CONCERNING THE WAVE-INTENSIVE BEAM LIMITING STATE IN PLASMA

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Vol. 36, No. 2, pp. 8-12, 1981

UDC 533.922

This paper examines the wave-beam state in an infinite electron plasma at the stage when the particles are captured by the wave and undergo phase mixing. The state of the beam is described by nonlinear self-congruent equations, the plasma remains a linear, dispersing medium. Estimates are obtained, characterizing the redistribution of the energy of the initial beam at single-mode generation of the longitudinal field by a low-density relativistic electron beam. The maximum generation efficiency amounts to 10% and is attained for nonrelativistic beams at $(n_0/n_p)^{1/3} \approx 0.5$, where $n_{0,p}$ are the beam and plasma densities. The heating effectiveness at this stage is estimated. A comparison with results of numerical experiments is performed.

1. In their preceding paper [1] the authors obtained equations describing the nonlinear wave-beam state in a plasma which allows obtaining detailed information about the system. The present paper is concerned with analysis of these equations.

Using Eqs. (8) from [1] we obtain an expression which is an analog of the dispersion equation for the nonlinear wave-beam state in the case when $v=\omega/k$ and $\zeta=\beta^2_0$:

$$\frac{n_0}{n_p}=\frac{4\gamma_0}{\beta_0^3}(\beta_0^2-\beta^2)\left(\frac{X}{2}+\frac{Y}{\beta}\right), \tag{1}$$

where $n_{0,p}$ are the beam and plasma densities, whereas $\beta_0 c$, βc are the initial and final beam velocities.

The numerical solution of Eq. (1) is shown in Fig. 1. At $(n_0/n_p)^{1/3}\ll\gamma_0$ it follows from Eq. (1) that $\beta/\beta_0=1-0{,}5(n_0/n_p)^{1/3}\gamma_0^{-1}$. Dense beam may be perceptibly decelerated, so that $\beta\ll\beta_0$. In this case it follows for nonrelativisitc beams from (1) that $\beta\simeq 2\beta^4_0(n_0/n_p)$.

Another important characteristic of the state of the beam is the particle capture factor. Equations (8) from [1] yield:

$$\alpha^2=\frac{\beta X}{Y+0{,}5\beta X}\xrightarrow[\beta\ll\beta_0]{}1-\gamma_0\beta_0^2\left(\frac{n_0}{n_p}\right)^{1/3}. \tag{2}$$

Complete particle capture ($\alpha = 1$) is possible only in the limit of vanishing beam fluxes. At $\beta \simeq 0$ virtually all the particles slip relative to the wave. The obvious physical cause of the latter is that the capture conditions deteriorate significantly upon substantial restructuring of the beam ($\beta \ll \beta_0$).

The equilibrium amplitude of the field is:

$$E_0 = \frac{2^{3/2}k\gamma_0}{\beta X\sqrt{\beta X+2Y}} \xrightarrow[\beta\ll\beta_0]{} 2k\gamma_0\beta_0^2\left(\frac{n_0}{n_p}\right)^{2/3}, \qquad (3)$$

where E_0 is in MV/cm. When the beam is decelerated, the amplitude of the field first increases, and then falls off. This falloff of the field, in spite of increasing beam intensity, is due to reduction in particle capture. The kinetic energy of the beam at this stage (see below for details) is primarily converted to the kinetic energy of the relative longitudinal motion of particles performing phase vibrations ("longitudinal temperature" of the beam).

In the wave coordinate system (Σ') the ratio of the energy of the field to the energy of the relative motion of particles in the beam is $W'_E/\Delta W'_b = \alpha^2/(1+0{,}5\,\alpha^2) \to 0$ as $\alpha \to 0$. True, Eq. (1) is valid when $E_0 < 0{,}25\,k\alpha/\gamma$ (nonrelativistic motion of particles in Σ'); in addition, it was assumed that $E_0 < 0{,}25\,k(\gamma-1)$ since the plasma electrons are not captured by the wave. Hence Eq. (3) describes properly the variation in E_0 over the entire range of possible values of β only for weakly relativistic beams with $\gamma_0 \leqslant 1{,}5-1{,}8$ (Fig. 2). In this case $\beta_1(E_{max}) \simeq 0{,}35\,\beta_0^2$, whereas $2E_{max} \simeq 0{,}3k(1+4\,\beta_0^2)^{1/2}$.

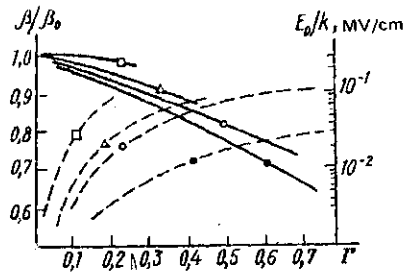

Fig. 1

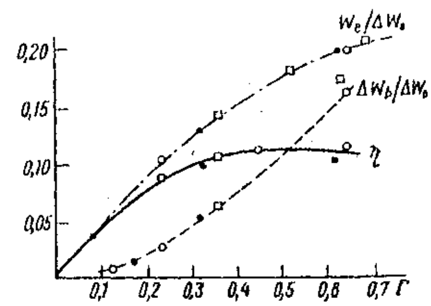

Fig. 2

Fig. 1. Nonlinear shift in beam velocity and generation frequency $\beta/\beta_0 = \omega/\omega_0$ (solid curve) and field amplitude E_0/k (dashed curve) at different values of $\Gamma = (n_0/n_p)^{1/3}$ □ — $\gamma_0 = 8$; $\beta_0 = 0{,}992$; △ — $\gamma_0 = 2$, $\beta_0 = 0{,}857$; ○ — $\gamma_0 = 1{,}5$; $\beta_0 = 0{,}745$; ● — $\gamma_0 = 1{,}1$; $\beta_0 = 0{,}417$.

Fig. 2. Energy of the field (solid curve), of plasma electrons W_e (dashed curve) and of the beam ΔW_b (dash-dot curve) for weakly relativistic beams in units of the initial beam kinetic energy $\Delta W_0 = mc^2(\gamma_0 - 1)n_b$ at different values of parameter $\Gamma = (n_0/n_p)^{1/3}$ $\gamma_0 = 1{,}5$(○), 1,3(□) and 1,1(●).

Equations (1) and (3) can be used for clarifying additional characteristics of the nonlinear wave-beam state. At $\beta_0 > \beta > \beta_1(E_{max})$, $\frac{\partial\omega}{\partial E_0} < 0$, which produces conditions for modulation instability of the system. The total energy of the (beam + + field + plasma) system is equal to the initial energy of the particle, which are "introduced" upon generation into a unit of volume: $W_S = mc^2\gamma_0 n_b = W_0\beta_0/\beta$. Hence $\mathrm{sgn}\frac{\partial W_s}{\partial E_0} = -\mathrm{sgn}\frac{\partial\beta}{\partial E_0}$, i.e., $\frac{\partial W_S}{\partial E_0} > 0$ at $\beta > \beta_1(E_{max})$, but $\frac{\partial W_S}{\partial E_0} < 0$ at $\beta < \beta_1(E_{max})$. In this state, in spite of the inception of temperature scatter in the beam and the exact equality $\beta c = \omega/k$, the system remains one with negative energy, i.e., particularly susceptible to different decay processes. The instability of the nonlinear wave is actually confirmed by numerical experiments of Thode and Sudan [2], although it should be noted that the energy scatter in the beam should slow down the development of instabilities.

2. Let us consider in more detail the redistribution of the beam energy occurring upon generation of the field. The entire energy which the beam particles are capable of losing is equal (per unit volume) to $\Delta W_0 = mc^2(\gamma_0 - 1)n_b$. The relative fraction of energy conserved in translational motion of the beam upon attaining equilibrium is:

$$\frac{W_b}{\Delta W_0} = \frac{\gamma - 1}{\gamma_0 - 1}. \qquad (4)$$

The fraction of the energy of relative motion of particles in the beam (which incorporates captured and noncaptured particles)

$$\frac{\Delta W_b}{\Delta W_0} = \gamma_0 \frac{\beta X + Y}{\gamma_0 - 1}. \qquad (5)$$

The energy stored directly in the field defines the efficiency η of generation. An estimate of the efficiency follows directly from the law of conservation of momentum in Σ' [1]:

$$\eta = \frac{E_0^2}{16\pi\Delta W_0} = \frac{\gamma_0\beta^3(\beta_0 - \beta)}{(\gamma_0 - 1)\,|\beta_0^2(2 - \beta^2) - \beta^4|}. \qquad (6)$$

Finally, a part of the beam's energy is transmitted directly to the plasma electrons oscillating in the field of the wave:

$$\frac{W_e}{\Delta W_0} = \frac{E_0^2}{16\pi\Delta W_0}\left[\frac{\partial(\omega\varepsilon)}{\partial\omega} - 1\right] = \eta\frac{\beta_0^2}{\beta^2}. \qquad (7)$$

The generation efficiency is of particular interest. The value of η as a function of n_0/n_p first increases, and then monotonically falls off, which confirms to the predictions of Thode and Sudan [2]. At low fluxes

$$\eta = \left(\frac{n_0}{n_p}\right)^{1/3} \begin{cases} 0{,}5 & \text{at} \quad \gamma_0 \simeq 1, \\ 0{,}25\gamma_0 & \text{at} \quad \gamma_0 \gg 1. \end{cases} \qquad (8)$$

The dependence of η on the system parameters was frequently discussed. For weakly-relativistic beams it is best to estimate η in units of parameter $\Gamma = (n_0/n_p)^{1/3}$, since the results here depend only on the value of β/β_0. The maximum value of η corresponds, as can be easily seen from Eq. (6), to $\beta \approx 0.75\beta_0$, i.e., $\Gamma \approx 0.5$ and attains the value of $\eta_{max} \approx 0.11$. Here $E_0 \simeq 0{,}2\,k\beta_0^2$ MV/cm, $E_0 < E_{max}$, and $\alpha^2 \approx 0.75$. The beam loses about 40% of the energy of translational motion.

The energy transmitted to the plasma electrons is, at $\gamma_0 \geq 1$, described by the simple ratio $W_e/\Delta W_0 \simeq \beta(\beta_0 - \beta)/\beta_0^2$ with a maximum approximately equal to 0.25 at $\beta \approx 0.5\beta_0$; here everywhere $W_e \gg \Delta W_0\eta$. The specific energy transmitted to the individual plasma electrons is:

$$\frac{W_e}{n_p} = \frac{n_b \beta_0^2}{n_p \beta^2} mc^2 (\gamma_0 - 1) \eta. \qquad (9)$$

At $\eta \approx \eta_{max}$ the plasma electrons acquire an energy equal to ≈3% of the initial kinetic energy of the beam particles, and 18-20% of the incident-beam energy is stored in the plasma as a whole. Thus, for $\gamma_0 \approx 1.5$ the energy of the vibrational motion of the plasma ("longitudinal temperature") is 5 to 10 keV.

The energy ΔW_b of relative particle motion in the beam at densities of $0 < \Gamma < 0.5$ is proportional to Γ^2 and remains smaller than the energy transmitted to both the plasma and the field, becoming equal to the latter at $\Gamma \simeq 0{,}4-0{,}5$*. Then, at $\Gamma \geqslant 1{,}5-2$, ΔW_b, it, increasing monotonically, accumulates virtually the entire initial, energy of the beam. At $\eta \simeq \eta_{max}$, i.e., at $\Gamma \simeq 0{,}5$, $\Delta W_b \simeq (0{,}12-0{,}14)\,\Delta W_0$ (see Fig. 2).

Nevertheless, the specific energy of relative motion of particles in the beam (beam "temperature") $\Delta W_b/n_b$ exceeds the analogous value for plasma. Their ratio is $\Delta W_b n_p / W_e n_b \sim \Gamma^{-2} \leqslant 4$ at $\eta \simeq \eta_{max}$, so that the beam "heats up" more effectively than the plasma electrons.

3. For relativistic beams, following the results of the linear theory, the generation efficiency is estimated in units of parameter $S = \gamma_0 \beta_0^2 (n_0/2n_p)^{1/3}$. At $S \leq 1/3$ to 1/4 we have $\eta = \text{const} \cdot S$. According to Eq. (6), const = 0.32. Kovtun and Rukhadze [3] found that const = 0.16 to 0.32. Judging by the results of numerical experiments by Thode and Sudan [2], Eq. (6) describes with sufficient accuracy the increase in η at low S, whereas the results of Kovtun and Rukhadze [3] are somewhat on the low side. The semiqualitative equation $\eta = 0{,}5\,S(1+S)^{-5/2}$, suggested by Thode and Sudan [2] for describing data of numerical experiments overestimates η at low S.

For $\gamma_0 \gg 1$ the redistribution of the incident-beam energy observed upon excitation of the field differs from that considered above. As was noted, $\eta \approx 0.32S$. Since here the relative variation in the mean velocity of beam particles is rather small: $\beta_0 - \beta \simeq 0{,}6\, S/\gamma_0^2\beta_0^2$, then the amount of energy accumulated in the plasma is close to the energy of the field, i.e., $W_e \simeq \Delta W_0 \eta$, including for S > 1. For the beam $\Delta W_b/\Delta W_0 \simeq 2S^2(\gamma_0\beta_0)^{-2}$ the specific energy (per one particle) of vibrational motion of the plasma electrons is $W_e/n_p \simeq 0{,}6\, mc^2 S^4 (\gamma_0\beta_0)^{-2}$, and respectively for the beam $\Delta W_b/n_b \simeq 2mc^2 S^2/\gamma_0^2\beta_0^2$.

At $S \geq 0.5$ the expressions obtained here must be refined, since they are valid only for relatively weak fields (nonrelativistic motion in the beam in Σ'). Hence the limiting value of $\eta_{max} \approx 0.16$, which follows from (6), is on the high side at $\gamma_0 \gg 1$ (the numerical experiments of Thode and Sudan point to $\eta \approx 0.11$ at $S \approx 0.6-0.7$).

Nevertheless it is possible to draw an important conclusion even on the basis of our present relationships. As was noted by various investigators, it is particularly convenient to use relativistic beams for generation of the field. This conclusion follows from the linear theory, where the generation efficiency is proportional to γ_0. Hence Thode and Sudan considered the feasibility of employ-

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*This explains why the nonlinear theory, which employs the hydrodynamic approximation [3] and makes no allowance for the internal kinetics of the beam ($\alpha = 1$), yields satisfactory results at low fluxes.

ing relativistic beams for heating of plasma as early as at the single-mode generation stage. Unfortunately, this prediction could not be confirmed. Under nonlinear conditions the maximum generation efficiency does not depend on γ_0 and does not exceed 0.1. The total losses of translational energy of the incident beam amount to ≈30%, and are distributed, at $\eta \simeq \eta_{max}$ approximately equally between the field, plasma and the beam. Moreover, the energy of the vibrational motion of the plasma electrons (per particle), interpreted here as the electron temperature of the plasma (designated below as T_e in units of mc^2), falls off with increasing γ_0. This is associated with reduction in the value of n_0/n_p at which the maximum amount of energy is released in the field. For example, at $\gamma_0 = 10$ and $n_0 = 3\cdot10^{-4}n_p$ we have $T_e = 5\cdot10^{-4}$.

4. The above shows that nonlinear analysis of the finite wave-beam state makes possible to obtain a number of important analytic estimates. These results should be generalized to the case of generation of strong fields, together with analysis of plasma waveguide systems, which are most frequently encountered in experiments. It is of particular interest to consider the limiting wave-beam states for surface waves.

The authors wish to thank A. A. Kolomenskii and A. A. Rukhadze for discussion of results.

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7 February 1979 Nuclear Physics Research Institute