PARAMETRIC SPIN-WAVE EXCITATION IN A FERRITE SINGLE CRYSTAL WITH HIGH ANISOTROPY

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A theoretical discussion is presented of the parametric excitation of spin waves in ferrite single crystals with high crystallographic anisotropy; it is shown that there are regions of sharp change in the spectrum and in the threshold fields that are related to the effects of the ellipticity and the overlap between the characteristic regions in the anisotropic spectrum for the thermal spin waves; the phenomena are analyzed for Ni ferrite.

The crystallographic-anisotropy energy has a considerable effect on the conditions for parametric excitation of first-order spin waves ($\omega_k = \omega/2$, where ω_k is the spin precession frequency and ω is the frequency of the pumping field) [1-6].

It has been shown [4-8] that the threshold field and spectrum of the parametric spin waves become anisotropic for material of low anisotropy ($|K_1|/M < M$, where K_1 is the first anisotropy constant and M is the magnetic moment), not only as regards the magnetization direction but also as regards polarization and orientation of the UHF pumping field h.

For example, the <111> axis differs from the <001> axis in that there is anisotropy in the threshold fields and in the spectrum when h is perpendicular to the $(1\bar{1}0)$ plane.

Under certain excitation conditions, only the parallel component of the UHF field determines the threshold field and the spectrum of the parametric spin waves (direction of the magnetization, magnitude of the steady field H_0, orientation of h relative to H_0, and closeness of ω to the ferromagnetic-resonance frequency).

If the crystallographic anisotropy is relatively high $\left(\frac{|K_1|}{M} \gg M\right)$, new features appear in the spectrum and in the threshold fields, which are considered here.

1. SPIN-WAVE SPECTRUM

The anisotropy energy affects the spectrum for thermal spin waves by removing the degeneracy in angle φ_k for a saturated ferrite single crystal or cubic crystal having low crystallographic anisotropy:

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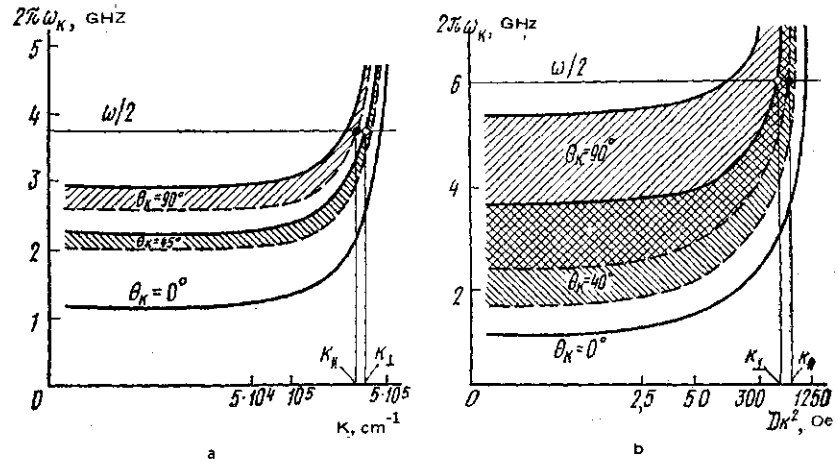

Fig. 1. Spin-wave spectra for spheres: YFG ($4\pi M$ = 1750 G, $|K_1|/M$ = 43 Oe, H_0 = 1000 Oe, D = 5 · 10^{-5} Oe·cm^2, <110>), b) Ni ferrite ($4\pi M$ = 3270 G, $|K_1|/M$ = 260 Oe, H_0 = 1500 Oe, <110>); solid lines for $\varphi_k=0°$, and 180°, broken for $\varphi_k=\pm 90°$

$$\omega_k = (A_k^2 - |B_k|^2)^{1/2} = [(\gamma H_1 + \omega_M \sin^2\theta_k)\gamma H_1 - N_{1*}(N_{1*} + \omega_M \sin^2\theta_k \cos 2\varphi_k)]^{1/2}, \quad (1)$$

where

$$\gamma H_1 = \omega_H - \widehat{\omega}_M N_z + \omega_{ex} k^2 + N_1;\ A_k = \gamma H_1 + \frac{\omega_M}{2}\sin^2\theta_k;$$

$$B_k = \frac{\omega_M}{2}\sin^2\theta_k e^{2i\varphi_k} + N_{1*};\ N_1 = -\omega_a\left(2 - 10\sin^2\theta + \frac{15}{2}\sin^4\theta\right);$$

$$N_{1*} = \frac{3}{2}\omega_a \sin^2\theta(2 - 3\sin^2\theta);\ \omega_M = 4\pi M\gamma;$$

$$\widehat{\omega}_M = \gamma M;\ \omega_H = \gamma H_{0z};\ \omega_{ex} = \gamma D M;\ \omega_a = \gamma\frac{|K_1|}{M};$$

$$H_{0z} = H_0 - \frac{1}{2}\left(\frac{K_1}{2M}\right)^2 \frac{1}{H_0}\sin^2\theta(3\sin^2\theta - 2);$$

where γ is the gyromagnetic ratio, k, θ_k, φ_k are the wave number, polar angle, and azimuthal angle for the propagation vector $\mathbf{k}$, H_0 lies in the $(1\bar{1}0)$ plane, θ is the angle between $\mathbf{H}_0$ and $\mathbf{M}$, and D is the exchange-interaction constant.

It follows from (1) that there is broadening in the lines of the $\omega_k(\mathbf{k})$ spectrum with $\theta_k \neq 0°$, i.e., instead of the $\omega_k(\mathbf{k})$, line corresponding to some value of θ_k in the isotropic case one gets a band of values $\omega_k(k, \varphi_k)$ for a given θ_k in the anisotropic specimen (Fig. 1).

Numerical minimization of the threshold field for this case shows that spin waves are excited with wave numbers respectively $k_\parallel$ and $k_\perp$ for longitudinal $(\mathbf{h}\parallel\mathbf{H}_0)$ and transverse $(\mathbf{h}\perp\mathbf{H}_0)$ forms of pumping ($\mathbf{h}$ lies in the plane $(1\bar{1}0)$, $H_0\parallel$<110>), as

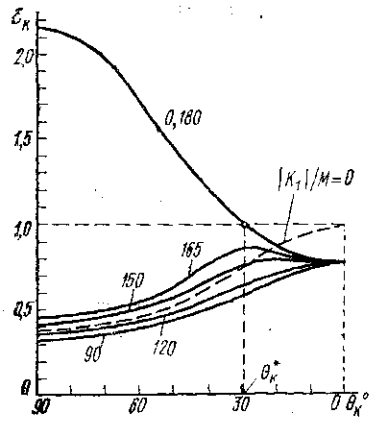

Fig. 2. θ_k dependence of the spin-wave ellipticity ε_k for various φ_k (sphere of Ni ferrite <110> axis). The numbers on the lines are the values of φ_k in degrees.

(see Fig. 1)

$$k_{\parallel} < k_{\perp}. \qquad (2)$$

If the pumping field is oriented at an arbitrary angle ψ (ψ is the angle between $\mathbf{h}$ and $\mathbf{H}_0$, with $\mathbf{h}$ still lying in the $(1\bar{1}0)$ plane), then we get spin waves with wave numbers k_ψ:

$$k_{\parallel} \leqslant k_{\psi} \leqslant k_{\perp}.$$

In a ferrite single crystal of high anisotropy ($K_1/M \geqslant M$) this broadening of the spectrum lines for θ_k = constant for a given φ_k becomes sufficient for the regions of the spectrum with $\theta_k = 90°$ and $\theta_k \leqslant 45°$ to overlap, which is characteristic of spin waves excited respectively by longitudinal and transverse forms of pumping (Fig. 1).

Here numerical minimization of the threshold shows that spin waves with

$$k_{\parallel} > k_{\perp} \qquad (3)$$

are excited, which conflicts with (2).

One therefore expects that particular features will appear in the spectrum and in the threshold fields of the parametric spin waves on going from longitudinal pumping to transverse.

2. SPIN-WAVE ELLIPTICITY

A major characteristic in research on parametric excitation mechanisms is the spin-wave ellipticity ε_k (the ratio of the semiaxes of the ellipse for the precession of the thermal spin-wave magnetization) [7]:

$$\varepsilon_k = \left(\frac{A_k - |B_k|}{A_k + |B_k|}\right)^{1/2} = f(k, \theta_k, \varphi_k). \qquad (4)$$

As we are discussing the parametric excitation of first-order spin waves, k can be eliminated from (4) on the basis of the condition $\omega_k(k, \theta_k, \varphi_k) = \omega/2$; the resulting $\varepsilon_k(\theta_k, \varphi_k)$ relation is shown in Fig. 2. Here H_0 defines the value of the wave number k when k > 0 or the limits to the values of θ_k and φ_k, when k = 0.

Figure 2 shows $\varepsilon_k(\theta_k)$ for $K_1 = 0$ (isotropic case). It is clear that ε_k for $K_1 = 0$ is determined only by the action of the dipole-dipole field, and $\varepsilon_k = 1$ for $\theta_k = 0°$ (circular precession).

In the anisotropic case, ε_k becomes dependent on φ_k, and we have $(\varepsilon_k)_{\varphi_k=0°,180°} = 1$

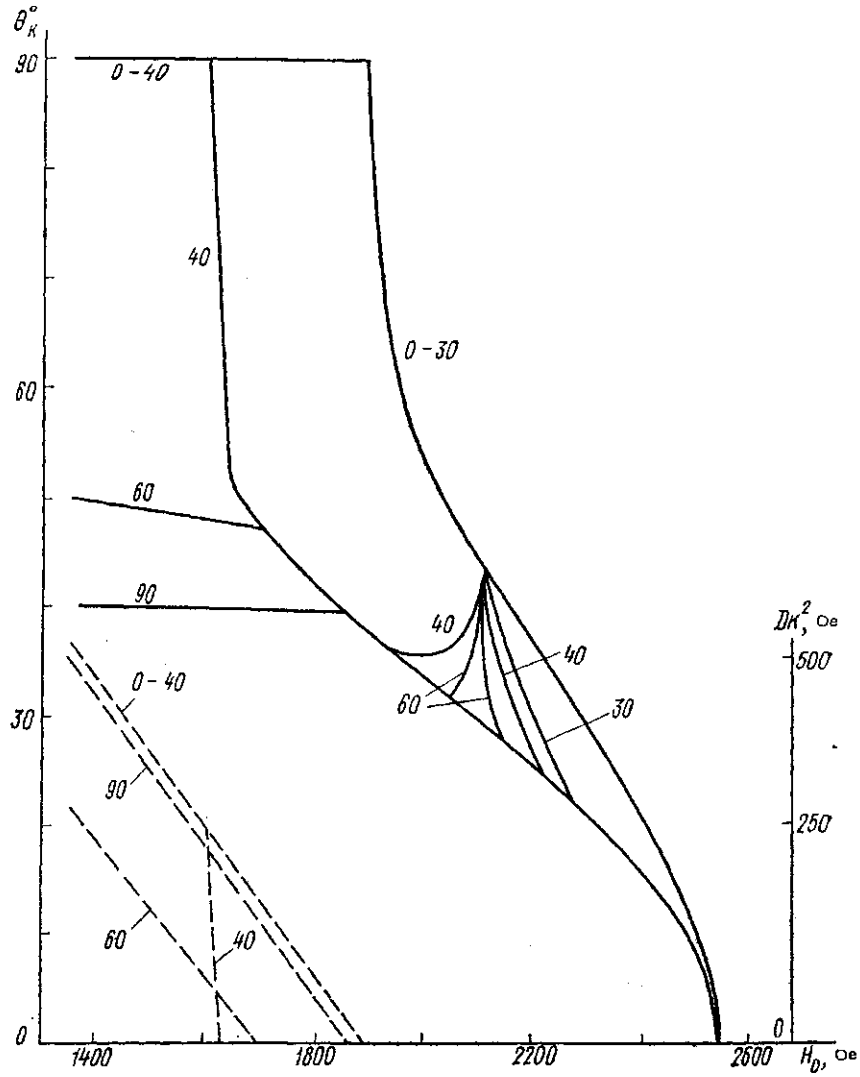

Fig. 3. The H_0 dependence of θ_k (solid lines) and D_k^2 (broken lines) are for various values of ψ (sphere of Ni ferrite, <110> axis, pumping frequency 9.4 GHz).

for $\varphi_k = 0°$, and 180° with $\theta_k = \theta_k^*$ (for the <110> axis), where $\sin^2\theta_k^* = 3\frac{|K_1|}{M} \Big/ 4\pi M$,

This feature of $\varepsilon_k(\theta_k, \varphi_k)$ for $\varphi_k = 0°$, 180° and $\theta_k = \theta_k^*$ (for the <110> axis) is due to balancing of the effects from the dipole-dipole interaction and the crystallographic anisotropy on the spin precession, which becomes circular.

Here the circular precession (ε_k = 1) corresponds to some angle $\theta_k^* \neq 0°$, which differs substantially from the case of an isotropic specimen, where ε_k = 1 corresponds to spin waves with θ_k = 0.

On the other hand, the condition ε_k = 1 denotes the absence of a transverse

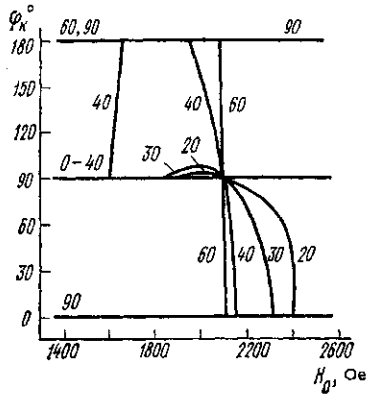

Fig. 4

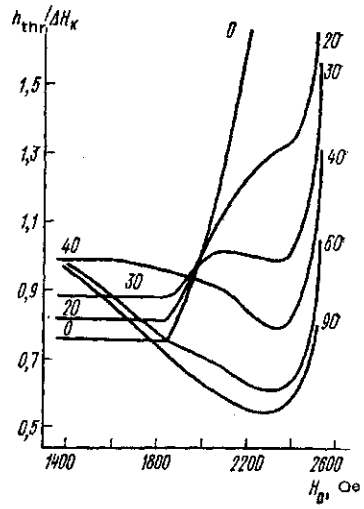

Fig. 5

Fig. 4. The H_0 dependence of φ_k for various ψ (Ni ferrite sphere, <110> axis, 9.4 GHz).

Fig. 5. The H_0 dependence of $h_{thr}/\Delta H_k$ for various ψ (Ni ferrite sphere, <110> axis, 9.4 GHz).

component in the inherent magnetic field of the spin wave in the isotropic and anisotropic cases. This means that such a spin wave cannot be excited parametrically via the double frequency-conversion mechanism of [9].

Therefore, particular interest attaches to the region of H_0 and the excitation conditions that provide parametric excitation of spin waves of ellipticity close to $\varepsilon_k = 1$ (direction and polarization of the UHF pumping field), because here one can expect to find special features in the spectrum and threshold fields for the parametric spin waves.

3. THRESHOLD-FIELD CALCULATION FOR A SPHERE OF Ni FERRITE

We consider the conditions for parametric excitation of first-order spin waves in a spherical single crystal of Ni ferrite ($|K_1|/M$ = 260 Oe, $4\pi M$ = 3270 G).

A computer was used in numerical minimization of the expressions for the threshold fields for the case $\mathbf{H}_0 \| <110>$, where the linearly polarized field $\mathbf{h}$ was oriented in the $(1\bar{1}0)$ plane at an arbitrary angle ψ to $\mathbf{H}_0$ [7]:

$$h_{thr} = \frac{\Delta H_k}{2}\frac{\omega}{|W|}, \text{ where } W = \frac{T_1\omega + T_2 Y}{XY - \omega^2}\sin\psi + B_k\cos\psi;$$

$$T_1 = \frac{\omega_M}{2}\sin\theta_k\cos\theta_k\left[\left(A_k + \frac{\omega}{2}\right)e^{i\varphi_k} - \right.$$

$$\left. - \frac{B_k^2}{|B_k|^2}\left(A_k - \frac{\omega}{2}\right)e^{-i\varphi_k} + 2iB_k\sin\varphi_k\right];$$

$$T_2 = \frac{\omega_M}{2}\sin\theta_k\cos\theta_k\left[\left(A_k + \frac{\omega}{2}\right)e^{i\varphi_k} + \right.$$

$$+\frac{B_k^2}{|B_k|^2}\left(A_k-\frac{\omega}{2}\right)e^{-i\varphi_k}-2B_k\cos\varphi_k\Big];$$

$$X=\omega_H-\omega_a;\quad Y=\omega_H+2\omega_a.$$

Figures 3-5 show the results for a sphere of Ni ferrite when the pumping field frequency was 9.4 GHz (it is assumed that ΔH_k = const, where ΔH_k is the spin-wave damping parameter); the numbers on the lines are the values of ψ in degrees.

The parametric spin-wave spectrum has the following characteristic features.

The $Dk^2(H_0)$ relation (Fig. 3) for the various ψ shows that (3) applies for Ni ferrite; in addition, the value of $(Dk^2)_\psi$ decreases on going from ψ = 90° to ψ = 0° (for H_0 = constant) zero, and then there is a jump to the value $(Dk^2)_{\psi=0°}$ (for the same H_0).

This variation in $(Dk^2)_\psi$ is completely explained by the above overlap between the regions of the anisotropic spectrum for specimens with high anisotropy.

There is a sharp change in all the parameters θ_k, φ_k, D_k^2 of the wave vector $\mathbf{k}(k, \theta_k, \varphi_k)$ for the parametric spin waves and also in the relative threshold field $h_{thr}/\Delta H_k$ for ψ = 40° when $H_0 \approx 1600$ Oe (Figs. 3-5) on account of transition from parametric excitation with preferential action of the longitudinal component (for H_0 < 1600 Oe) to the joint action of the longitudinal and transverse pumping fields (for H_0 > 1600 Oe). This phenomenon is also seen in ferrite specimens with low anisotropy.[7].

Figures 3-5 show that there are also sharp changes in θ_k and φ_k (Figs. 3 and 4) and H_0 = 2100 Oe, while there are less sharp changes in $h_{thr}/\Delta H_k$ (Fig. 5) for ψ of 20°, 30°, 40°, and 60°. The physical reason for this phenomenon is related to the impossibility of parametric excitation for spin waves with circular polarization (ε_k = 1), which has been discussed above.

Calculations show that ε_k = 1 for $\theta_k^* \approx 30°$, $\varphi_k = 0°$, 180° for Ni ferrite for H_0 = = 2100 Oe (under the above magnetization conditions); minimization results show that spin waves are excited near H_0 = 2100 Oe whose ellipticity is close to 1. The values of θ_k and φ_k alter rapidly as H_0 = 2100 is approached ($\varphi_k \to 90°$, $\theta_k \to 45°$), since parametric excitation of spin waves with $\varphi_k = 0°$, 180° and $\theta_k = \theta_k^* = 30°$ is impossible, as has been shown above.

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