THEORY OF THE TEMPERATURE SUPERLATTICE IN A SEMICONDUCTOR WITH HOT ELECTRONS

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The production of an electron-temperature superlattice in a hot-electron semiconductor is considered in the linear approximation for the case where the absorption coefficient for the heating light is dependent on the electron temperature. The distribution of the electron temperature under static conditions has been derived, and also for free convection in the electron gas. The electromagnetic-field distribution has also been derived for the latter case. The critical light intensity at which the superlattice arises has been calculated.

INTRODUCTION

The origin of a temperature superlattice in a semiconductor with unipolar conductivity has been considered [1] in the linear approximation for conditions of a fairly high gradient in the electron (or hole) temperature. It is assumed that the free-carrier gas is heated by the field of the light wave during intraband light absorption. The electron gas is considered incompressible, while the absorption coefficient γ is independent of the electron temperature T. Here the problem of [1] is considered, but for conditions where one cannot neglect the dependence of the absorption coefficient on the electron temperature. The assumptions are otherwise those of [1]. In particular, we assume that the thickness of the specimen in the direction of the light beam greatly exceeds the reciprocal of the absorption coefficient for a weak electromagnetic wave γ_0^{-1}, while

$$\gamma_0^{-1} \gg \lambda_0^{-1} = \sqrt{\frac{2}{3}\varkappa_0\tau_0}.$$

It is readily seen what effect is to be expected. This relationship with $\frac{d\gamma}{dT} > 0$ raises the absorption coefficient at the illuminated surface somewhat by comparison with the case when $\gamma(T)$ is neglected. Then there is an increase (decrease) in the amount of light energy absorbed at the surface (reaching the depth of the specimen). One therefore expects somewhat greater variations in the electron temperature. Correspondingly, instability sets in for $\frac{d\gamma}{dT} > 0$ at light intensities lower than would be expected when $\gamma(T)$ is neglected. The situation is the converse for $\frac{d\gamma}{dT} < 0$.

1. BASIC EQUATIONS

The initial equations are as in [1], the only difference lying in the ex-

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pression for the light-wave energy absorbed by unit volume in unit time.

We introduce the following symbols: n is the free-carrier concentration (the carriers are subsequently taken to be electrons, although all formulas are written for particles with positive charge), $\mathbf{u}$ is the speed of the convective motion, $\varkappa$ is the thermal diffusivity, τ is the energy relaxation time, $\mathbf{q}$ is the electron energy flux density, I is the light-energy flux, T and T_0 are the electron temperature and the lattice temperature, $\nabla\varphi$ is the strength of the steady electric field, $\mathbf{E}_\sim$ is the strength of the light-wave field, $\mathbf{j}_\sim$ is the density of the current due to the electromagnetic wave, and σ is the conductivity.

If the electron gas is incompressible and nondegenerate, while a power-law relationship applies for the dependence on energy W for the momentum relaxation time $(\tau_p(W) \sim W^r)$, then the transport equation for the energy takes the form

$$\frac{3}{2} n \frac{\partial T}{\partial t} + \left(\frac{5}{2} + r\right) n \operatorname{div}(\mathbf{u}T) - n \operatorname{div}(\varkappa \nabla T) + \\ + en(\mathbf{u}, \nabla\varphi) + \frac{3}{2} n \frac{T - T_0}{\tau} = n\langle \dot{W} \rangle. \tag{1}$$

The term $n\langle\dot{W}\rangle$ describes the mean energy acquired by unit volume of the electron gas in unit time from the light-wave field.

If the light wave is monochromatic, i.e.,

$$\mathbf{E}_\sim = \operatorname{Re}[\mathbf{E}_0 e^{-i\omega t}], \quad \mathbf{j}_\sim = \operatorname{Re}[\mathbf{j}_0 e^{-i\omega t}].$$

then

$$n\langle\dot{W}\rangle = \langle \mathbf{j}_\sim \mathbf{E}_\sim \rangle = \frac{1}{2}\sigma_r(T)|E_0|^2.$$

Therefore, to obtain an expression for $n\langle\dot{W}\rangle$, we have to solve Maxwell's equations on the basis that the conductivity σ $(\sigma = \sigma_r + i\sigma_i)$ is dependent on the electron temperature. We assume that this relationship is known from kinetic theory [2].

The boundary conditions of (1) are as in [1]; if the Oz axis coincides with the direction of the light beam, then

$$z = 0: \quad -n\varkappa \frac{\partial T}{\partial z} = -n\nu(T - T_0),$$
$$z \to \infty: \quad T \to T_0,$$

where $\nu > 0$ is a phenomenological coefficient. We assume at once (in contrast to [1], where the condition was imposed at the end) that the heat transfer at the illuminated boundary is small, so $\nu_0' = \nu_0 \varkappa_0^{-1} \lambda_0^{-1} \ll 1$; we put

$$T = T_0(1 + \xi)$$

and assume that the heating is small, i.e., $\xi \ll 1$; therefore, linearization with respect to ξ is performed subsequently. On this basis we restrict ourselves in Taylor series of expansion of $\sigma(\xi)$ to terms up to the first order in ξ, i.e.,

$$\sigma(\xi) = \sigma_0 + \sigma_0 \dot{\sigma}_0 \xi,$$

where $\dot{\sigma}_0$ denotes the logarithmic derivative, $\dot{\sigma}_0 = \left(\frac{d \ln \sigma}{d \ln T}\right)_{T=T_0}$.

2. STATIC CASE

The solution is sought in the following form when the instability has not yet set in:

$$\mathbf{u} = 0, \ \xi = \xi_s(z), \ \varphi = \varphi_s(z), \ \mathbf{E}_0 = \mathbf{E}_{s0}(z) \| Ox.$$

Then (1) becomes

$$-nT_0 \xi_s'' - nT_0 \frac{(\xi_s')^2}{1+\xi_s} \varkappa + \frac{3}{2} nT_0 \frac{\xi_s}{\tau} = \frac{1}{2} \sigma_r(\xi_s) |E_{s0}|^2. \qquad (2)$$

Joint solution of Maxwell's equations and (2) gives

$$E_{s0}(z) = E_m e^{i q \bar{n}_0 z - \gamma_0 z/2} \exp\left\{ i \frac{q}{2} \frac{\frac{d\varepsilon(0)}{d\xi}}{\varepsilon(0)} \int_0^z \xi_s dz' - \frac{1}{4} \frac{\frac{d\varepsilon(0)}{d\xi}}{\varepsilon(0)} [\xi_s(z) - \xi_s(0)] \right\}$$

and (in the approximation linear in ξ_s)

$$\xi_s(z) = \frac{\sigma_{r0} |E_m|^2}{3nT_0 \tau_0^{-1}} [e^{-\gamma_0 z} - (\nu_0' + \gamma_0 \lambda_0^{-1}) e^{-\lambda_0 z}].$$

Here

$$\bar{n}_0 + i\gamma_0/2q = \sqrt{\varepsilon(\omega, 0)}, \ q = \omega/c, \ \varepsilon(\omega, \xi) = \varepsilon_l(\omega) + i4\pi\sigma(\omega, \xi)/\omega.$$

By $\varepsilon_l(\omega)$ we denote the part of the dielectric constant due to polarization of the atoms and vibrations of lattice ions, which is independent of the electron temperature; $E_m = E_{s0}(0)$. The value of E_m is determined from the condition for continuity of the tangential component of the electric field at z = 0.

3. TEMPERATURE SUPERLATTICE

To determine the stability of the static state for occurence of a superlattice we impose a small perturbation on the static solution assuming that

$$\xi = \xi_s + \delta\xi, \ \varphi = \varphi_s + \delta\varphi, \ \mathbf{u} = \mathbf{u}_s + \delta\mathbf{u} \, (\mathbf{u}_s = 0),$$

and $\mathbf{A}_0 = \mathbf{A}_{s0} + \delta\mathbf{A}_0 \, (\mathbf{A}_\sim = \mathrm{Re}[\mathbf{A}_0 e^{-i\omega t}]$ is one of the vectors $\mathbf{E}_\sim, \mathbf{H}_\sim, \mathbf{D}_\sim, \mathbf{j}_\sim$), where

$$\delta\xi, \ \delta\varphi, \ \delta u \backsim e^{st} \cos \mathbf{kr},$$

and the coefficients of proportionality are respectively $f_1(z)$, $f_2(z)$, $\mathbf{f}(z)$; by $\mathbf{k}$ and $\mathbf{r}$ we denote two-dimensional vectors in the plane of x and y, while s is a quantity with the dimensions of reciprocal time. The form to be used for $\delta\mathbf{E}_0$, $\delta\mathbf{H}_0$, $\delta\mathbf{D}_0$ and

δj_0, will be clear from what follows. The instant of onset of instability is defined by the condition Re s = 0. We are interested in stationary convection, so we put Im s = 0 also.

The vectors δj_0, δE_0 and E_{s0} are related by

$$\delta j_0 = \sigma_s \delta E_0 + \sigma_s \dot{\sigma}_s \delta \xi E_{s0}, \tag{3}$$

which is obtained by varying $j_0 = \sigma E_0$ about the static solution.

We have to solve Maxwell's equations in order to find δE_0; we vary the latter about the static solution to get

$$\mathrm{rot}\, \delta H_\sim = \frac{4\pi}{c} \delta j_\sim + \frac{\varepsilon_l}{c} \frac{\partial \delta E_\sim}{\partial t}; \tag{4}$$

$$\mathrm{rot}\, \delta E_\sim = -\frac{1}{c} \frac{\partial \delta H_\sim}{\partial t}; \tag{5}$$

$$\mathrm{div}\, \delta H_\sim = \mathrm{div}\, \delta E_\sim = 0. \tag{6}$$

Here we have used the fact that the electron temperature ξ, the free carrier concentration n, and the space charge density ρ^{free} are independent of time when $\omega\tau_{ec} \gg 1$.

From (3) and (6) we have that δE_0 should be sought in the form

$$\delta E_{0x},\ \delta E_{0y} \backsim \cos \mathbf{kr},\ \delta E_{0z} \backsim \sin \mathbf{kr};$$

where the coefficients of proportionality are correspondingly $\mathcal{E}_{0x}(z)$, $\mathcal{E}_{0y}(z)$ and $\mathcal{E}_{0z}(z)$. The equations (4), (5), and (6) give

$$\left(\frac{d^2}{dz^2} + \mathcal{K}_s^2\right) \mathcal{E}_0 = \Psi_s, \tag{7}$$

where

$$\mathcal{K}_s^2 = \frac{\omega^2}{c^2}\left[\varepsilon_l + i\frac{4\pi\sigma_s}{\omega} - \frac{k^2c^2}{\omega^2}\right],\ \Psi_s = \left\{\Psi_s = -i\frac{4\pi\omega}{c^2}\sigma_s\dot{\sigma}_s f_1 E_{s0},\ 0,\ 0\right\}.$$

The boundary condition for (7) with $z \to \infty$ is

$$\mathcal{E}_0|_{z\to\infty} \to 0.$$

The values of the constants $\mathcal{E}_{m\alpha} = \mathcal{E}_{0\alpha}|_{z=0}$ $(\alpha = x, y, z)$ are determined from the boundary conditions for Maxwell's equations at z = 0; these conditions are obtained if we vary the usual boundary conditions of electrodynamics about the static solution (for constants $\mathbf{E}_\sim^{inc}$ and $\mathbf{H}_\sim^{inc}$). We have

$$\delta E_{m\alpha}^{ref} = \delta E_{m\alpha} \quad (\alpha = x, y); \tag{8}$$

$$\varepsilon_d \delta E_{mz}^{ref} = \varepsilon_l \delta E_{mz}; \tag{9}$$

$$\delta H_{m\beta}^{ref} = \delta H_{m\beta} \quad (\beta = x, y, z). \tag{10}$$

Here ε_d is the dielectric constant of the insulator bounding the semiconductor.

The relation between the vectors $\delta\mathbf{E}_m$ and $\delta\mathbf{H}_m$ required to solve (8)-(10) is defined as usual from (5) and (7); (7) is solved from perturbation theory. We represent $\mathscr{E}_0$ as

$$\vec{\mathscr{E}}_0 = \vec{\mathscr{E}}_0^{(1)} + \vec{\mathscr{E}}_0^{(2)}, \text{ where } |\vec{\mathscr{E}}_0^{(2)}| / |\vec{\mathscr{E}}_0^{(1)}| \sim \xi_s \ll 1.$$

Then for $\vec{\mathscr{E}}_0^{(1)}$ we have the following boundary-value problem:

$$\left(\frac{d^2}{dz^2} + \mathscr{K}_0^2\right)\vec{\mathscr{E}}_0^{(1)} = \mathbf{\Psi}_0;$$

$$\mathscr{E}_{0\alpha}^{(1)}\big|_{z=0} = \mathscr{E}_{m\alpha}^{(1)} \quad (\alpha = x, y, z);$$

$$\mathscr{E}_{0\alpha}^{(1)}\big|_{z\to\infty} \to 0,$$

where $\mathscr{K}_0^2 = \mathscr{K}_s^2 (\xi_s = 0)$.

The quantities $\vec{\mathscr{E}}_{0\alpha}^{(2)}$ will not be determined, so superscript 1 will be omitted in future. The solution to this boundary-value problem is

$$\mathscr{E}_{0x} = \mathscr{E}_{mx} e^{i\mathscr{K}_0 z} + \frac{i e^{i\mathscr{K}_0 z}}{12\mathscr{K}_0}\left[\int_0^\infty dz' e^{i\mathscr{K}_0 z'}\Psi_0(z') - \right.$$

$$\left. - \int_0^z dz' e^{-i\mathscr{K}_0 z'}\Psi_0(z') - e^{-i2\mathscr{K}_0 z}\int_z^\infty dz' e^{i\mathscr{K}_0 z'}\Psi_0(z')\right];$$

$$\mathscr{E}_{0\alpha} = \mathscr{E}_{m\alpha} e^{i\mathscr{K}_0 z} \quad (\alpha = y, z).$$

Then from (5) we find for z = 0 that

$$(k_y\mathscr{E}_{mz} - \mathscr{K}_0\mathscr{E}_{my})\cos\mathbf{kr} = q\delta H_{mx}; \tag{11}$$

$$\left(\mathscr{K}_0\mathscr{E}_{mx} - \int_0^\infty dz' e^{i\mathscr{K}_0 z'}\Psi_0(z') - k_x\mathscr{E}_{mz}\right)\cos\mathbf{kr} = q\delta H_{my}; \tag{12}$$

$$-(k_x\mathscr{E}_{my} - k_y\mathscr{E}_{mx})\sin\mathbf{kr} = q\delta H_{mz}. \tag{13}$$

Estimation of the integral in the form of (12) shows that it is small in relation to the parameter $\gamma_0/\lambda_0 \ll 1$, so it should be omitted. By virtue of (6), the quantities $\mathscr{E}_{mx}$, $\mathscr{E}_{my}$ and $\mathscr{E}_{mz}$ are not independent, but instead are related by

$$-k_x\mathscr{E}_{mx} - k_y\mathscr{E}_{my} + \mathscr{K}_0\mathscr{E}_{mz} = 0. \tag{14}$$

To write (11)-(14) for the reflected wave, we have to put $\sigma = 0$ in all the formulas and replace ε_l by ε_d and $\mathscr{K}_0$ by $-Q_0 = -\omega c^{-1}(\varepsilon_d - k^2c^2\omega^{-2})^{1/2}$.

It is readily seen that all the necessary conditions are met only if

$$\mathscr{E}_{m\alpha} = \mathscr{E}_{m\alpha}^{\text{ref}} = \delta H_{m\alpha} = \delta H_{m\alpha}^{\text{ref}} = 0 \quad (\alpha = x, y, z).$$

This result should not cause surprise, since the boundary conditions cannot be

satisfied by considering only two waves, as in ordinary linear optics.

It is convenient to solve the energy-balance equation in dimensionless variables; we assume that length is measured in units of λ_0^{-1}, time in units of τ_0, and velocity in units of $u_0=3/(5+2r)\lambda_0\tau_0$, with electric field measured in $3T_0/2e\tau_0u_0$, and the light intensity in $3nT_0/2\lambda_0\tau_0$. Then we vary (1) about the static solution, linearize with respect to ξ, and neglect terms small on the parameters γ_0 and γ_0^2, to get

$$f_1'' - [k^2 + 1 - (2\dot{\tau}_0 + \dot{\varkappa}_0)\,\xi_s]\,f_1 = -\dot{\sigma}_{r0}\gamma_0 I_m e^{-\gamma_0 z} f_1 - \\ - \gamma_0 I_m e^{-\gamma_0 z}(\mathscr{E}_{0x}/E_{s0} + \text{K. C.}). \tag{15}$$

If $\dot{\sigma}_{r0}=0$ (15) becomes the corresponding equation of [1]; we solve (15) for two regions: $z \ll \gamma_0^{-1}$ (where $\xi_s \simeq \gamma_0 I_m$) and $z \gg \gamma_0^{-1}$ ($\xi_s \simeq \gamma_0 I_m e^{-\gamma_0 z}$). Estimation of the right side of the equation (where f_1 is taken as the zeroth approximation, namely the function found in [1]) shows that it is small, so we discard it. In the region $z \ll \gamma_0^{-1}$ we get

$$f_1(z) = A\cos\lambda z + B\sin\lambda z, \tag{16}$$

where

$$\lambda^2 = (2\dot{\tau}_0 + \dot{\varkappa}_0 + \dot{\sigma}_{r0})\,\gamma_0 I_m - k^2 - 1 = q_0^2 - k^2 - 1.$$

For the region $z \gg \gamma_0^{-1}$ (on the basis of the condition $T \to T_0$ for $z \to \infty$) we get

$$f_1(z) = CJ_p\left(\frac{2q_0}{\gamma_0}\,e^{-\gamma_0 z/2}\right), \tag{17}$$

where $p = 2\gamma_0^{-1}(1+k^2)^{1/2}$.

We find the wave vector for the superlattice k and the critical intensity I_m^{cr} from the boundary condition to (15) for z = 0 and the condition for the solutions of (16) and (17) to link up at some point $z_0 \lesssim \gamma_0^{-1}$. It is found that the results for k and I_m^{cr} are independent of the linkup point z_0 (V. L. Bonch-Bruevich directed my attention to this).

The first of these conditions takes the form

$$z = 0: \quad f_1' = \nu'_{\text{eff}} f_1,$$

where

$$\nu'_{\text{eff}} = \nu_0'[1 - (2\dot{\nu}_0' + \dot{\varkappa}_0)\,\gamma_0 I_m]\,(1 - \dot{\varkappa}_0\gamma_0 I_m)^{-1}. \tag{18}$$

From (16)-(18) and conditions for linkup at z_0 we get $\gamma_0 I_m(2\dot{\tau}_0 + \dot{\varkappa}_0 + \dot{\sigma}_{r0}) = 1 + k^2$ to terms of order $\sim \gamma_0^2$ and therefore

$$k = \sqrt{\frac{I_m}{I_m^{cr}} - 1},$$

where

$$I_m^{cr} = \frac{1}{\gamma_0 (2\dot{\tau}_0 + \dot{\varkappa}_0 + \dot{\sigma}_{r0})} \tag{19}$$

is the minimum value of the intensity at which the instability sets in.

Phenomenologically speaking, the incorporation of the electron-temperature dependence of the absorption coefficient makes itself felt in that the formula $I = I_m e^{-\gamma z}$, used in [1] no longer applies. Maxwell's equations have to be solved to determine the true dependence. If instead of this we simply put $I = I_m \exp\left\{-\int_0^z \gamma(z')\,dz'\right\}$ (which follows directly from definition of the absorption coefficient), we obtain the same result, only with the difference that $\dot{\sigma}_{r0}$ replaces $\dot{\gamma}_0$ in (19). However, this approach is less convincing than solution of the equations of electrodynamics, since then there is not procedure for calculating $\gamma(T)$, and the quantities γ_0 and $\dot{\gamma}_0$ along with it.

We see from (19) that for $\dot{\sigma}_{r0}=0$, i.e., the absorption coefficient is independent of the electron temperature, we get the corresponding formula of [1]; for $\dot{\sigma}_{r0}>0$, as would be expected, we get a value of I_m^{cr} somewhat less than that of [1], while for $\dot{\sigma}_{r0}<0$ we get a somewhat larger value. If the electron momentum in the semiconductor is dissipated mainly at charged impurities, while the energy is dissipated in piezoelectric acoustic phonons, then $\gamma_0 I_m^{cr}$ will be 1/5, not 1/3.5 (as would be the case for $\dot{\sigma}_{r0}=0$).

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