MIXING OF THE $2S_{1/2}$ AND $2P_{1/2}$ LEVELS IN HYDROGEN

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A calculation is performed on proton-recoil effects in the mixing of the $2S_{1/2}$ and $2P_{1/2}$ levels in hydrogen due to the weak interaction.

Out of the effects due to the weak interaction between an electron and the nucleus in an atom, the main interest attaches to effects related to spatial parity violation. A detailed discussion of this topic is to be found in [1]. Here we consider the mixing of the $2S_{1/2}$ and $2P_{1/2}$ levels in hydrogen. These are separated only by the Lamb shift, so the mixing coefficient attains a value such that one might observe the parity violation in transitions involving these levels. Calculations have been performed [1] on the mixing coefficient in the infinitely heavy nucleus approximation. The quasi-potential approach [2] enables one to incorporate proton recoil.

The quasi-potential for a weak interaction of an electron with a proton can be derived in the same way as in [3]. In the first approximation, the quasi-potential is equal to the scattering amplitude at zero relative particle energy, and the quasi-potential equation goes over to the usual Schrodinger equation in the momentum representation in the relativistic limit. Here the weak-interaction quasi-potential V_Z is a small correction to the Coulomb part of the quasi-potential for one-photon exchange, and quantum-mechanical perturbation can be used, where the initial approximation is taken as the solution to Schrodinger's equation with the Coulomb potential.

The scattering amplitude contains the matrix element for the neutral weak current, which is written as follows in the standard theory of electroweak interactions [4]:

$$\langle \mathbf{q}|J_\mu^Z|\mathbf{p}\rangle = (\sqrt{2}G)^{1/2} m_Z \bar{u}(\mathbf{q})\{g_V\gamma_\mu + g_A\gamma_\mu\gamma^5 + g_M(i/(2m))\sigma_{\mu\nu}k^\nu + g_T\gamma^5 k_\mu\}u(\mathbf{p}), \quad (1)$$

where $k = p - q$, $k^0 = 0$, m_Z is the mass of the Z boson, m is the mass of the relevant fermion, u(p) are the Dirac spinors, $G=1{,}027\cdot10^{-5}\ m_p^{-2}$ is the Fermi weak interaction constant, and g_V, g_A, g_M, and g_T are the neutral weak vector, axial, magnetic, and tensor form factors correspondingly.

We restrict ourselves to the nonrelativistic approximation for V_Z. We give the values of the form factors in (1) for zero momentum transfer (θ_W is the Weinberg angle): for the electron $g_V^e=-1/2+2\sin^2\theta_W$, $g_A^e=-1/2$, $g_M^e=(\alpha/(2\pi))g_V^e$ (this is calculated by analogy with the anomalous magnetic moment of the electron); and for the proton $g_V{}^p = -g_V{}^e$, $g_A{}^p = -\lambda g_A{}^e$, $g_M{}^p = (1/2)(\varkappa_p - \varkappa_n) - 2\varkappa_p \sin^2\theta_W$, where λ = 1.25 and κ_p and κ_n are the anomalous magnetic moments of the proton and neutron. The term $g_T\gamma^5 k_\mu$ does not make a contribution to the potential in the nonrelativistic approximation.

Then we have for the P-odd part of V_Z that

$$V_Z^0 = \sqrt{2}\, G \left\{ \frac{g_V^e g_A^p}{2\mu} (\mathbf{p} + \mathbf{q}) \cdot \sigma_p - \frac{g_A^e g_V^p}{2\mu} (\mathbf{p} + \mathbf{q}) \cdot \sigma_e - \right.$$
$$\left. - \left[\frac{g_A^p (g_V^e + g_M^e)}{2m_e} + \frac{g_A^e (g_V^p + g_M^p)}{2m_p} \right] i (\mathbf{k} \cdot [\sigma_e \times \sigma_p]) \right\},$$

where $\mu = m_e m_p/(m_e + m_p)$, and σ are Pauli matrices (the potential acts in the space of two-component wave functions).

This potential leads to mixing of states opposite in parity, in particular the $2S_{1/2}$ and $2P_{1/2}$ levels in hydrogen. It is necessary to allow for the hyperfine splitting of these levels; each consists of two components with different values of the total momentum F. States with identical values of F are mixed; the energy difference between these states is denoted by Δ_F. Then the mixing coefficient is

$$\eta_F = (1/\Delta_F) \langle 2P_{1/2}, F | V_Z{}^0 | 2S_{1/2}, F \rangle,$$

where F = 0 or 1.

The potential matrix element $V_Z{}^0$ can be calculated from the Wigner-Eckart theory by the use of a reduction formula for the reduced matrix element for the direct product of the tensor operators [5]. To calculate the reduced matrix elements of operators p and q, we use the explicit forms of the Coulomb wave functions for the $2S_{1/2}$ and $2P_{1/2}$ states in the momentum representation. We have finally that

$$\eta_F = -\frac{\sqrt{6}\, G \alpha^2 R_N \mu^2}{16\pi \Delta_F} \left\{ g_1 + \frac{(-1)^F \cdot 2}{2F+1} \left[\frac{1}{2} \lambda g_1 + \right.\right.$$
$$\left.\left. + \frac{\mu}{m_e} \left(1 + \frac{\alpha}{2\pi} \right) \lambda g_1 + \frac{\mu}{m_p} (g_1 + g_2) \right] \right\}, \qquad (2)$$

where we have introduced the symbols

$$g_1 = -g_V{}^e g_A{}^e = 1/4 - \sin^2\theta_W, \qquad g_2 = g_A{}^e g_M{}^p = (1/4)(\varkappa_p - \varkappa_n) - \varkappa_p \sin^2\theta_W,$$
$$R_H = R_\infty (1 + m_e/m_p)^{-1}$$

being the Rydberg constant.

For $m_p \to \infty$, the result coincides with [1]. Allowance for recoil effects leads not only to the replacement of m_e by μ but also to an additional term due to the neutral weak magnetism of the proton (the last term in (2)).

From (2) we have

$$\eta_0 = -1{,}166 \cdot 10^{-10} (g_1 + 2{,}3 \cdot 10^{-4} g_2), \quad \eta_1 = 0{,}615 \cdot 10^{-11} (g_1 + 1{,}45 \cdot 10^{-3} g_2).$$

For $\sin^2\theta_W = 0{,}23$, the contribution from the recoil effects is 0.6% for η_0 and 4% for η_1.

Experiment so far only sets an upper bound to the mixing [6]: $|\eta|<1{,}5\cdot10^{-8}$.

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