SYNCHROTRON NEUTRINOS AT ULTRAHIGH ENERGIES

A. V. Borisov and I. A. Kelekhsaeva

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The probability of emission of a pair of muon neutrinos by an electron moving in a constant magnetic field is determined within the framework of the standard theory of electroweak interactions. The dependence of this probability on the electron energy and the external magnetic field is investigated.

The emission of a neutrino-antineutrino pair, $\nu\bar{\nu}$, by an electron in a magnetic field ($e \to e\nu\bar{\nu}$), may be referred to as synchrotron neutrino emission (SNE) in which the neutrino pair $\nu\bar{\nu}$ is the analog of the photon. The emission of $\nu\bar{\nu}$-pairs is of major importance in astrophysical applications, as was first pointed out in [1]. Different mechanisms for the emission of neutrinos were examined in [2]. SNE by relativistic electrons was investigated in [3] within the framework of the four-fermion V-A theory of weak interactions in a relatively weak magnetic field $H \ll H_0 = m^2c^3/\hbar e = 4.41 \cdot 10^{13}$ G. The astrophysical aspects of SNE were discussed in [4,5]. A generalization of SNE theory to ultrastrong fields, $H \gtrsim H_0$, was given in [6] (V-A variant) and in [7] (low-energy limit of the standard Weinberg-Salam theory of the electroweak interaction [8]). Polarization effects in SNE in the same contact limit of the Weinberg-Salam theory that was used in [7] were investigated in [9].

The SNE process was analyzed in [3-9] at relatively low energies at which the interaction is effectively the four-fermion point interaction. In this paper, we consider the case of high energies for which the weak interaction ceases to be of the contact type and is due to the exchange of the massive vector bosons $W^{\pm}$ and Z, which leads to a significant change in the energy dependence of the probability for the process. We shall confine our attention to the emission of a pair of muon neutrinos by an electron, in which case only one diagram with a neutral Z-boson exchange ($m_Z \simeq 93$ GeV [8]) contributes to the SNE amplitude: the electron emits a virtual Z-boson which decays into the $\nu_\mu\bar{\nu}_\mu$-pair (the neutrino is assumed to be massless).

The amplitude for the process $e \to e\nu_\mu\bar{\nu}_\mu$ is (we are using a system of units in which $\hbar = c = 1$, and a pseudo-Euclidean metric with the signature (+ – – –):

$$M = \frac{G}{\sqrt{2}} m_Z^2 \int d^4q\,\delta^{(4)}(k + k' - q)\,[\bar{u}(k')\,\gamma^\mu(1+\gamma^5)\,u(-k)] \times \frac{g_{\mu\nu} - q_\mu q_\nu/m_Z^2}{q^2 - m_Z^2 + im_Z\Gamma_Z} \cdot 2\pi\delta(\varepsilon' + q_0 - \varepsilon)\, j^\nu, \quad (1)$$

where $G = 1.17 \cdot 10^{-5}$ GeV^{-2} is the Fermi constant, q is the 4-momentum of the Z-boson, $\Gamma_Z \simeq 2.5$ GeV is the total decay width of the Z-boson [8], k and k' are the 4-momenta of the neutrinos $\bar{\nu}_\mu$ and ν_μ, u represents their bispinor amplitudes, and the electron current

$$j^\mu = \int \bar{\psi}_{n'}(\mathbf{r})\,\gamma^\mu (g_V + g_A\gamma^5)\,\psi_n(\mathbf{r})\,e^{-i\mathbf{q}\mathbf{r}}\,d^3x$$

is the determined by the exact weight functions of the electron ψ_n and $\psi_{n'}$ in the initial and final states (energies ε and ε') in the constant uniform magnetic field (the explicit form of these functions can be found, for example, in [10]). The vector and axial constants in the current are expressed in terms of the Weinberg angle $\theta_W (\sin^2\theta_W \simeq 0{,}225)$:

$$g_V = -1/2 + 2\sin^2\theta_W, \quad g_A = -1/2.$$

The probability for the process can be deduced from (1):

$$w = \frac{G^2 m_Z^4}{3(2\pi)^4} \sum_f \int d^3q \, \frac{[(qj^*)(qj) - q^2 (j^*j)]}{(q^2 - m_Z^2)^2 + (m_Z\Gamma_Z)^2}, \tag{2}$$

where f is the set of quantum numbers describing the state of the final electron and the range of integration is defined by $|\mathbf{q}| \leqslant q_0 = \varepsilon - \varepsilon'$.

We shall confine our attention to the case of ultrarelativistic electron energies ($\varepsilon \gg m$) and relatively weak magnetic fields, $H \ll H_0$, so that the initial and final states of the electron can be treated quasiclassically. The dependence of the total probability of the process on the electron energy and the magnetic field is then determined by the single invariant parameter (according to the general theory given in [10,11]):

$$\chi = \frac{p_\perp}{m}\frac{H}{H_0} = \frac{e}{m^3}\left[-(F_{\mu\nu}p^\nu)^2\right]^{1/2}, \tag{3}$$

where $p_\perp$ is the transverse (relative to H) momentum of the electron and $F_{\mu\nu}$ is the external field tensor. Using the well-known quasiclassical asymptotic behavior of the matrix elements from the theory of synchrotron radiation [12], we find that the probability (2) is given by the following integral representation:

$$w = \frac{\sqrt{\pi}\, G^2 m_Z^4}{3(2\pi)^4}\frac{m^2}{\varepsilon}\int_0^\infty \frac{du}{(1+u)^3}\int_0^\infty \frac{dx}{(x-\Delta)^2 + \delta^2}\left[a\Phi_1 - 2b\left(\frac{\chi}{u}\right)^{2/3}\Phi'\right] \tag{4}$$

where $\Phi'(t) = d\Phi(t)/dt$, $\Phi_1(t) = \int_t^\infty dx\cdot\Phi(x)$, $\Phi(t) = (2\sqrt{\pi})^{-1}\int_{-\infty}^{\infty} dx\,\exp[-i(tx + x^3/3)]$ is the Airy function,

$$t = (u/\chi)^{2/3}(1+x) \tag{5}$$

$$a = -\frac{1}{2}(g_V^2 + g_A^2)u^2x^2 + (2g_A^2 - g_V^2)(1+u)\,x,$$

$$b = \frac{1}{2}(g_V^2 + g_A^2)(2 + 2u + u^2)\,x + g_A^2(1+u); \tag{6}$$

$$\Delta = (m_Z/m)^2(1+u)/u^2, \quad \delta = (\Gamma_Z/m_Z)\Delta;$$

and

$$u = \frac{\varepsilon - \varepsilon'}{\varepsilon'}, \quad x = \frac{q^2}{m^2}\frac{1+u}{u^2},$$

i.e., u is the same quantity as in the theory of synchrotron radiation, whereas x is related to the invariant mass $(q^2)^{1/2}$ of the neutrino pair. At high energies, for which $\chi \gg 1$, the main contribution to the integral in (4) is provided by region $u \lesssim 1$, $x \lesssim \chi^{1/3}$ (see (5)). This means that for $1 \ll \chi \ll (m_Z/m)^3$ and $x \ll \Delta \sim (m_Z/m)^2$, i.e., $q^2 \ll m_Z^2$, we find that, since $\delta/\Delta = \Gamma_Z/m_Z \ll 1$, the factor

$$R = [(x-\Delta)^2 + \delta^2]^{-1}, \tag{7}$$

that determines the contribution of the Z-boson propagator to (4) becomes $R \simeq \Delta^{-2}$. As a result, (4) becomes identical with the well-known expression deduced in the effective four-fermion theory (see, for example, [9,11]).

It is clear from (4) that, when $q^2 \sim m_Z^2$, the differential probability dw/dudx exhibits a resonance in the region $|x-\Delta| \lesssim \delta$, i.e., $|q_2 - m_Z^2| \lesssim m_Z\Gamma_Z$. Hence, when $\chi \sim (m_Z/m)^3$, we can find a representation for the probability in the form of a single integral if we use the resonance approximation for the factor given by (7), i.e.,

$$R \simeq \frac{\pi}{\delta}\delta(x-\Delta), \tag{8}$$

and also recall that, when $\chi \gg 1$, the main contribution is provided by $u \sim 1$. Substituting (8) in (4), we obtain the probability w_R of the process in the resonance region $(\tilde{\chi} = (m_Z/m)^3\chi \sim 1)$:

$$w_R = \frac{\pi^{3/2}}{6(2\pi)^4}(g_V^2 + g_A^2)\frac{G^2 m_Z^7}{\varepsilon\Gamma_Z}F(\tilde{\chi}),$$
$$F(\tilde{\chi}) = \int_0^\infty \frac{du}{(1+u)^2}\left[-\Phi_1(y) - 2\left(1 + 2\frac{1+u}{u^2}\right)\frac{1}{y}\Phi'(y)\right],$$
$$y = \left(\frac{u}{\tilde{\chi}}\right)^{2/3}\frac{1+u}{u^2}, \tag{9}$$

where $\tilde{\chi} \sim 1$ for $F(\tilde{\chi}) \sim 1$.

At ultrahigh energies $(\chi \gg (m_Z/m)^3)$, the main contribution to the integral (4) is provided by the term proportional to $\Phi'(t)$. The corresponding asymptotic behavior of the probability can be obtained by setting $t = 0$, $R \simeq x^{-2}$, and integrating with respect to x in the logarithmic approximation $(x \lesssim x_{max} \sim \chi^{2/3})$. The integral with respect to u can then be evaluated exactly and the final result is

$$w = \frac{14\Gamma(2/3)}{9(6\pi)^3}(g_V^2 + g_A^2)G^2 m_Z^4\frac{m^2}{\varepsilon}(3\chi)^{2/3}\ln\chi, \tag{10}$$

which is significantly different from the well-known result for $1 \ll \chi \ll (m_Z/m)^3$ [9, 11], namely, $w \sim \chi^2 \ln\chi$. This difference is due to the propagator effect: it follows from (5) and (6) that $q^2 \sim \chi^{2/3}m^2$ in the significant region, i.e., the factor (7) is $R \sim R_e \sim \chi^{-4/3}$ and χ^2 must be replaced with $R_e\chi^2 \sim \chi^{2/3}$, which is in agreement with (10): $w \sim \chi^{2/3}\ln\chi$.

Let us now compare the intensities I_ν and I_γ of the neutrino pairs and gamma rays emitted by an electron in a magnetic field at ultrahigh energies. The neutrino-pair intensity can be found from (4) by multiplying the integrand by the neutrino-pair energy $q_0 = u\varepsilon/(1+u)$:

$$I_\nu = \frac{4}{3}\frac{\Gamma(2/3)}{(9\pi)^3}(g_V^2 + g_A^2)G^2 m_Z^4 m^2(3\chi)^{2/3}\ln\chi. \tag{11}$$

The intensity I_γ for $\chi \gg 1$ is given by [10]:

$$I_\gamma = (2^4/3^5)\,\Gamma(2/3)\,\alpha m^2 (3\chi)^{2/3}.$$

From this and from (11) we obtain the intensity ratio

$$r = \frac{I_\nu}{I_\gamma} = \frac{g_V^2 + g_A^2}{36\pi^3\alpha} G^2 m_Z^4 \ln\chi \simeq 4\cdot 10^{-4} \ln\chi, \qquad (12)$$

where we have substituting numerical values for the electroweak interaction constants. This shows that the ratio (12) is a logarithmic function of the particle energy, whereas for $1 \ll \chi \ll (m_Z/m)^3$ the result is $r \simeq 2\cdot 10^{-24}\cdot\chi^{4/3}\ln\chi$.

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