CALCULATION OF THE IMAGINARY PART OF THE OPTICAL POTENTIAL IN THE COLLECTIVE MODEL FOR SPHERICAL NUCLEI

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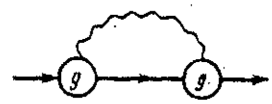
The paper is concerned with the angular dependence of the imaginary part of the optical potential for even-even nuclei in the low-energy region with allowance for excitation of lower collective states of the target nucleus. It is shown that the principal contribution to Im V_{opt} is made by the low collective states 2^+ and 3^- of the target, which completely exhaust the entire imaginary part of the optical potential in the low-energy region. According to calculations, the contribution to Im V_{opt} due to formation of the compound nucleus is small.

In its time the optical model was an important achievement of the theory of the atomic nucleus; its success showed that the scattering of nucleons by nuclei is, to some approximation, rather satisfactorily described within the limits of the concept of a complex potential, i.e., it can be reduced to a single-particle problem.

Over the more than 20 years of the existence of the optical model an enormous amount of work was performed both for clarifying its theoretical meaning and on semimicroscopic parameterization of potentials. The flow of work concerned with the optical model is still continuing, but there is one interesting aspect of this problem which, in our opinion, has not been sufficiently explored. We are referring to the effect of the nuclear structure on the calculation of the imaginary part of the optical potential [1-3]. The formalism and computational technique of the present study are close to those used in [1-3]. The imaginary part of the optical potential is due to the feasibility of inelastic collisions, and also of resonance formation of an excited complex, with a relatively long lifetime. Among predominantly inelastic processes (within the framework of direct collisions) between a nucleon and the nucleus in the low-energy region we count collisions, as a result of which the lower collective states of the target nucleus are excited. We are starting with the assumption that the optical potential can be subdivided into two parts: one part (V^s_{opt}) is associated with summation over noncollective states of the target nucleus, depends little on time and changes little from one nucleus to another; and the second part (V^Q_{opt}), which is controlled by the specifics of the collective excitations for the given target nucleus:

$$V_{opt} = V^s_{opt} + V^Q_{opt}(E, A).$$

Subsequently we shall be primarily interested in the contribution, to the imaginary part of the optical potential, from this type of inelastic processes. As shown by Van Giai with his coworkers [1] and by Migdal [4], the general expression for the optical potential has the form:

$$V_{opt} = V^{s}_{opt} +$$  (1)

where block g is related to the residue of the vertex part of Γ over the particle-hole channel [4]:

$$g = UGGg, \tag{2}$$

U is the irreducible vertex part along the particle-hole channel. Block g can be written in the form:

$$g = \Gamma^{\omega} \chi_s, \tag{3}$$

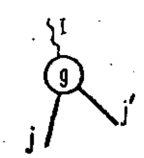
where Γ^{ω} is the amplitude of scattering in the medium [4], whereas χ_s is obtained from the expression

$$\chi_s = A \Gamma^{\omega} \chi_s. \tag{4}$$

Here A is the integral of the product of the pole parts of single-particle Green's functions. General expression (1) for the part of the optical potential associated with the contribution from inelastic scattering with excitation of lower collective states of the target nucleus, and also with the excitation of the intermediate nucleus in the bound particle + lower collective excitation state is (in the j representation):

$$V^{Q}_{opt}(jj', E) = \sum_{j_1} \frac{g^{I}_{jj_1}(\omega_s)\, g^{I}_{j_1 j'}(\omega_s)}{E - \omega_s - \varepsilon_{j_1} + i\Delta/2},$$

where j and j' is the set of quantum numbers of the states of the continuous spectrum, j_1 corresponds to the discrete and continuous spectrum of the intermediate nucleon, Δ is the energy interval of the averaging; ω_s is the collective excitation energy, and E is the input-channel energy. In the microscopic theory the resultant expression for block $g^{I}_{jj'}$ is:

 $$= \sum_{j_1 j_2} \chi^{I}_{j_1 j_2} \sqrt{2I+1} \langle j_1 \bar{j}_2;\ I | \bar{V} | j \bar{j}';\ I \rangle. \tag{5}$$

where

$$\chi^{I}_{j_1 j_2} = \langle I \| (a^{+}_{j_1} a_{j_2})_I \| 0 \rangle,$$

$\bar{V}$ is the operator of the effective nuclear interaction (in the particle-hole channel). The matrix element of paired interaction $\langle j_1, \bar{j}_2; I | \bar{V} | j \bar{j}', I \rangle$ is calculated from standard formulas in the shell model. Block $g^{I}_{jj'}$ can be written with sufficient accuracy, by introducing collective variable nuclei [5]:

$$g^{I}_{jj'} = \frac{V_0 \beta_I}{2\sqrt{\pi}} (-1)^{j-1/2} \langle l | F_I | l' \rangle \sqrt{(2j+1)(2j'+1)} \times \tag{6}$$

$$\times \left\langle j \frac{1}{2} j' - \frac{1}{2} \middle| I 0 \right\rangle, \qquad (6)$$

where β_I is a deformation parameter with multipolarity I,

$$F_I(r) = \frac{V_I(r)}{V_0 \sqrt{2I+1}}$$

the form factor for the vibrational nuclei:

$$V_I(r) = -V_0 \frac{R_0}{a} e(1+e)^{-2},$$

$e = \exp\{(r-R_0)/a\}$; V_0, $R_0 = r_0 A^{1/3}$, a are the parameters of the Woods-Saxon potential.

The imaginary part of the optical potential for vibrational nuclei is due to the fact that, as a result of the interaction, the particle leaves the input channel and is emitted in one of the inelastic channels, when the target nucleus is in the excited vibrational state (IM):

$$\mathrm{Im}\, V_{\mathrm{opt}}(ljE) = -\frac{1}{2j+1} \sum_I \frac{\hbar\omega_I}{2C_I} \sum_{l'j'} |\langle lj \| Y_I \| l'j' \rangle|^2 \times$$

$$\times \left\{ \frac{1}{2} \sum_n \frac{\Delta}{(E - E_I - \varepsilon_{nl'j'})^2 + \Delta^2/4} \langle \chi_{ljE} | K(r) | \Phi_{nl'j'} \rangle \times \right. \qquad (7)$$

$$\times \langle \Phi_{nl'j'} | K(r') | \chi_{ljE} \rangle + \pi \langle \chi_{ljE} | K(r) | \Phi_{\varepsilon_I l'j'} \rangle \langle \Phi_{\varepsilon_I l'j'} | K(r') | \chi_{ljE} \rangle,$$

where $\varepsilon_I = E - E_I$, E_I is the energy of excitation of the collective state with moment I. The first term in Eq. (7) is controlled by the contribution from the formation of the intermediate nucleus, whereas the second term is governed by the contribution from inelastic scattering channels. Quantity Im $V_{opt}(ljE)$ under study is simply related to the absorption cross section:

$$\sigma_{\mathrm{abs}}^{ljE} = -\frac{2}{\hbar J} \mathrm{Im}\, V_{\mathrm{opt}}(ljE), \qquad (8)$$

where J is the flux of the incident particles; for function χ_{ljE}, satisfying the norming condition

$$<\chi_{ljE} | \chi_{ljE'}> = \delta(E - E'),$$

this flux is equal to

$$J = \frac{k^2}{\hbar (2\pi)^3}.$$

Here **k** is the wave number of the incident flux.

The total absorption cross section $\sigma_{\mathrm{abs}}^{E}$ is related to $\sigma_{\mathrm{abs}}^{ljE}$ by the expression

$$\sigma_{\mathrm{abs}}^{E} = 4\pi \sum_{lj} \left(j + \frac{1}{2} \right) \sigma_{\mathrm{abs}}^{ljE}. \qquad (9)$$

For functions χ and Φ, specified in the form of plane waves, we obtain for Im $V_{opt}(\mathbf{k}, \mathbf{k}', \theta)$ the following (contribution from inelastic channels):

$$\mathrm{Im}\, V_{opt}(\mathbf{k}, \mathbf{k}', \theta) = -\frac{m}{\pi^2\hbar^2} \sum_{l_1 l_2} Z_l \frac{(2l+1)(2l_2+1)}{\sqrt{4\pi(2l+1)}} \times$$
$$\times \langle l 0 l_2 0 | l_1 0 \rangle^2 Y_{l_1 0}(\theta) \frac{\beta_l^2}{2l+1} J_{l_1 l_2}(kZ_l) J_{l_2 l_1}(kZ_l), \quad (10)$$

where

$$J_{l_1 l_2}(kZ_l) = \int j_{l_1}(kr) V_l(r) j_{l_2}(Z_l r) r^2 dr, \; Z_l = \sqrt{(2m/\hbar^2)(E - E_l)}.$$

$\hbar\omega_l/2C_l = \beta^2_l/2l+1$ controls the deformability of the nucleus, $\mathbf{k}'$ is the wave vector of the departing flux, E_I is the nucleus excitation energy, ω_I is the frequency of the surface vibrations of the nucleus, $j_l(kr)$ are spherical Bessel functions, and θ is the scattering angle. In this paper we are investigating angular relations for Im V_{opt} for even-even nuclei in the low-energy region with allowance for the excitations of lower collective states.

The ^{40}Ca Nucleus. In our calculations we included the contributions of the following collective excitations of the target nucleus: 3^- - 3.73 MeV (0.358); 2^+ - 3.3 MeV (0.143); 5^- - 4.48 MeV (0.132); 2^+ - 8.0 MeV (0.309); 4^+ - 8.0 MeV (0.254); 3^- - 15.73 MeV (0.38); 2^+ - 16.0 MeV (0.250); 5^- - 16.48 MeV (0.053); 1^- - 18 MeV (0.087). The numbers in the parentheses represent distortion parameters β_I. The study was performed for incident neutron energies of 7, 14.6 and 20 MeV.

Figure 1 - Ia shows the calculation of Im V_{opt} for the ^{40}Ca nucleus at E = = 14.6 MeV (1 is the theoretical curve in the plane-wave approximation, whereas 4 is the phenomenological local optical potential [6]). Figure 1 - Ib compares the results obtained in the plane-wave (curve 1) and distorted intermediate wave (3) approximations. Curve 2 corresponds to the nonlocal phenomenological potential [7]. The distorted wave approximation improves the calculations in the small-angle region. The principal contribution in the plane-wave approximation at θ = = 0 is given by the 3^- (41%) and 2^+ (35%) levels. The principal contribution in the distorted-wave approximation at θ = 0 is made by the 3^- (23%), 2^+ (46%) and 4^+ (21%). The contributions to Im V_{opt} due to formation of the composite nucleus (the phonon + quasi-bound particle intermediate state) for the energies under study were shown by calculations to be effectively low ($\leqslant$ 10%). Figure 1 - Ic shows calculations of Im $V_{opt}(\theta)$ at E = 20 MeV (1) and 7 MeV (1') in the plane-wave approximation. The principal contribution at 20 MeV (θ = 0) is given by the 3^- (20%), 2^+ (19%), and 3^- (21%) levels. At 7 MeV these are 3^- (69%) and 2^+ (30%).

The ^{90}Zr Nucleus. The values of Im V_{opt} at neutron energies of 7, 14.5 and 20 MeV were calculated from 14 excited levels:

2^+, 2,18 MeV (0,071); 2^+, 3,31 MeV (0,028);
2^+, 3,84 MeV (0,037); 3^-, 2,75 MeV (0,125);
3^-, 5,12 MeV (0,031); 3^-, 5,65 MeV (0,053);
3^-, 5,78 MeV (0,030); 4^+, 3,07 MeV (0,032);
4^+, 4,07 MeV (0,020); 4^+, 4,35 MeV (0,044);
4^+, 5,38 MeV (0,028); 4^+, 5,48 MeV (0,030);
5^-, 2,31 MeV (0,053); 5^-, 3,96 MeV (0,033).

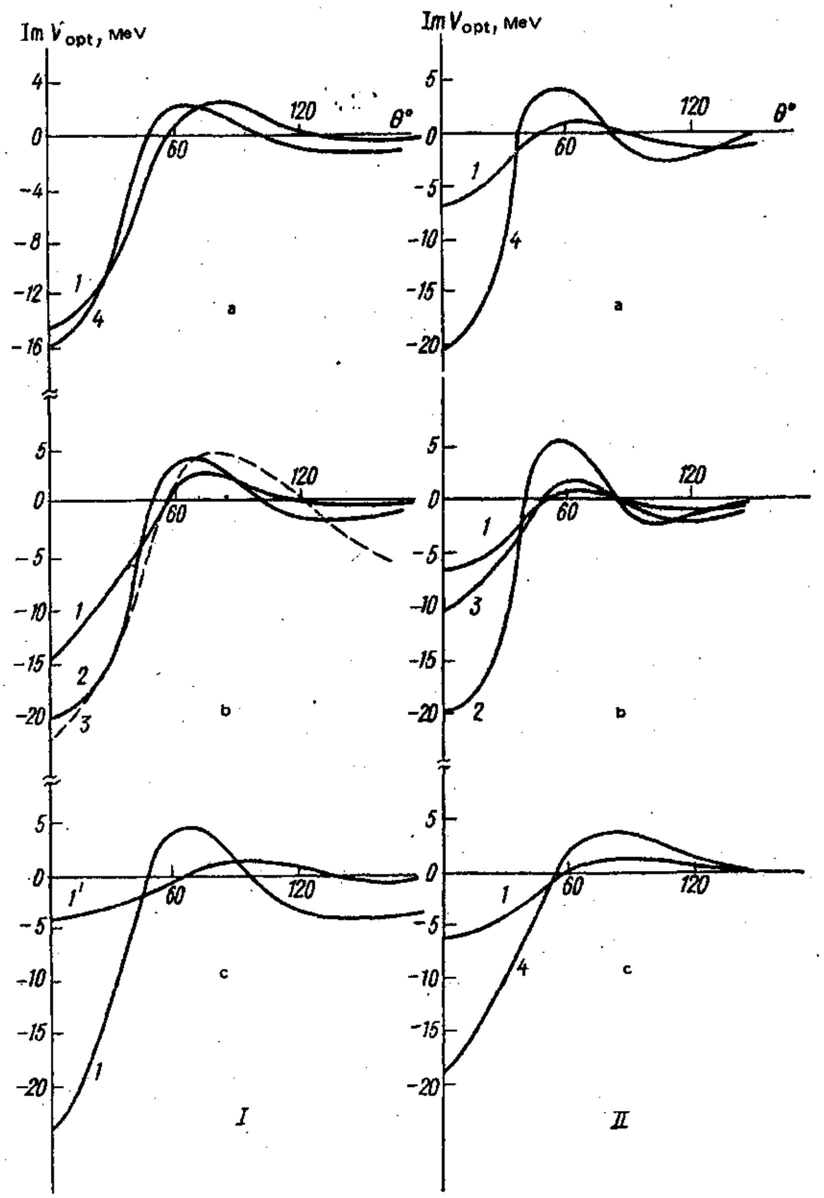


Fig. 1. Im $V_{opt}(\theta)$ for ^{40}Ca (I) and ^{90}Zr (II) at E = 7.0 (c) and 14.6 MeV (*a* and b). 1) Calculation from the collective model in the plane-wave approximation; 2) nonlocal phenomenological potential; 3) calculation in the distorted-wave approximation; 4) local phenomenological potential.

Figure 1 - IIa shows results in the plane-wave approximation at E = 14.5 MeV (1 is the theoretical curve, 4 is the phenomenological local potential [8]). Figure 1 - IIb shows Im $V_{opt}(\Theta)$ in the plane-wave approximation (1), in the distorted-wave approximation (3), and the nonlocal phenomenological potential [4,8] (2). The principal contribution in the distorted-wave approximation at $\theta = 0$ to Im V_{opt} is given by the 2^+ level (15%), 2^+ (27%), 3^- (36%) and 3^- (11%). The results for 7 MeV in the plane-wave approximation are plotted in Fig. 1 - IIc (1 is the

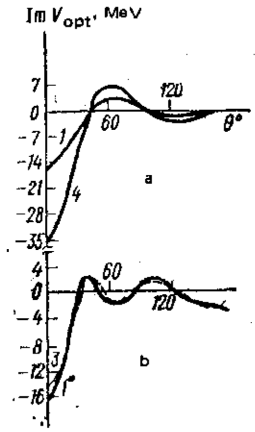


Fig. 2. Plot of Im $V_{opt}(\theta)$ for ^{208}Pb at E = 7 (a) and 20 MeV (b). For legend see Fig. 1.

theoretical curve, 4 is the local phenomenological potential [8]). The principal contribution is made by the 3^- (52.7%) and 2^+ (21%) levels. Similar results were obtained at 20 MeV. The principal contributions are from the 2^+ level (17%) and the 3^- level (43%). Calculations show that the contribution to Im V_{opt} from the formation of the composite nucleus is small and amounts to not more than 10%.

The ^{208}Pb Nucleus. The levels included were: 2^+ – 4.06 MeV (0.071); 3^- – 2.62 MeV (0.119); 4^+ – 430 MeV (0.099); 5^- – 3.20 MeV (0.069). Figure 2a shows the results at E = 7 MeV in the plane-wave approximation (curve 1); curve 2 corresponds to the local phenomenological potential [8]. The contribution of the individual levels are: 2^+ (13%), 3^- (49%), 4^+ (28%) and 5^- (10%).

Contributions to Im V_{opt} due to formation of the composite nucleus at E = 7 MeV are not more than 10%. Similar results were obtained for E = 14.3 and 20 MeV. Calculations in the plane-wave and distorted-wave approximations yield close results (see Fig. 2b). The greatest contribution is given by the 3^- level (47%). Analysis of Im V_{opt} for the nuclei under study shows that the main contribution is given by low collective states 2^+ and 3^- of the target nucleus and almost fully exhaust the entire "observed" imaginary part of the phenomenological optical potential in the low-energy region.

REFERENCES

1. N. Van Giai, J. Sawicki, and N. Vinh-Mau, "Microscopic theory of the optical-model potential and the hole-particle model in nuclear spectroscopy," Phys. Rev., vol. 141, pp. 913-926, 1966.

2. F. A. Zhivopistsev and E. S. Rzhevskii, "Concerning the contribution of collective excitation of the target nucleus to the imaginary part of the optical potential," Vestn. Mosk. un-ta. Ser. Fiz. Astron. [Moscow University Physics Bulletin], vol. 15, no. 6, pp. 661-666, 1974.

3. A. Lev, W. Beres, and M. Divadeenam, "Complex nonlocal optical potential for neutron scattering from ^{208}Pb," Phys. Rev., vol. 96, pp. 2416-2434, 1974.

4. A. B. Migdal, Theory of Finite Fermi Systems and Properties of Atomic Nuclei [in Russian], Moscow, 1965.

5. A. Bohr and B. R. Mottelson, "Rotational states in even-even nuclei," Phys. Rev., vol. 90, pp. 717-719, 1953.

6. F. E. Bjorklund, S. Fernbach, and N. Sherman, "Optical model of nucleus with absorbing surface," Phys. Rev., vol. 101, pp. 1832-1833, 1956.

7. C. A. Engelbrecht and H. Fiedeldey, "Nonlocal potentials and energy dependence of the optical model for neutrons," Ann. Phys., vol. 42, pp. 262-295, 1967.

8. F. Perey and B. A. Buck, "A nonlocal potential model for the scattering of neutrons by nuclei," Nucl. Phys., vol. 32, pp. 353-380, 1962.

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