

Therefore, the Laguerre functions have the asymptotic behavior [10]:

$$I_{nn'}(x) \simeq \frac{(-1)^{n'}}{\sqrt{n'!\sqrt{2\pi n}}} D_{n'}(\eta),$$

where the parabolic cylinder functions

$$D_{n'}(\eta) = 2^{-n'/2} e^{-\eta^2/4} H_{n'}(\eta/\sqrt{2})$$

are functions of the argument

$$\eta = 2\sqrt{n'+g} - \frac{M^2}{mE}\sqrt{g}.$$

Using the asymptotic behavior as given above, we can write the inverse lifetime (13) as

$$\tau^{-1} = \frac{e^2\rho(\pi)^{3/2}}{mE}\left(\frac{B_0}{\mathscr{H}}\right)^{1/2}\frac{1}{n'!}\left[n' D^2_{n'-1}(\eta) + D^2_{n'}(\eta) - \frac{2n'}{\sqrt{n'+g}} D_{n'}(\eta) D_{n'-1}(\eta)\right].$$

In particular, if the positron is located in the ground state ($n' = 0$, $\zeta' = 1$), then

$$\tau_0^{-1} = \frac{2e^2}{mE}\pi^{3/2}\rho\left(\frac{B_0}{\mathscr{H}}\right)^{1/2} e^{-\frac{B_0}{\mathscr{H}}(1 - M^2/2mE)^2}.$$

As the field $\mathcal{H}$ increases, the lifetime τ_0 decreases, reaching a minimum at

$$\mathcal{H} = 2B_0\left(1 - \frac{M^2}{2mE}\right)^2 = 2\,\frac{m^2}{e_0}\left(1 - \frac{M^2}{2mE}\right)^2.$$

As $\mathcal{H}$ is increased further, the lifetime grows as $\mathcal{H}^{1/2}$.

In the case of the creation of a ρ^0 meson of mass M = 770 MeV by an electron of energy $E \sim 10^7$ GeV, the parameter M^2/mE is around 0.1. The magnetic field in the surface layer of neutron stars can reach values $\mathcal{H} \sim 10^{13}$ G [11] (in the internal layers, according to some estimates [12,13], values $\mathcal{H} \lesssim 10^{17} - 10^{18}$ G are possible). Under such conditions the effects of an external magnetic field on the creation and decay of a massive photon with the formation of e^+e-pairs, as investigated here, can become important.

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