SCATTERING INDICATRIX OF RAYS IN A PLANE-LAYERED ISOTROPIC MEDIUM WITH ANISOMERIC INHOMOGENEITIES

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We present a theoretical analysis of how the parameters of the scattering indicatrix of rays on exit from an isotropic refracting medium with ellipsoidal inhomogeneities depend on the azimuth and length of the airways, the magnetic inclination at the point of reflection, and the geometry of the scattering inhomogeneities.

A statistical description of the scattering of waves in a randomly inhomogeneous medium such as the ionosphere can be carried out using a scheme which is familiar in the theory of Brownian motion and rests on the investigation of the Einstein-Fokker equation [1,2]. Using this statistical scheme, the authors of Ref. [3] obtained a solution $V(\chi, \psi/\zeta)$ to the Einstein-Fokker equation which characterizes the probability that a ray traversing the scattering medium on the path ζ has a direction given by the coordinates $\chi=\theta-\theta_0$ and $\psi=\varphi-\varphi_0$. The fluctuations of the polar θ and azimuthal ψ angles are measured relative to the trajectory described by Snell's law

$$n_0 \sin\theta_0 = \sin\theta_{00}, \quad \varphi_0 = 0,$$

where $n_0(z)$ is the refractive index characterizing the regular refraction of the medium, and θ_{00} is the original angle of incidence.

This paper is devoted to a theoretical analysis of the parameters of the scattering indicatrix of the rays upon exit from a linear ionospheric layer whose scattering inhomogeneities are of the form of ellipsoids of rotation. In accordance with the voluminous experimental data (for example, those of Refs. [4,5], we assume that the scattering inhomogeneities are protracted in the direction of the magnetic lines of force of the Earth.

To investigate the scattering of rays in a refracting medium, it is advantageous to transform from the variable θ to the variable $\xi = \ln \mathrm{tg}(\theta/2)$ [6]. Therefore, in what follows we will use the variable $\eta = \xi - \xi_0$ where $\xi_0 = \ln \mathrm{tg}(\theta_0/2)$ in place of χ.

The scattering indicatrix is defined by the cross section of equal probability $V_0(\eta, \psi/\zeta)$ = const. According to Ref. [3], the equation of the scattering indicatrix is of the form

$$\sigma_\psi^2\eta^2 + \sigma_\eta^2\psi^2 - 2R\sigma_\eta\sigma_\psi\eta\psi = \text{const}, \tag{1}$$

where σ_η^2 and σ_ψ^2 are the dispersions of the fluctuations of variables η and ψ, and R is the correlation coefficient of these variables. Equation (1) describes an ellipse inscribed in a rectangle with sides $2\sigma_\eta$ and $2\sigma_\psi$. The semimajor and semiminor axes of the characteristic ellipse (1) are

$$\sigma_{1,2}=\left\{\frac{1}{2}\left[\sigma_\eta^2+\sigma_\psi^2\pm\sqrt{(\sigma_\eta^2-\sigma_\psi^2)^2+4R^2\sigma_\eta^2\sigma_\psi^2}\right]\right\}^{1/2},$$

and its prolateness is $e = \sigma_1/\sigma_2$.

The orientation of the scattering indicatrix is determined by the relation

$$\operatorname{tg} 2\varphi_e=\frac{2R\sigma_\eta\sigma_\psi}{\sigma_\eta^2-\sigma_\psi^2},$$

where the angle φ_e is measured counterclockwise from the η axis, which lies in the plane of the airway, to the semimajor axis of the ellipse.

For this model of the medium, in the case $\overline{\varepsilon_1^2}=\text{const}$, where ε_1 is the fluctuational part of the dielectric constant, the parameters of the scattering indicatrix at the exit of the rays from the layer can be written in the form [3]

$$\sigma_\eta^2=\overline{\varepsilon_1^2}\,\frac{z_0}{a}\,\frac{\sqrt{\pi}}{\sin\theta_{00}}\,I_1(\alpha,\beta,\theta_{00},e),\quad \sigma_\psi^2=\overline{\varepsilon_1^2}\,\frac{\sqrt{\pi}\,z_0}{a}\,I_2(\alpha,\beta,\theta_{00},e),$$
$$R=I_3(\alpha,\beta,\theta_{00},e),$$

where I_i (i = 1, 2) are rather unwieldly functions which obey the relation

$$\begin{gathered}I_i(\pi+\beta)=I_i(\pi-\beta)=I_i(-\beta)=I_i(\beta),\\ I_3(\pi+\beta)=I_3(-\beta)=-I_3(\beta),\\ I_i(\pi-\alpha)=I_i(\alpha),\ I_3(\pi-\alpha)=-I_3(\alpha).\end{gathered}\tag{2}$$

Here α is the angle between the direction of the magnetic lines of force and the vertical to the surface of the Earth in the region where the wave is reflected ($I = (\pi/2 - \alpha)$ is the angle of magnetic inclination), β is the azimuth of the airway measured clockwise (if one is looking in the direction of the vertical) from the plane of the magnetic meridian, and e is the ratio of the axes of the ellipsoids of rotation.

The results of a computer evaluation of the functions I_i (i = 1, 2) for fixed values of α and β (θ_{00} is the initial angle of incidence) are given in the tables. The parameter e was chosen in accordance with the experimental data of Refs. [4,5]. The data obtained are sufficient to discover the basic relationships governing the variation of $\hat{e}$ and φ_e for an arbitrary choice of the parameters which determine them.

Table 1 gives the values of the prolateness $\hat{e}$ of the scattering indicatrix. Analysis of these results implies that the parameter $\hat{e}$ depends substantially on the prolateness e of the actual inhomogeneities. This dependence is manifested most clearly in airways in the polar region (α = 0°) and in the propagation of a wave in a plane perpendicular to the magnetic meridian (β = 90°). In these airways the prolateness of the scattering indicatrix increases as the angle of incidence θ_{00} of the wave increases, and at the geomagnetic equator (α = 90°) for

$\beta = 90^{\circ}$ the two prolatenesses are equal, $\hat{e} = e$. At the geomagnetic pole ($\alpha = 0^{\circ}$) the parameter $\hat{e}$ does not depend on the azimuth of the airway.

Table 1

$\bar{\varepsilon}_1^2$=const		e = 10					e = 5				
		α, deg									
θ_{ee}, deg	β, deg	0	20	40	60	90	0	20	40	60	90
10	0	3,23	1,60	1,91	2,06	2,13	1,98	1,46	1,67	1,82	1,89
	30	3,23	1,37	1,18	1,10	1,03	1,98	1,39	1,20	1,12	1,04
	60	3,23	2,22	2,12	2,19	2,28	1,98	1,85	1,91	2,05	2,31
	90	3,23	3,26	3,93	5,58	10,0	1,98	2,27	2,92	3,89	5,0
20	0	5,48	1,80	1,73	1,92	1,97	2,86	1,66	1,53	1,57	1,74
	30	5,48	2,27	1,36	1,19	1,11	2,86	1,89	1,32	1,17	1,08
	60	5,48	3,57	2,69	2,57	2,60	2,86	2,56	2,32	2,29	2,32
	90	5,48	4,55	4,93	6,57	10,0	2,86	3,0	3,48	4,25	5,0
40	0	8,28	6,25	1,81	1,66	1,77	4,16	3,33	1,65	1,47	1,54
	30	8,28	6,32	2,70	1,67	1,36	4,16	3,52	2,17	1,53	1,33
	60	8,28	7,14	4,63	3,77	3,40	4,16	3,97	3,45	3,02	2,85
	90	8,28	7,81	8,13	8,66	10,0	4,16	4,17	4,35	4,65	5,0
60	0	9,52	8,59	5,22	1,73	1,57	4,76	4,33	3,07	1,53	1,35
	30	9,52	8,70	5,65	3,03	2,02	4,76	4,44	3,46	2,30	1,74
	60	9,52	9,09	7,35	5,83	4,90	4,76	4,65	4,25	3,86	3,55
	90	9,52	9,34	9,86	9,78	10,0	4,76	4,81	4,95	4,85	5,0

Table 2

$\bar{\varepsilon}_1^2$=const		e = 10					e = 5				
		α, deg									
θ_{ee}, deg	β, deg	0	20	40	60	90	0	20	40	60	90
10	0	90	90	90	90	90	90	90	90	90	90
	30	90	62	61,5	61,5	90	90	69	68,5	69	90
	60	90	49,5	31,5	18		90	52	32	17,5	0
	90	90	49	30,5	17,5		90	50	30,5	17	0
20	0	90	90	90	90	90	90	90	90	90	90
	30	90	65	45	34	0	90	69	52	39,5	0
	60	90	60	38	22	0	90	61	39	22	0
	90	90	59	38	22	0	90	59,5	38	21,5	0
40	0	90	90	90	90	90	90	90	90	90	90
	30	90	76,5	54,5	31,5	0	90	77	56	34	0
	60	90	69	48	27,5	0	90	69	47,5	27,5	0
	90	90	66,5	47	26,5	0	90	66,5	46,5	26,5	0
60	0	90	90	90	90	90	90	90	90	90	90
	30	90	79	64	41	0	90	79	64	42	
	60	90	71,5	52	31,5	0	90	71,5	52	31,5	0
	90	90	69	48,5	28,5	0	90	69	48,5	28,5	0

In the polar region, $\hat{e}$ increases as the airway deviates from the plane of the magnetic meridian, while in medium latitudes and above the geomagnetic equator the prolateness for angles of incidence $\theta_{00} \leq 40°$ initially decreases as β increases, then begins to increase, reaching an absolute maximum at $\beta = 90°$. For an equatorial airway in the plane of the magnetic meridian, the parameter $\hat{e}$ decreases as the angle of incidence θ_{00} of the wave on the layer increases.

The values of the parameter φ_e, which determines the orientation of the scattering indicatrix relative to the plane of the airway, are given in Table 2. The results obtained lead to the conclusion that the orientation of the indicatrix is practically independent of the prolateness e of the inhomogeneities. For propagation of a wave above the geomagnetic pole or in the plane of the magnetic meridian, the major axis of the indicatrix is perpendicular to the plane of the airway. As α and β increase, the parameter φ_e decreases, the rate of decrease falling as the angle of incidence θ_{00} of the wave on the layer increases. For equatorial airways the characterisitic ellipse, which for $\beta = 0$ is protracted in a direction perpendicular to the direction of propagation, at first degenerates into a circle as the azimuth of the airway is increased, then begins to protract along the direction of propagation.

Taking relation (2) into account, one can generalize these results for airways with azimuths greater than 90°.

The results of this paper can be used to obtain information on the parameters of the inospheric inhomogeneities by radio methods, for example, the prolateness or the deviation of the direction of prolateness of the actual inhomogeneities from the lines of force of the Earth's magnetic field.

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