SPATIALLY POLARIZED FILTRATION OF ELECTROMAGNETIC WAVES

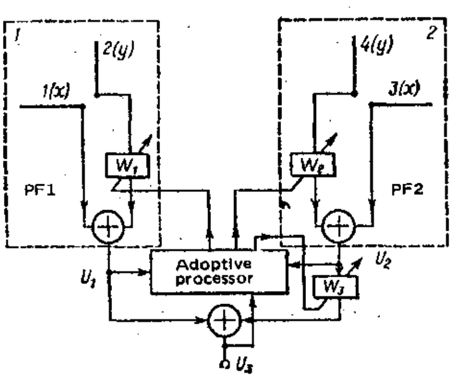
Yu. V. Berezin and A. N. Talitskii

Vestnik Moskovskogo Universiteta. Fizika,
Vol. 36, No. 2, pp. 12-15, 1981

UDC 621.396

The paper examines a spatial-polarization filter (SPF) employed for supression of two-beam, partially scattered interference. The operation of the filter is based on the assumption that two interference beams have different polarizations and incidence angles. Two versions of operation of such a filter are analyzed for the case of supression of a two-beam concentrated interference in receiving a partially scattered two-beam signal in the presence of additive noise. Analytic expressions, controlling the possible gain in the signal/noise ratio when using this filter are obtained.

Selection of electromagnetic waves of a given frequency $E_1(\omega_0)$ and $E_2(\omega_0)$ is possible either on the basis of the difference in their angle of approach or in their polarization. There are no other physical parameters which would differentiate such two waves. Waves are selected on the basis of the aforementioned attributes by means of spatial [1] and polarization [2] filters. These two selection methods have their advantages and disadvantages, which limit the region of their applicability. It is possible to combine the angular and polarization properties of the magnetic field and to thus develop a spatial-polarization method for selection of one from several waves of the same frequency. The incidence of one wave on the ionosphere induces several reflected waves. The physical causes for their appearance are different - there occur multiple reflections from different ionospheric layers, with each such reflection consisting of two magnetoionic components.



It is of practical and scientific importance to select individual waves (modes, magnetoionic components). Simultaneous polarization and angular filtration of electromagnetic waves can be used for controlling the multiwave nature of ionospheric signals. The structure of the spatial-polarization filter is depicted in the figure. Two polarization filters (PF1 and PF2), consisting each of a pair of crossed antennas and a system for weight processing of the signal are placed at a distance d apart and combined into a single spatial filter (SpF) by a third weight-processing system. Such a spatial-polarization filter has three degrees of freedom: two in the phasor space [3] and one in the space of angles. We shall investigate the capabilities of the spatial-polarization filter in selecting of partially scattered vectorial waves. Let four partially scattered waves of a given frequency, $\mathbf{E}_1(\omega_0)$, $\mathbf{E}_2(\omega_0)$, $\mathbf{E}_3(\omega_0)$, $\mathbf{E}_4(\omega_0)$, differing by angles of approach and by polarization arrive at observation points 1 and 2. Waves $\mathbf{E}_1$ and $\mathbf{E}_2$ shall be termed a two-beam signal, and waves $\mathbf{E}_3$ and $\mathbf{E}_4$ shall be termed two-beam error. We shall assume that both the signal and the error are stationary, uniform, mutually noncorrelated processes. In addition, additive noise, which is fully nonpolarized and delta correlated, is present at the antenna-array input. The supression functions of the individual beams, in processing two-beam interference by a spatial-polarization filter can be subdivided into the spatial and polarization branches. Two versions are possible:

1. The polarization filters are used for supressing the first error beam and the interference part of the field, whereas the spatial filter is used for supressing the second error beam.

Let us consider the first version of operation of the composite filter. The orthogonal antennas are designated by the subscript i (i running from 1 to 4; i = 1,2 are antennas in point 1, whereas i = 3,4 are antennas in point 2), the signal and error beams are designated by subscript k = 1,2. The two-beam signal and error fields are described by column vectors, with $A_i=\|a_{ik}(t)\|$ representing the signal, and $B_i=\|b_{ikl}(t)\|$ - the error. The sum of potentials of the first error beam at the output of polarization filter 1 and of additive noise $n_1(t)$ is

$$U_1(t)=b_{11}(t)+n_1(t)+w_1[b_{21}(t)+n_2(t)],$$

where w_1 is the complex weight coefficient of weight processing system 1.

The corresponding mean output power of polarization filter 1

$$<P_1(t)>=<U_1(t)U^*_1(t)>=<|b_{11}(t)|^2>+ \\ +\sigma^2_n+|w_1|^2(<|b_{21}(t)|^2>+\sigma^2_n)+2\,\mathrm{Re}[<b_{11}(t)b^*_{21}(t)>w^*_1], \quad (1)$$

where

$$\sigma^2_n=<|n_1(t)|^2>=<|n_2(t)|^2>.$$

It is known [4] that minimum of mean power (1) can be attained by proper selection of w_1. For this it is necessary that

$$w_1=w_{10}=-\frac{R}{p(1+r)}, \quad (2)$$

where $p=\sigma_{21}/\sigma_{11}$, $\sigma^2_{ik}=<|b_{ik}(t)|^2>$, $r=\sigma^2_n/\sigma^2_{21}$, whereas $R=<b_{11}(t)b^*_{21}(t)>/\sigma_{11}\sigma_{21}$ is the coefficient of correlation of orthogonal projections of the first error beam in point 1.

As a result of uniformity of the processes under study $w_{20}=w_{10}=w_0$, i.e., the polarization filters should be controlled simultaneously. The spatial filter is optimized on the basis of minimum of the mean power of the two-beam error and noise at the composite-filter output, given by the expression

$$<P_3(t)>=(1+|w_3|^2)[M_{11}+|w_0|^2M_{22}+\sigma^2_n(1+|w_0|^2)+2\mathrm{Re}(w^*_0M_{12})]+ \\ +2\mathrm{Re}[w^*_3(M_{13}+w_0M_{23}+w^*_0M_{14}+|w_0|^2M_{24})], \quad (3)$$

where $M_{jl}=e'B_{jl}e$, $e'=(1\quad 1)$ is the transposed column vector $e=\begin{pmatrix}1\\1\end{pmatrix}$, $B_{jl}=<B_jB^*_l>$ are correlation matrices of the error field; j, l = 1-4.

It can be shown that the minimum of $<P_3(t)>$ is attained at

$$w_{3opt}=w_{30}=-\frac{R_{B13}+P_B(w_0R_{B23}+w^*_0R_{B14})+P^2_B|w_0|^2R_{B24}}{1+r_1(1+|w_0|^2)+2P_B\,\mathrm{Re}(w^*_0R_{B12})+|w_0|^2P^2_B}, \quad (4)$$

where $P_B = \sqrt{M_{22}}/\sqrt{M_{11}}$ is the modulus of the phasor of the summary error field; $P_{B12} = M_{12}/\sqrt{M_{11}M_{22}}$ is the coefficient of correlation of orthogonal projections of the error field in a single point in space; $R_{B13}(d) = M_{13}/M_{11}$ and $R_{B24}(d) = M_{24}/M_{22}$ are coefficients of spatial correlation, respectively, of the x and y projections of the error field; $R_{B23}(d) = M_{23}/\sqrt{M_{11}M_{22}}$ and $R_{B14}(d) = M_{14}/\sqrt{M_{11}M_{22}}$ are coefficients of mutual spatial correlations of orthogonal projections of the error field; $r_1 = \sigma^2_n/M_{11}$ is the relative power of the noise.

Using Eqs. (3) and (4) we shall determine the minimum mean error and noise power at the output of the composite filter

$$\langle P_3(t)\rangle_{\min} = (1-|w_{30}|^2)\,[M_{11} + |w_0|^2 M_{22} + \sigma^2_n(1+|w_0|^2) + 2\,\mathrm{Re}(w^*_0 M_{12})]. \qquad (5)$$

The efficiency of the composite-filter operation is defined by number μ, which controls the gain in the signal-to-noise ratio at the output of the composite filter as compared with the maximum value of the S/N ratio at its inputs (the error referred to here is the two-beam concentrated error plus noise). Using Eq. (5) and calculating the mean output power of the two-beam signal of an optimized composite filter, we obtain

$$\mu = \mu_0 \left\{ \frac{(1+|w_{30}|^2)\,[1 + 2P_A\,\mathrm{Re}\,(w_0^* R_{A12}) + |w_0|^2 P_A^2]}{L\,(1-|w_{30}|^2)} + \frac{2\,\mathrm{Re}\,[w_{30}^*(R_{A13} + w_0 R_{A23} + w_0^* R_{A14} + |w_0|^2 R_{A24}]}{L\,(1-|w_{30}|^2)} \right\}, \qquad (6)$$

where

$$\mu_0 = \left(\frac{N_{11}}{M_{11}}\right) \Big/ \left(\frac{N_{jj}}{M_{jj} + \sigma_n^2}\right)_{\max}, \quad N_{jl} = e'\,\langle A_j A_l^*\rangle\, e,$$

$$L = 1 + r_1(1+|w_0|^2) + 2P_B\,\mathrm{Re}\,(w_0^* R_{B12}) + |w_0|^2 P_B^2.$$

Quantities P_A and R_{Ajl} for the signal have the meaning analogous to that of values of P_B and R_{Bjl} for the error. This means that Eq. (6) defines the (power) gain of the S/N ratio, which can be obtained when using the composite filter for supressing the two-beam error (when the functions of polarization filters 1 and 2 and of the spatial filter are divided as specified above). The value of μ depends on functions, P_A, P_B, R_{Ajl}, R_{Bjl}, which are controlled by the properties of the signal and error fields. Analytic expressions for these functions can be obtained by using model representations of partially scattered fields. Without going into a detailed analysis of μ, we shall perform estimates of the capabilities of the composite filter.

We introduce coefficients of transmission q_s of the signal and q_n of the error, as ratios of the mean filter output and input powers. Then $\mu = q_c/q_n$. Since the transmission factor is $q_{SpPF} = q_{PF} \times q_{SpF}$, we represent the efficiency of the composite filter as $\mu = \mu_{PF} \times \mu_{SpF}$. The efficiency of the polarization filter in supression of a single-beam error [2] is estimated on the average by the value of $\mu'_{PF} \sim 10^2$. As a function of the number K, equal to the ratio of intensities of error beams $1 \leq \mu_{PF} \leq \mu'_{PF}$.

The gain in the S/N ratio μ'_{SpF}, obtained upon supression of a single-beam error by means of a spatial filter with one degree of freedom [4,5] is about

10^2-10^3 over a wide range of error angles of approach. Here $1 \le \mu_{SpF} \le \mu'_{SpF}$, depending on the value of K. In the particular cases of $K \ll 1$ and $K \gg 1$ (the error becomes a single-beam error), the value of μ of the composite filter is controlled by the effectiveness of one of the component filters. It can be easily shown that at K = 1 and noncorrelated error beams we have $\mu \approx 2\mu'_{PF}\,\mu'_{SpF}/(\mu'_{PF} + \mu'_{SpF})$. Accordingly one should expect that the composite filter will be quite efficient ($\gtrsim 10^2$) in supressing a two-beam partially scattered error. If the polarization filter is used not only for supressing the first error beam, but also that part of the field which is produced by beam interference (second version of composite-filter operation), the above discussion and Eq. (6) retain their validity, and only the expression for w_0 changes. It follows from general considerations and from the above equations that the value of μ for the composite filter is in each case a complex function of all the properties of the vector fields of the signal and the error and its investigation comprises a problem on its own. Analysis of the capabilities for supressing a two-beam error by means of the composite filter shows that this problem can be solved by two methods which do not basically differ from one another. However, the difference in weight coefficients w_0 for the two versions of the composite-filter operation is, in general, responsible for differences in its efficiency in these cases.

REFERENCES

1. B. Widrow, P. E. Montey, L. J. Griffiths, and B. B. Goode, "Adaptive antenna systems," IEEE Proceedings, vol. 55, no. 12, pp. 2143-2159, 1967.
2. Yu. V. Berezin, V. D. Gusev, and V. I. Smirnov, "Experimental study of the supression of a single magnetoionic component in the reflection of waves from the F2 layer," Geomagnetizm i aeronomiya, vol. 11, no. 2, pp. 258-262, 1971.
3. D. B. Kanareikin, N. F. Pavlov, and V. A. Potekhin, Polarization of Radar Signals [in Russian], Moscow, pp. 35-36, 1966.
4. W. F. Gabriel, "Adaptive arrays - an introduction," IEEE Proceedings, vol. 64, no. 2, pp. 239-272, 1976.
5. R. L. Riegler and R. T. Compton, Jr., "An adaptive antenna array for error supression," IEEE Proceedings, vol. 61, no. 6, pp. 748-758, 1973.

13 February 1979

Department of General Physics
and Wave Processes