EFFECTS OF SPATIAL DISPERSION IN THE NONLINEARITY ON THE NONSTATIONARY SELF-FOCUSSING OF A LASER BEAM IN A LIQUID CRYSTAL*

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The finite-element method has been used in a program for numerical solution of the equations describing laser self-focussing in the isotropic phase for nematic liquid crystals. It is shown that the spatial dispersion in the nonlinear susceptibility arising from the thermal and orientation mechanisms stabilizes the time envelope of the laser pulse, while sharpening the leading edge and reducing the characteristic self-focussing length.

1. Spatial dispersion in the cubic nonlinearity has a considerable effect on the self-focussing of a high-power light beam in the isotropic phase of a liquid crystal ($T > T^*$, where T^* is the temperature of the phase transition from nematic crystal to isotropic liquid) [1-3]. The most important mechanisms are the thermal one (change in the relaxation time of the order parameter when the liquid crystal is heated by the laser radiation [4]) and the orientation mechanism (increase in the correlation radius for the fluctuations in the order parameter near the transition region). The qualitatively new effects (all of them in some way related to increased stability in self-focussing) are due to the spatial dispersion in the nonlinearity as follows: 1) increased stability of the self-focussing against division of the beam into separate filaments (the effect has been observed by experiment [5]) and 2) quasi-waveguide propagation occurs in the steady state for a certain range in the parameters of the radiation and the medium, which is accompanied by stabilization of the minimum width of the focussed beam as the radiation power increases [6].

The qualitative features of the self-focussing that occur in the steady state [3] persist also in the nonstationary state; here of course the effects of the spatial dispersion are weaker. A new effect of spatial dispersion in the nonlinearity in the nonstationary state is of interest in controlling the output parameters of laser, namely stabilization of the time envelope with sharpening of the leading edge and reduction in the characteristic self-focussing length.

2. The self-focussing in the isotropic phase of a liquid crystal is described by a parabolic equation for nonlinear diffraction, the relaxation equation for the order parameter Q, and the thermal-conduction equation [2, 3]:

$$\left(\frac{\partial}{\partial z}+\frac{i}{2k}\Delta_\perp+\frac{1}{v}\frac{\partial}{\partial t}+\delta\right)E=-\frac{4\pi ik}{3n^2}\chi_a QE; \qquad (1)$$

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$$\eta \frac{\partial Q}{\partial t} + a(T - T^*) Q = b \Delta_\perp Q + 2\chi_a |E|^2/9; \qquad (2)$$

$$C\rho \frac{\partial T}{\partial t} = \varkappa \Delta_\perp T + \delta cn |E|^2/8\pi. \qquad (3)$$

Here k is wave number, v is group velocity, δ is absorption coefficient, $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$; n is the linear refractive index, χ_a is the anisotropy in the susceptibility, η is the viscosity, a is a constant independent of T, $b = a(T - T^*) L_\kappa^2/4$; L_c is the correlation radius for the fluctuations in Q, and $\varkappa$, C, and ρ are respectively the thermal conductivity, the specific heat, and density of the liquid crystal. We assume that the slow amplitude E of the electromagnetic wave at the boundary of the medium (z = 0) takes the form

$$E(t, r, z = 0) = E_0 \exp(-r^2/2r_0^2 - 2t^2/\tau_i^2), \quad r^2 = x^2 + y^2. \qquad (4)$$

The program of [3] for numerical solution of (1)-(3) can be used with certain minor changes for the nonstationary state, which occurs if the length τ_i of the incident pulse is comparable with the relaxation time of the order parameter $\tau_r = \eta/a(T - T^*)$ and is much less than the time needed to establish the temperature distribution $\tau_T = C\rho r_0^2/4\varkappa$. We consider separately the thermal mechanism of spatial dispersion in the nonlinearity ($\delta \neq 0$, $L_c = 0$) and the orientation mechanism ($\delta = 0$, $L_c \neq 0$)

3. In the first case, the dynamics of the self-focussing may be defined from the following parameters: $\alpha = P/P_1$ the ratio of the power in the incident beam $P = cnE_0^2 r_0^2/8$ to the threshold power for self-focussing without allowance for the spatial dispersion: $P_1 = 27\pi cn^3 a(T_0 - T^*)/2(8\pi k\chi_a)^2$ (T_0 is the thermostat temperature), $\beta = P_1/P_2$ the spatial-dispersion parameter for the thermal mechanism ($P_2 = 2\pi\varkappa(T_0 - T^*)/\delta$), and $\theta_r = \tau_r/\tau_i$, $\gamma = 4\tau_T/\tau_r$. Experiments with liquid crystals are usually performed for fixed values of τ_i and τ_T, while τ_r may be varied by more than two orders of magnitude by smooth variation of T_0. The numerical solution gives the normalized laser intensity $|E|^2$ at the axis ($\eta = 0$) in terms of the dimensionless time $\theta = (t - z/v)/\tau_i$ for various values of the parameters and of the dimensionless length $\xi = z/L_c$.

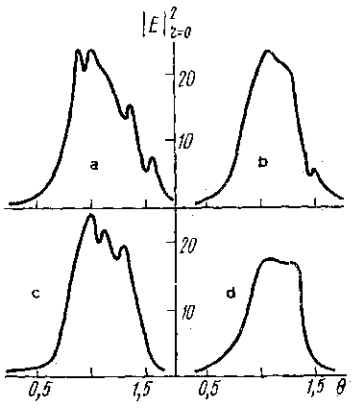

Fig. 1. Stabilization of the time envelope of a laser pulse for values of β of: a) 0; b) 0.2; c) 0.4; d) 1.

Figure 1 shows a series of typical curves for various β ($\alpha = 12$, $\theta_r = 0{,}1$; $\gamma = 10^4$, $\xi = 1$); the value $\beta = 0$ (Fig. 1a) corresponds to self-focussing in a medium with slow Kerr nonlinearity. Characteristic oscillations appear on the time profile [7], which are absent if ξ and α are small. The amplitude and frequency of these increase with α. The distortion increases and the pulse broadens as θ_r is reduced. The spatial dispersion also stabilizes the time envelope of the pulse. Figure 1 shows that the oscillations are gradually suppressed as β increases; the pulse is smooth when $\beta = 1$. The suppression of the oscillations begins at the end of the pulse; the rate of smoothing increases as

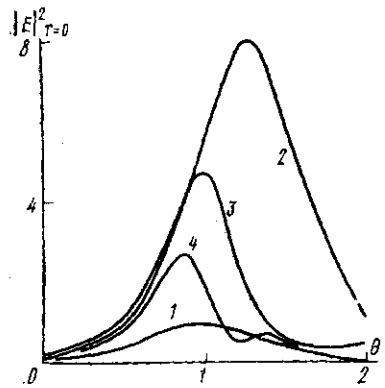

Fig. 2

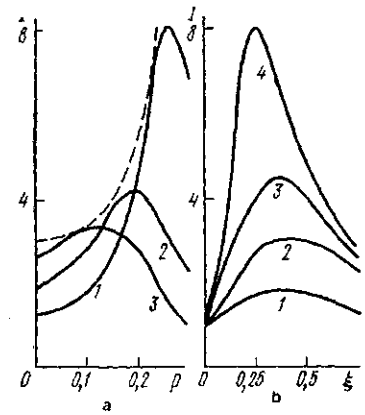

Fig. 3

Fig. 2. Behavior of the time envelope of a laser pulse in the presence of spatial dispersion in the nonlinearity: 1) $\xi = 0$; 2) $\xi = 0.25$; 3) $\xi = 0.5$; and 4) $\xi = 0.75$.

Fig. 3. Maximum intensity in a pulse in relation to : a) spatial-dispersion parameter for ξ of: 1) 0.25; 2) 0.5; 3) 0.75; b) in relation to dimensionless length for α of: 1) 6; 2) 10; 3) 14; 4) 20.

θ_r is reduced. The spatial dispersion ($\beta \neq 0$) results in the first oscillations occurring when α are ξ are larger than in the case of the slow Kerr nonlinearity. These results are the time analog of the stabilization of self-focussing against division into filaments [2] and may be used in producing smooth high-intensity pulses.

4. If the main spatial dispersion mechanism is the orientation one ($\delta=0$, $L_c \neq 0$), then the dynamics of the self-focussing may be determined by α, θ_r, and $p=(L_c/2r_0)^2$. If p is of the order of 10^{-2}, the variation in the time envelope as ξ increases is as in a medium with ordinary Kerr nonlinearity, for which there is sharpening of the trailing edge. If p is sufficiently large, one gets some special features. The peak intensity shifts towards the start of the pulse from a certain ξ_p onwards, i.e., the leading edge is sharpened. Figure 2 shows a series of characteristic curves ($\alpha=40$, $\theta_r=2$, $p=0{,}2$). The oscillations (see Section 3) are absent for these values of the parameters.

One can obtain the maximum possible pulse intensity in two ways for given ξ and θ_r: by optimizing the spatial-dispersion parameter and by raising α. Figure 3a shows the maximum intensity I ($\alpha=28$, ξ, $\theta_r=2$, p, $r=0$) as a function of p for various values of ξ. The value of p_{opt} defined from $\partial T(p)/\partial p=0$, decreases as ξ increases. The I (ξ) relation is shown in Fig. 3b for various α ($\theta_r=2$, $p=0{,}2$). Therefore, there is a marked increase in I on increasing p and α for ξ small.

The dashed line in Fig. 3 shows the absolute maximum intensity $I_M(\alpha, \theta_r, p)$ as a function of p($I_M=I(\xi_M)$, where ξ_M is defined by $\partial I(\xi)/\partial\xi=0$) . Here I_M increases with p, while the peak point ξ_M shifts to smaller ξ, i.e., spatial dispersion in the nonlinearity causes an increased rate of self-focussing. Also, I_M is proportional to the square of α. These calculated time envelopes can be compared with experiment [6] for self-focussing in liquid crystals with $0{,}1<\theta_r<10$, there

is qualitative agreement.

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