DETERMINATION OF THE STRENGTH FUNCTIONS FOR GIANT MULTIPOLE RESONANCES IN THE SEMIMICROSCOPIC MODEL OF NUCLEAR VIBRATIONS

A. N. Gol'tsov, B. S. Ishkhanov, and V. N. Orlin

Vestnik Moskovskogo Universiteta. Fizika,
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A method is described for calculating the strength functions for giant multipole resonances (GMR). The fragmentation of particle and hole excitations is taken into account, as is the coupling of the GMR to the continuum. The method is used to calculate the photoabsorption cross section of the nucleus ^{12}C.

1. INTRODUCTION

One of the main problems in the detailed escription of giant multipole resonances (GMR) is the problem of the coupling between the entrance (1p1h) states and the 2p2h, 3p3h, ... nuclear excitations. The coupling of the 1p1h-states to the more complex configurations leads to the three different types of damping of normal vibrations, shown schematically in Fig. 1, namely, particle fragmentation (a), hole fragmentation (b), and renormalization of the particle-hole interaction (c). The first two processes have been investigated experimentally in one-nucleon transfer reactions. Spectroscopic information obtained from these reactions can be used directly to calculate the structure and the decay parameters of GMR, in the same way as in the semimicroscopic vibrational model (SVM) [1-5]. In particular, it is shown in [5] how the random phase approximation (RPA) must be modified to take into account process (b). The particle fragmentation process can be taken into account in a similar way.

In this paper, we consider the evaluation of the GMR strength functions using the SVM theory. In addition to fragmentation processes such as (a) and (b), we also take into account the coupling of entrance states to the continuum. The method that we employ is close to that used in the quasiparticle-phonon model [6]. However, in contrast to [6], the basis wave functions are taken to be the initial shell configurations rather than calculated phonon states. This enables us to express the GMR strength function in terms of quantities that can be estimated from spectroscopic data.

2. DESCRIPTION OF THE MODEL

Consider the eigenstates $|\Psi_i\rangle$ of the nuclear Hamiltonian H, excited by some external field $\mathscr{F}(\Gamma)$ of multipolarity $\Gamma \equiv \{J_\Gamma, T_\Gamma\}$. We shall follow the RPA theory for nuclei with unfilled shells, formulated in [7], and assume that these states can be written in the form

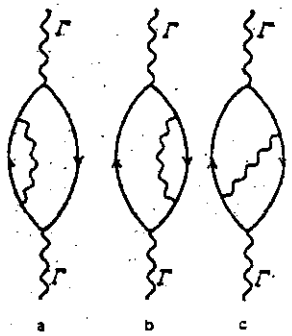

Fig. 1. Diagrams illustrating the damping of normal vibrations. Wavy lines represent one-phonon excitation.

$$|\Psi_i\rangle = Q_i^+|\Psi_0\rangle = \sum_k \{x_k(i)\,A_k^+|\Psi_0\rangle - y_k(i)\,A_{\tilde{k}}|\Psi_0\rangle\} + \\ + \sum_k \sum_{\alpha_k} \{u_{\alpha_k}(i)\,G_{\alpha_k}^+|\Psi_0\rangle - v_{\alpha_k}(i)\,G_{\tilde{\alpha}_k}|\Psi_0\rangle\} \equiv |\Psi_i^{(1)}\rangle + |\Psi_i^{(2)}\rangle, \quad (1)$$

where Q_i^+ is the operator representing the creation of a phonon with excitation energy ω_i, x, y, u, v are the coefficients in the expansion of the phonon over the basis states, $|\Psi_0\rangle$ is the ground state of the nucleus (assuming for simplicity that $J_0 = T_0 = 0$ in the ground state), A_k^+ and A_k are the creation and annihilation operators for the entrance states in the field $\mathcal{F}(\Gamma)$, so that

$$\langle\Psi_i|\mathcal{F}(\Gamma)|\Psi_0\rangle = \langle\Psi_i^{(1)}|\mathcal{F}(\Gamma)|\Psi_0\rangle$$

$G_{\alpha_k}^+$ and G_{α_k} are the creation and annihilation operators for states into which the state $A_k^+|\Psi_0\rangle$ decays, and the hat over the state index represents the transformation of a particle into a hole [8].

The states $A_k^+|\Psi_0\rangle$, $A_{\tilde{k}}|\Psi_0\rangle$, $k = 1, 2, \ldots$ form the basis for the normal vibrations of the nucleus. They can be the 1p1h-excitations [1] or the more complex states (1p1h + 2p2h + ...) if fragmentation effects are taken into account in the basis $\{A_k^+|\Psi_0\rangle, A_{\tilde{k}}|\Psi_0\rangle\}$. Some of these states have a particle in the continnum. At this stage of our calculation, we shall suppose that the continuum has been discretized in some way. This is not a significant restriction because discretization can be abandoned in the final stage of our calculation (see section 3).

The states $G_{\alpha_k}^+|\Psi_0\rangle$, $G_{\tilde{\alpha}_k}|\Psi_0\rangle$ correspond to the configurations (2p2h + 3p3h + ...). For processes a) and b) of Fig. 1, we may suppose that different states $A_k^+|\Psi_0\rangle$, $k = 1, 2, \ldots$ decay to the noncrossing groups of states $\{G_{\alpha_1}^+|\Psi_0\rangle\}$, $\{G_{\alpha_2}^+|\Psi_0\rangle\}, \ldots$. The $A_k^+|\Psi_0\rangle \to G_{\alpha_k}^+|\Psi_0\rangle$ transitions are then used to describe the spreading of the 1p1h-states that was not taken into account in the basis $\{A_k^+|\Psi_0\rangle, A_{\tilde{k}}|\Psi_0\rangle\}$ (for example, because of the lack of spectroscopic data, as is the case for the quasidiscrete particle excitations). We shall now assume that the states $G_{\alpha_k}^+|\Psi_0\rangle$ have been chosen so that they are the eigenstates of the Hamiltonian H in the absence of the coupling to the states $A_k^+|\Psi_0\rangle$.

Substituting (1) in the equation of motion

$$[H, Q_i^+]\,|\Psi_0\rangle = \omega_i |\Psi_0\rangle$$

and using the approximations adopted in SVM [1,5], we obtain

$$x_k(i) = \frac{\varkappa M_k \langle \Psi_i | \mathcal{F}(\Gamma) | \Psi_0 \rangle}{\omega_i - \omega_k - \sum\limits_{\alpha_k} \frac{V_{k,\alpha_k}^2}{\omega_i - \omega_{\alpha_k}}},$$

$$y_k(i) = \frac{c\varkappa M_k \langle \Psi_i | \mathcal{F}(\Gamma) | \Psi_0 \rangle}{\omega_i + \omega_k - \sum\limits_{\alpha_k} \frac{V_{k,\alpha_k}^2}{\omega_i + \omega_{\alpha_k}}}, \qquad (2)$$

$$u_{\alpha_k}(i) = \frac{V_{k,\alpha_k}}{\omega_i - \omega_{\alpha_k}} x_k(i),$$

$$v_{\alpha_k}(i) = -\frac{V_{k,\alpha_k}}{\omega_i + \omega_{\alpha_k}} y_k(i),$$

where $\varkappa$ is a constant characterizing the coupling of the field $\mathcal{F}(\Gamma)$ to the corresponding nuclear moment (multipole-multipole constant), $M_k = \langle \Psi_0 | A_k \mathcal{F}(\Gamma) | \Psi_0 \rangle$ is the amplitude for the transition $|\Psi_0\rangle \to A_k^+ |\Psi_0\rangle$ under the influence of the field $\mathcal{F}(\Gamma)$ (expressed in terms of the one-particle matrix elements of the operator $\mathcal{F}(\Gamma)$ and the spectroscopic factors of states populated in the one-nucleon transfer reactions [1,5]), ω_k, ω_{α_k} are the excitation energies of the states $A_k^+|\Psi_0\rangle$ and $G_{\alpha_k}^+|\Psi_0\rangle$, $V_{k,\alpha_k} \equiv \langle \Psi_0 | A_k V G_{\alpha_k}^+ | \Psi_0 \rangle$ is the matrix element describing the coupling of the states $A_k^+|\Psi_0\rangle$ and $G_{\alpha_k}^+|\Psi_0\rangle$, V is the effective interaction, and $c = \pm 1$ is a constant characterizing the properties of the operator $\mathcal{F}(\Gamma)$ under the particle-hole conjugation [8].

The amplitude for the transition $|\Psi_0\rangle \to |\Psi_i\rangle$ produced by the field $\mathcal{F}(\Gamma)$ can be written in the form [1,5]:

$$\langle \Psi_i | \mathcal{F}(\Gamma) | \Psi_0 \rangle = \sum_k M_k (x_k(i) - c y_k(i)).$$

Substituting (2) into this expression, we obtain the secular equation describing the excitation energies ω_i of the states $|\Psi_i\rangle$:

$$D(\omega_i) = 1 - \varkappa \sum_k M_k^2 \left[\left(\omega_i - \omega_k - \sum_{\alpha_k} \frac{V_{k,\alpha_k}^2}{\omega_i - \omega_{\alpha_k}} \right)^{-1} - \right.$$
$$\left. - \left(\omega_i + \omega_k - \sum_{\alpha_k} \frac{V_{k,\alpha_k}^2}{\omega_i + \omega_{\alpha_k}} \right)^{-1} \right] = 0. \qquad (3)$$

The transition amplitude $\langle \Psi_i | \mathcal{F}(\Gamma) | \Psi_0 \rangle$ is determined by the normalization condition

$$\langle \Psi_i | \Psi_i \rangle = \sum_k (x_k^2 - y_k^2) + \sum_k \sum_{\alpha_k} (u_{\alpha_k}^2 - v_{\alpha_k}^2) = 1.$$

Substituting (2) into this expression, we obtain

$$\langle \Psi_i | \mathcal{F}(\Gamma) | \Psi_0 \rangle^2 = \frac{1}{\varkappa \dfrac{\partial D(\omega)}{\partial \omega}}\Bigg|_{\omega=\omega_i}.$$

3. DETERMINATION OF THE STRENGTH FUNCTION

The properties of the individual states $|\Psi_i\rangle$ are very laborious to determine because the number of such states is in general very large. It is possible, however, to take an average over the states $|\Psi_i\rangle$. Let us introduce the strength function for the $\mathcal{F}(\Gamma)$-excitation of a nucleus, namely,

$$S(E) = \sum_i \frac{f_i}{\omega_i} E\,\rho(E-\omega_i),$$

where

$$f_i = \langle \Psi_i | \mathcal{F}(\Gamma) | \Psi_0 \rangle^2 \omega_i = \frac{\omega}{\varkappa \dfrac{\partial D(\omega)}{\partial \omega}}\Bigg|_{\omega=\omega_i} \qquad (4)$$

is the oscillator strength of the transition $|\Psi_0\rangle \to |\Psi_i\rangle$ produced by the field $\mathcal{F}(\Gamma)$,

$$\rho(E-\omega_i) = \frac{1}{2\pi} \frac{\Delta}{(E-\omega_i)^2 + (\Delta/2)^2}$$

is the strength function [8], and Δ is the interval over which the average is evaluated.

The function S(E) describes the energy distribution of the oscillator strengths of the $\mathcal{F}(\Gamma)$-transitions. It is important to note that the averaging interval Δ has no effect on the integrated properties of this distribution if the following conditions is satisfied:

$$\Delta \ll \varkappa M_k^2. \qquad (5)$$

It follows from (3) and (4) that f_i is the residue of the function $\Pi(\omega) = \omega/[\varkappa D(\omega)]$ at the pole ω_i. Hence the strength function can be written as a contour integral in the complex plane [8,9]. By evaluating this integral, we obtain

$$S(E) = -\frac{1}{\pi} \operatorname{Im}\left\{\frac{E}{\varkappa D(\omega)_i}\right\}\Bigg|_{\omega=E+i\Delta/2}.$$

Substituting (3) in this expression, and using some simple simplifications, we obtain

$$S(E) = \frac{1}{2\pi} \frac{EB(E)}{A^2(E) + \frac{1}{4}\varkappa^2 B^2(E)}. \qquad (6)$$

where

$$A(E) = 1 - \sum_k \varkappa M_k^2 \left[\frac{E-\omega_k-\Delta\omega_k}{(E-\omega_k-\Delta\omega_k)^2 + \frac{1}{4}(\Gamma_k+\Delta)^2} - \frac{E+\omega_k}{(E+\omega_k)^2 + \frac{1}{4}\Delta^2}\right], \qquad (7)$$

$$B(E)=\sum_k M_k^2\left[\frac{\Gamma_k+\Delta}{(E-\omega_k-\Delta\omega_k)^2+\frac{1}{4}(\Gamma_k+\Delta)^2}-\frac{\Delta}{(E+\omega_k)^2+\frac{1}{4}\Delta^2}\right]. \qquad (8)$$

The quantities

$$\Gamma_k=\Delta\sum_{\alpha_k}\frac{V_{k,\alpha_k}^2}{(E-\omega_{\alpha_k})^2+\frac{1}{4}\Delta^2}$$

and

$$\Delta\omega_k=\sum_{\alpha_k}\frac{V_{k,\alpha_k}^2(E-\omega_{\alpha_k})}{(E-\omega_{\alpha_k})^2+\frac{1}{4}\Delta^2}$$

describe, respectively, the width and the energy shift of the state $A_k^+|\Psi_0\rangle$ due to its coupling to the states $G_{\alpha_k}^+|\Psi_0\rangle$ [8]. For process (a) or (b) of Fig. 1, they can be estimated from spectroscopic data.

It is clear from (7) and (8) that the functions A(E), B(E) have no singularities when $\Delta \neq 0$. This means that summation over the discrete set of "continuum" states can be replaced by integration over the continuum for these functions with $\Delta \neq 0$.

When (6) is used in practice, it is important to remember that it was obtained in an approximation in which only the "particle" and "hole" mechanisms were taken into account in the damping of normal vibrations. Of course, this restricts the range of its applicability. For example, it is hardly suitable for the description of GMR in the region of intermediate-mass vibrational nuclei for which there are strong coherent effects (see diagram (c) of Fig. 1) due to the coupling between charge and surface vibrations of the nucleus. However, (6) may be useful in the description of GMR of light nuclei ($A \lesssim 40$) for which the coupling between the charge and surface vibrations appears to be much weaker. It is also important to note that the restrictions that we have imposed have enabled us to parametrize the strength functions in terms of the quantities $(M_k^2, \omega_k, \Gamma_k, \Delta\omega_k)$ that can be estimated from spectroscopic data.

4. APPLICATION TO THE PHOTODISINTEGRATION OF ^{12}C

To check the validity of the assumptions made in the derivation of (6), we have used it to calculate the photoabsorption cross section of ^{12}C. This cross section is related to the strength function for the giant dipole resonance (GDR) by

$$\sigma_{abs}(E)=\frac{4\pi^2e^2}{3\hbar c}S(E).$$

The calculation was performed as follows. The one-particle wave functions were calculated for the Woods-Saxon potential whose geometric parameters were taken from [10], and whose depth and spin-orbital coupling constant was deduced from a fit to the spectroscopic data for each $j = l \pm 1/2$ douplet [11].

The entrance states $A_k^+|\Psi_0\rangle$, $k=1, 2, \ldots$ were taken to be states of the form "particle above a finite nucleus in a particular state" [5]. All final states over which the holes $(1p_{1/2})^{-1}$, $(1p_{3/2})^{-1}$ and $(1s_{1/2})^{-1}$ were spread were taken into account. The experimental strength function [12] was substituted for the $(1s_{1/2})^{-1}$ hole in (6): it was assumed that the square of the matrix element $M_k^2=\langle\Psi_0|A_k\mathcal{F}(\Gamma)|\Psi_0\rangle^2$ for the $1s_{1/2}\rightarrow 1p_{1/2,3/2}$ transitions was distributed over the energy ω_k in accordance with

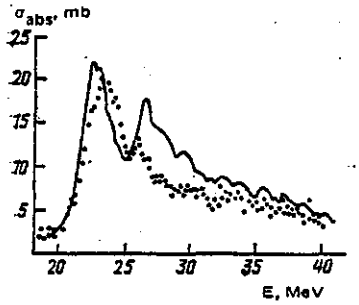

Fig. 2. Photoabsorption cross section of the nucleus ^{12}C. Points) experimental [14]; solid curve) calculated.

the one-hole strength function. The width Γ_k = 6 MeV was then used to describe the coupling of the high-energy one-particle resonance $1d_{3/2}$ to more complicated configurations. The chosen Γ_k falls into the energy range 5-10 MeV given by the optical model [10,13]. The small width Γ_k = 0.5 MeV was also introduced for the quasidiscrete state $1d_{5/2}$. All the quantities $\Delta\omega_k$ were assumed zero. The calculation was carried out with the dipole-dipole constant κ reported in [1]. The averaging interval Δ was assumed to be 0.5 MeV.

The results of this calculation are shown in Fig. 2. As can be seen, the two photoabsorption peaks at E_γ = 22.5 and 25-27 MeV are in agreement with experimental data (see also the calculations reported in [1,7,15,16]). The first peak is mostly due to the configuration $\Psi_{i=0}\,(A-1)\,(1d_{5/2})^1$ (the index i = 0, 1, 2, ... labels states of the final nucleus, beginning with the ground state, in order of increasing energy (whereas the second peak corresponds to the configuration $\Psi_{i=1}(A-1)\,(1d_{3/2})^1$. It also contains a contribution due to the configuration $\Psi_{i=3}(A-1)\,(1d_{5/2})^1$, that is responsible for the narrow peak at E_1 = 26 MeV. The main contribution to the photoabsorption cross section at E ≥ 30 MeV is due to $1s_{1/2}\rightarrow 1p_{1/2,3/2}$ transitions. On the whole, the calculations reproduce the size and position of the experimental resonances quite well, but the calculated cross section is too high in the region of the second peak. An analogous result was previously reported in [1,7] in the 1p-1h approximation. This discrepancy is a measure of coherent processes such as (c) of Fig. 1 that have not been taken into account in the present calculation.

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