PHOTOCONDUCTIVITY AND PHOTO-EMF UNDER RELAXATION CONDITIONS. PART 2

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Vol. 39, No. 6, pp. 10-14, 1984

UDC 539.21:537.1;548:537.1

A simple model is considered that provides a qualitative explanation of the intensity and temperature dependence of the photocurrent observed for various materials.

A study has been made [1] of calculating the voltage-current characteristics (VCC) of an illuminated specimen under conditions where the Maxwell time in the specimen greatly exceeds the characteristic carrier recombination time. That paper yielded the equilibrium carrier distribution in a semiconductor when one of the contacts of the n-type semiconductor injects folds.

Consider a specimen having a cross sectional area S = 1 on which falls light of intensity i, while a voltage v is applied to it (symbols as in [1]). We assume that i and j are small in a certain sense, and the corresponding criteria are given below.

Under these conditions,

$$u^{-1} = v + m, \quad m = i\,\frac{g(v) + g(v^{-1})}{g(v^{-1}) + v^{-2} g(v)}.$$

Here we have introduced the symbol

$$g(v) = g^c(v) = g_0 v^6,$$

and v is given in Eq. (31) of [1].

For m we get the equation

$$\frac{d}{dx}\left\{\frac{jv}{v^2+1} + \frac{1}{v}\frac{dv}{dx} + \frac{v^2}{v^2+1}\frac{d}{dx}\frac{m}{v}\right\} = g(v^{-1}) - g(v)\left(1 - \frac{m}{\Delta v}\right) + g_0^c. \tag{1}$$

We seek the solution to (1) in the form

$$v = u_0 e^{y+z}, \tag{2}$$

where $z|_{0,L} = 0$ (see (21) and (25) of [1]).

Then we have the following equation for z:

$$\frac{d}{dx}\left\{\frac{dz}{dx} + \frac{j}{2\,\mathrm{ch}\,\theta} + i\,\frac{e^{-\theta}}{2\,\mathrm{ch}\,\theta}\frac{d}{dx}\frac{\mathrm{ch}\,\theta/\Delta}{\mathrm{ch}\,\theta}\right\} = i\,\frac{g(v)}{\Delta}\,\frac{\mathrm{ch}\,\theta/\Delta}{\mathrm{ch}\,\theta}. \tag{3}$$

$$\theta \equiv -y - \ln u_0. \tag{4}$$

On the basis of (2), one can continue z over the segment [0,2L] and perform a Fourier-series expansion:

$$z = \sum_{n=1}^{\infty} A_n \sin \frac{\pi n x}{L}. \tag{5}$$

As the coefficient functions in (3) differ appreciably from zero only in a certain region within the specimen, they can also be taken as zero at the boundaries, and can be continued in an odd fashion over the segment [0,2L]. Then

$$-A_n \frac{\pi^2 n^2}{L} + 2\int_0^L \sin \frac{\pi n x}{L} \frac{d}{dx}\left\{\frac{j}{2\,\mathrm{ch}\,\theta} + i\frac{e^{-\theta}}{2\,\mathrm{ch}\,\theta}\frac{d}{dx}\frac{\mathrm{ch}\,\theta/\Delta}{\mathrm{ch}\,\theta}\right\}dx - \\ -2\int_0^L \frac{i}{\Delta}\frac{e^{\theta/\Delta}\,\mathrm{ch}\,\theta/\Delta}{\mathrm{ch}\,\theta}\sin\frac{\pi n x}{L}\,dx = 0, \tag{6}$$

whence (see Appendix I)

$$A_n \frac{\pi^2 n^2}{L} = 2\left(-j\frac{\pi n}{2L}\Delta x \cos\frac{\pi n x}{L} - \frac{i}{\Delta}\Delta x \sin\frac{\pi n x_1}{L} + \right. \\ \left. + \frac{i\pi n}{2L}\Delta x \cos\frac{\pi n x_1}{L} - \frac{\pi i}{2}\sin\frac{\pi n x_1}{L}\right). \tag{7}$$

Here x_1 is defined from the condition

$$\theta = 0, \tag{8}$$

and Δx is the width of the maximum in the coefficient functions near x_1, which is derived from the condition

$$\Delta x \cdot \frac{dy}{dx} \approx 1.$$

We use (4) and formula (31) of [1] to get

$$\zeta_0 \ln\frac{v_0}{u_0}\frac{\Delta x}{L}\left(\frac{\pi}{2} - \zeta_0\right)\frac{1}{\cos^2(\pi/2 - \zeta_0)(1 - x_1/L)} \approx 1.$$

As $\zeta_0 \ll 1$, we use (8) to get

$$\Delta x \approx \frac{4L}{\pi}\zeta_0 \frac{\ln v/u_0}{\ln^2 u_0}, \\ \frac{\pi x_1}{2L} = -\zeta_0 \frac{\ln v_0}{\ln u_0} \ll 1. \tag{9}$$

We have thus found the coefficients A_n in (5). However, we are interested in the VCC. Switching on the light and field redistributes the charges and produces an additional potential ΔV, which is related to the above z:

$$\Delta V = z + \int_0^x \left\{\frac{j}{2\,\mathrm{ch}\,\theta} + i\frac{e^{-\theta}}{2\,\mathrm{ch}\,\theta}\frac{d}{dx}\frac{\mathrm{ch}\,\theta/\Delta}{\mathrm{ch}\,\theta}\right\}dx. \tag{10}$$

The boundary condition for (10) takes the form

$$\Delta V(L) = eV, \quad z(L) = 0.$$

Consequently, the VCC equation can be written as

$$eV = \int_0^L \left\{ \frac{j}{\operatorname{ch}\theta} + i \frac{e^{-\theta}}{2\operatorname{ch}\theta} \frac{d}{dx} \frac{\operatorname{ch}\theta/\Delta}{\operatorname{ch}\theta} \right\} dx. \tag{11}$$

Equations (3) and (10) enable one to construct a scheme for iteration on i and j to solve system (20) of [1]. In fact, we represent the A_n as a series:

$$A_n = A_n^{(0)} i + A_n^{(1)} j + A_n^{(2)} i^2 + A_n^{(3)} j^2 + A_n^{(4)} ij + \ldots, \tag{12}$$

and then the latter coefficients can be found from (3) as above, while (11) enables one to calculate the VCC with the desired accuracy.

We restrict ourselves to terms quadratic in i and j. We see from (11) that in this case it is sufficient to know only $A_n^{(0)}$ and $A_n^{(1)}$ from (12), i.e., as θ in (11) we must take

$$\theta = -y - z - \ln u_0.$$

We proceed as previously to get

$$eV = j\Delta x_{ij}/2 - \pi i. \tag{13}$$

Here

$$\Delta x_{ij} \frac{\partial\theta}{\partial x}\Big|_{x=x_1+\delta x} \sim 1,$$

where δx is the peak displacement in the presence of the field and light. As i and j are small, we get

$$\Delta x_{ij} = \frac{4L}{\pi} \zeta_0 \frac{\ln u_0^{-1} + \ln v_0 - z(x_1)}{\ln^2 u_0 - 2z(x_1) \ln u_0^{-1}} \tag{14}$$

and

$$\zeta_0 \ln \frac{v_0^{-1}}{u_0} \frac{\delta x}{x_1} \operatorname{ctg} \frac{\pi x_1}{2L} = z(x_1). \tag{15}$$

Here x_1 is defined from (9). To find $z(x_1)$, we have to sum the series of (5) with the A_n from (7). On performing the summation (see Appendix II), we get

$$z(x_1) = (i - j)\Delta x \left(\frac{1}{4} - \frac{x_1}{L}\right) - i\left(\frac{2\Delta x}{\Delta} + \pi\right) x_1 \left(\frac{1}{2} - \frac{x_1}{L}\right) \tag{16}$$

or on the basis that $x_1/L \ll 1$,

$$z(x_1) \approx j\frac{\Delta x}{4} - \frac{\pi x_1}{2} i.$$

We substitute into (14) and (15) and use (13) to get

$$V = \frac{1}{2} jL\zeta_0 \left[\frac{\ln u_0^{-1} + \ln v_0}{\ln^2 u_0} + \left(i\zeta_0 \frac{\ln v_0}{\ln u_0^{-1}} - j\zeta_0 L \frac{\ln u_0^{-1} + \ln v_0}{\pi \ln^2 u_0} \right) \frac{\ln u_0^{-1} - 2\ln v_0}{\ln^3 u_0^{-1}} \right] - \pi i. \tag{17}$$

We introduce the symbol

$$R_0 = \frac{1}{2} L\zeta_0 \frac{\ln u_0^{-1} + \ln v_0}{\ln^2 u_0}.$$

Then (17) is rewritten as

$$V = jR_0\left(1 - i\frac{R_0}{L} - j\frac{R_0}{2\Delta\varphi}\right) - \pi i. \tag{18}$$

We see from (18) that R_0 has the meaning of the resistance of the specimen (S = 1) for i and j → 0 (on the ohmic part of the VCC).

Under the standard conditions used in photoconductivity measurement, the voltage produced by the load resistor r is given by

$$rj + V(j) = V_b.$$

Here V_b is the battery voltage and V(j) is the voltage across the specimen. We used (18) and the fact that usually $r \ll R_0$ to get

$$-jR_0\left(1 - i\frac{R_0}{L} - j\frac{R_0}{2\Delta\varphi}\right) + \pi i = V_b.$$

Then

$$j_{1,2} = \frac{\Delta\varphi}{R_0}\left[\left(1 - \frac{i}{L}\right) \pm \sqrt{\left(1 - \frac{i}{L}\right)^2 - 2\frac{\pi i - V_b}{\Delta\varphi}}\right] = \frac{\Delta\varphi}{R_0}\left[1 \pm \sqrt{B(V_b; i)}\right]. \tag{19}$$

Out of the two roots, we have to select the one smaller in magnitude (our argument applies for small currents).

We see from (19) that j is of the order of i for low light intensities, $j \sim 1$ for $\sqrt{B}\,\pi i/\Delta\varphi \sim 1$ (the calculation is correct for $i < \Delta\varphi/\pi$).

We now consider the temperature dependence of the photocurrent. At low light intensities,

$$|j| \approx (-\pi i + V_b)/R_0. \tag{20}$$

Experiment [2] shows that

$$R_0 \sim e^F.$$

Here F is the distance from the c band to the Fermi level in units of kT. As

$$i = sJ/(\alpha n_i),$$

the temperature dependence of the photocurrent for small i is

$$j \sim e^{E^+},\ E^+ \equiv \Delta\varphi. \tag{21}$$

Then j has an exponential temperature dependence with a positive exponent (in principle, E^+ could contain terms linear in temperature due to the change in gap width with temperature, but usually the contribution from these is unimportant).

As the light intensity increases, the term in the expression for the VCC corresponding to the photo-EMF increases (the analog of the rectifier photo-EMF). The increase in the photo-EMF leads to the working joint jumping to the other branch of the VCC, on which

$$j = \frac{\Delta\varphi}{R_0}\left[\left(1 - \frac{i}{L}\right) - \sqrt{\left(1 - \frac{i}{L}\right)^2 - 2\frac{\pi i - V_b}{\Delta\varphi}}\right]. \qquad (22)$$

Here the temperature dependence of j is determined by that of R_0, i.e.,

$$j \sim e^{-E^-}, \quad E^- \equiv F. \qquad (23)$$

In experiments, E^- is usually less than F. It would, however, be difficult to expect within the framework of this simple model any better agreement with experiment. In fact, we have not incorporated the dependence of the center parameters (s and α) on the ionization energy and charge state. On incorporating this we evidently could improve the agreement between theory and experiment.

The author is indebted to Professor V. L. Bonch-Bruevich for fruitful discussions on the content of this paper.

Appendix I

We calculate the integral appearing in the last term on the left in (6):

$$J = \int_0^L \frac{e^{\theta/\Delta}\,\mathrm{ch}^{\theta/\Delta}}{\mathrm{ch}\,\theta} \sin\frac{\pi n x}{L}\,dx.$$

As $\Delta \gg 1$ by virtue of the formulation, the integrand has a sharp peak at $\theta = 0$. Therefore, approximately

$$J \approx \sin(\pi n x/L)\cdot\Delta x,$$

where x_1 is defined from the condition

$$\theta = 0,$$

and Δx is the peak width, which is defined by

$$\Delta x \left.\frac{\partial\theta}{\partial x}\right|_{x=x_1} \sim 1.$$

The other integrals in (6) are estimated similarly.

Appendix II

The summation is performed on the following formula:

$$z = \sum_{n=1}^{\infty} \dot{z}_n \approx \int_0^{\infty} z_n dn - z_0 = \int_0^{\infty} \frac{i-j}{\pi}\,\Delta x\,\frac{\sin(\pi n x/L)\sin(\pi n x_1/L)}{n}\,dn - (i-j)\frac{\Delta x\cdot x}{L} -$$

$$-\frac{iL}{\pi^2}\left(2\frac{\Delta x}{\Delta} + \pi\right)\int_0^{\infty}\frac{\sin(\pi n x/L)\sin(\pi n x_1/L)}{n^2}\,dn + i\frac{x}{L}x_1\left(\frac{2\Delta x}{\Delta} + \pi\right).$$

Subsequently, we delete terms containing the ratio $\Delta x/\Delta$, since $\Delta \gg 1$ and $\Delta x \ll 1$. The integrals appearing in the formula are calculated by standard methods, which gives us (16) given in the main text.

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16 November 1983

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