DIFFRACTION RADIATION PRODUCED DURING THE EXCITATION OF CYCLOTRON WAVES IN A RELATIVISTIC ELECTRON BEAM

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Deffraction radiation generated when fast and slow cyclotron waves are excited in a relativistic electron beam is examined theoretically. The focusing magnetic field H_0 is shown to play a fundamental role in this process and that it can be used to control the amplitudes of radiated harmonics in multimode diffraction generators.

INTRODUCTION

The development of powerful and ultrapowerful sources of coherent electromagnetic radiation for the microwave region, using relativistic electron beams and wide electrodynamic systems, is an urgent problem at present. The bulk-wave generator is an example of this in which multimode diffraction radiation is emitted by a relativistic electron beam traveling near a periodic surface (diffraction grating). The design of such devices relies on theoretical analyses of the properties of multimode coherent radiation from electron beams, taking into account both the longitudinal and lateral motion of electrons, since the experiment reported in [1] has shown that the radiated power depends on the strength of the focusing magnetic field.

The distribution of diffraction radiation from an electron beam in which transverse waves (synchronous waves) were first excited was investigated experimentally in [2]. Cyclotron waves (lateral velocity waves) were absent from the beam because the experimental conditions were too far from cyclotron resonance.

In this paper, we examine theoretically the radiation emitted by a relativistic electron beam under the conditions of cyclotron resonance at a Doppler-shifted frequency.

Interacting waves. We shall consider an electrodynamic system, analogous to systems used in multimode diffraction generators. We shall consider that physical processes are determined by the self-consistent interaction between the relativistic electron beam and the multimode diffracted field at a given frequency ω. The diffracting body will be a plane, perfectly conducting grating with a rectangular profile (grating l). The length of the grating in the longitudinal direction is L, but in the lateral direction the grating is infinite (Fig. 1).

Consider the effect of the focusing magnetic field H_0 on the diffraction field. The wave number spectrum for $H_0 = 0$ is given by

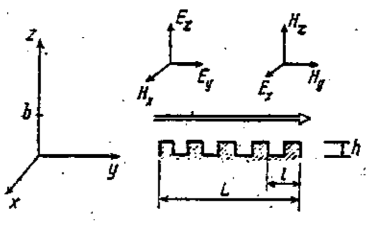

Fig. 1. The system under investigation.

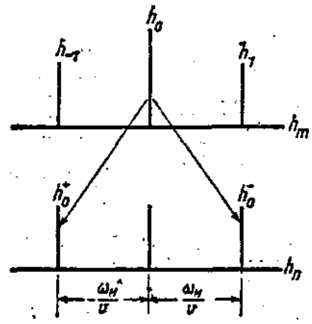

Fig. 2. Superposition of the spectra.

$$h_m = \frac{\omega}{v} + \frac{2\pi}{l} m, \qquad m = 0, \pm 1, \pm 2, \ldots,$$

where h_m is the longitudinal wave number and v is the velocity of electrons in the beam. When $H_0 \neq 0$, we have two additional spectra, namely,

$$h_n^{\pm} = \frac{\omega}{v} + \frac{2\pi}{l} n^{\pm} \mp \frac{\omega_H}{v}, \qquad n^{\pm} = 0, \pm 1, \pm 2, \ldots,$$

where ω_H is the cyclotron frequency. For each harmonic of number m there are two components with numbers n^+ and n^-, shifted by $-\omega_H/v$ and ω_H/v. The shift ω_H/v is due to the anomolous Doppler effect and the shift $-\omega_H/v$ to the normal Doppler effect.

When H_0 is suitably chosen, i.e., so that

$$\omega_H = \mp \frac{2\pi}{l} v (n^{\pm} - m), \qquad \mp (n^{\pm} - m) = 1, 2, \ldots, \tag{1}$$

spectra with numbers $n^{\pm}$ and m will coincide. Let $\omega_H = (2\pi/l)v$. The superposition of the spectra (Fig. 2) then occurs so that the longitudinal wave number h_{-1} of the m = 1 harmonic of the main spectrum coincides with the longitudinal wave number h_0^+ of the zero component of the Doppler-shifted original spectrum. The longitudinal wave number h_1 of the m = +1 harmonic of the main spectrum coincides with the longitudinal wave number h_0 of the zero component of the additional spectrum, shifted by the anomalous Doppler effect. In the linear approximation, this wave number matching leads to the excitation of fast and slow cyclotron waves in the beam. The condition for the validity of this approximation is, in this case, that the amplitudes of the diffracted waves must be small in comparison with the focusing magnetic field. In the terminology of the linear theory, the above superposition of the spectra corresponds to the excitation of a fast cyclotron wave by the -1 diffracted harmonic and to the excitation of a slow cyclotron wave by the +1 diffracted harmonic. It is well known [3] that a fast cyclotron wave transports negative power and a slow harmonic transports positive power. Consequently, when the magnetic field

H_0 is suitably chosen, the excitation of the fast cyclotron wave leads to the absorption of the -1 harmonic by the beam, whereas the excitation of the slow wave leads to the amplification of the +1 harmonic. We note that, according to (1), the absorption of the -1 and -2 diffraction harmonics occurs in fields H_0 that differ by a factor of 2.

The longitudinal and transverse motion of the electrons must be taken into account when we consider diffraction radiation emitted by a relativistic electron beam in a focusing magnetic field. This enables us to solve the problem of the transformation of a H-polarized electromagnetic wave into an E-polarized wave (see Fig. 1 in which we use the notation of [4]). The transformation is possible because the electron beam is uncoiled by the field of the H-polarized wave in the focusing magnetic field.

Diffractin theory. To analyze the longitudinal and transverse motion of electrons, we introduce electric and magnetic currents [5] that vary slowly in the longitudinal direction. The electric current density is given by

$$j_{\parallel}^{e} = v_{\parallel}\rho_0 e^{i\frac{\omega}{v_{\parallel}}y}\delta(z-b)\,e^{-i\omega t}, \tag{2}$$

and the magnetic current density by

$$j_{\parallel}^{m} = -\frac{i\omega}{2c}[\mathbf{r}_{\perp}\times\mathbf{v}_{\perp}]_{\parallel}\rho_0 e^{i\frac{\omega}{v_{\parallel}}y}\delta(z-b)\,e^{-i\omega t}, \tag{3}$$

where $v_{\parallel}$ is the longitudinal velocity of the electrons, $\mathbf{v}_{\perp}$ and $\mathbf{r}_{\perp}$ are, respectively, the transverse velocity and displacement of electrons, ω is the modulation frequency, ρ_0 is the constant component of the charge density, and b is the distance between the current and the grating (impact parameter). It is assumed in (3) that $v_{\perp}/c \ll 1$.

The electric current $j_{\parallel}^{e}$ excites H-polarized waves:

$$\Delta A_{\parallel}^{e} + \frac{\omega^2}{c^2}A_{\parallel}^{e} = -\frac{4\pi}{c}j_{\parallel}^{e},$$

$$A_{\parallel}^{e} = \frac{2\pi\rho_0}{\frac{\omega}{v_{\parallel}}\sqrt{1-\frac{v_{\parallel}^2}{c^2}}}\,\frac{v_{\parallel}}{c}\,e^{-\frac{\omega}{v_{\parallel}}\sqrt{1-\frac{v_{\parallel}^2}{c^2}}|z-b|}\,e^{i\frac{\omega}{v_{\parallel}}y}\,e^{-i\omega t},$$

$$H_x^{e} = 2\pi\rho_0\frac{v_{\parallel}}{c}\,e^{-\frac{\omega}{v_{\parallel}}\sqrt{1-\frac{v_{\parallel}^2}{c^2}}|z-b|}\,\mathrm{sign}(z-b)\,e^{i\frac{\omega}{v_{\parallel}}y}\,e^{-i\omega t},$$

$$H_y^{e} = H_z^{e} = 0, \quad \mathbf{E}^{e} = i\frac{c}{\omega}\,\mathrm{rot}\,\mathbf{H}^{e}.$$

The magnetic current $j_{\parallel}^{m}$ is the source of the E-polarized waves:

$$\Delta A_{\parallel}^{m} + \frac{\omega^2}{c^2}A_{\parallel}^{m} = -\frac{4\pi}{c}j_{\parallel}^{m},$$

$$A_{\parallel}^{m} = -i\,\frac{\pi[\mathbf{r}_{\perp}\times\mathbf{v}_{\perp}]_{\parallel}\rho_0}{\sqrt{c^2-v_{\parallel}^2}}\,\frac{v_{\parallel}}{c}\,e^{-\frac{\omega}{v_{\parallel}}\sqrt{1-\frac{v_{\parallel}^2}{c^2}}|z-b|}\,e^{i\frac{\omega}{v_{\parallel}}y}\,e^{-i\omega t},$$

$$E_x^{m} = i\frac{\pi}{c^2}\,[\mathbf{r}_{\perp}\times\mathbf{v}_{\perp}]_{\parallel}\rho_0\omega\,e^{-\frac{\omega}{v_{\parallel}}\sqrt{1-\frac{v_{\parallel}^2}{c^2}}|z-b|}\,\mathrm{sign}(z-b)\,e^{i\frac{\omega}{v_{\parallel}}y}\,e^{-i\omega t},$$

$$E_y^m = E_z^m = 0, \quad \mathbf{H}^m = -i\frac{c}{\omega}\,\mathrm{rot}\,\mathbf{E}^m,$$

where $A_{\parallel}^e$ and $A_{\parallel}^m$ are, respectively, the longitudinal components of the electric and magnetic vector potentials.

The diffraction of the H- and E-polarized fields by an infinite grating can be described by the mathematical theory of diffraction [4]. The diffracted field is the sum of an infinite number of harmonics that are damped in the transverse direction and an infinite number of plane waves. For the H-polarization,

$$H_x^e = \sum_m H_{xm}^e e^{p_m(z-h)} e^{ih_m y} e^{-i\omega t}, \qquad (4)$$

and for the E-polarization,

$$E_x^m = \sum_m E_{xm}^m e^{p_m(z-h)} e^{ih_m y} e^{-i\omega t},$$

where $p_m = i\sqrt{\omega^2/c^2 - h_m^2}$ is the transverse wave number, $h_m = \omega/v_\parallel + (2\pi/l)m$, $m = 0, \pm 1, \pm 2, \ldots$ is the longitudinal wave number, and H_x^e and E_x^m are the complex amplitudes of the two types of wave. Diffracted harmonics (plane waves) are characterized by the angle θ_m between the direction of propagation of the m-th harmonic and the plane of the grating, $\theta_m = \pm\arcsin(p_m c/\omega)$, where it is assumed that $\theta_m > 0$ when $h_m > 0$ (wave propagates in the direction of the current) and $\theta_m < 0$ when $h_m < 0$.

Numerical analysis of the self-consistent interaction. The complex amplitudes H^e_{xm} and E^m_{xm} are found from the boundary conditions that the electromagnetic field must satisfy on the surface of the perfectly conducting diffraction grating. These conditions lead to linear sets of algebraic equations, similar to those investigated in [4], which are solved by a computer using the truncation method. The dependence of H^e_{xm} and E^m_{xm} on the longitudinal coordinate corresponds to the slow variation in the amplitudes of the electric and magnetic currents.

The motion of electrons can be investigated with the aid of the relativistic equation [6] for the discrete model of the current:

$$\begin{cases} \dfrac{d\boldsymbol{\beta}}{dY} = \dfrac{e}{mc^2}\,\dfrac{\lambda}{\beta_y}\sqrt{1-\beta^2}\,(\mathbf{E} + [\boldsymbol{\beta}\times\mathbf{H}] - \boldsymbol{\beta}(\boldsymbol{\beta}\mathbf{E})), \\[2mm] \dfrac{dx}{dY} = \lambda\dfrac{\beta_x}{\beta_y}, \quad \dfrac{dz}{dY} = \lambda\dfrac{\beta_z}{\beta_y}, \quad \dfrac{dT}{dY} = 2\pi\dfrac{1}{\beta_y}, \end{cases} \qquad (5)$$

where $\beta = v/c$, $Y = y/\lambda$, $T = \omega t$, $\lambda = 2\pi c/\omega$, e is the charge of the electron, and m is the electron rest mass. We assume that the electron current enters the region of interaction ($Y = 0$) with zero transverse velocity when its distance from the grating is equal to the impact parameter and $T = 0$. On the right hand side of the equations of motion, the fields H and E are given as a set of diffraction harmonics and the focusing magnetic field is also taken into account. The solution of (5) is found numerically by the modified Euler method. The electric (2) and magnetic (3) currents are determined from the velocities and coordinates of electrons obtained in this way.

The interaction between the electron current and the field is analyzed on

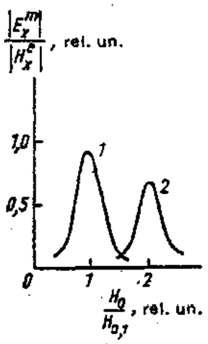

Fig. 3

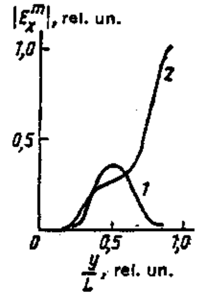

Fig. 4

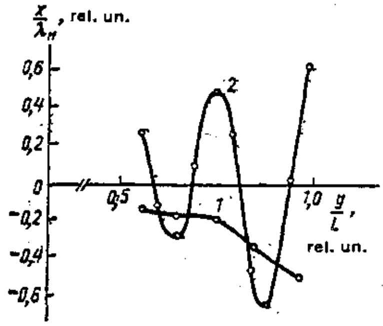

Fig. 5

Fig. 3. Transformation of H-polarized diffraction harmonics into E-waves.

Fig. 4. Distribution of the amplitude of the E-polarized field along the length of the system.

Fig. 5. Distribution of electrons in the beam at a given time (λ_H is the cyclotron wavelength): the fast (1) and slow (2) waves are shown.

a computer by successively specifying the current and the field and repeatedly solving the equations. The initial conditions correspond to a given electric current and no transverse motion of electrons. The transverse electric component E_z^e of the spatial harmonics of the H-polarized diffracted wave uncoils the current (in which the fast and slow cyclotron waves are excited), and a nonzero magnetic current appears. Its interaction with the diffraction grating produces the E-polarized diffracted wave.

In our calculations, we considered a two-mode diffracted field, i.e., we assumed that the spectrum was dominated by two plane waves, i.e., harmonics with numbers m = -1 and m = -2 in (4), where the m = -1 harmonic propagates in the direction of the current at the angle $\theta_{-1} = 86°$ to the plane of the diffraction grating, and the m = -2 harmonic propagates in the opposite direction at the angle $\theta_{-2} = -26°$. The product of the frequency by the period of the grating is held constant: $(\omega/c)l = 6.5$. The impact parameter b is chosen as follows. When b is large enough, the effect of the damped diffracted harmonics can be neglected because the radiation due to the slow cyclotron wave is negligible in comparison with the radiation by the fast cyclotron wave. The interaction with the current is largely determined by the propagating diffraction harmonics excited by the fast cyclotron wave (mostly plane waves with m = -1, -2).

The transformation of the waves was described by the amplitude E_x^m of the E-wave and by the ratio E_x^m/H_m^e of this amplitude to the amplitude of the H-polarized wave. Figure 3 shows E_x^m/H_x^e as a function of the focusing magnetic field. Curve 1 refers to the transformation of the -1 diffracted harmonic of the H-polarized wave into the E-wave and curve 2 represents the transformation of the -2 harmonic. The quantity H_{01} corresponds to the condition $\omega_H = (2\pi/l)v_{\parallel}^-$. The fact that curves 1 and 2 have maxima is explained by the excitation of a fast cyclotron wave by the diffracted harmonics with m = -1 and m = -2, respectively. When $H_0 = H_{01}$, the power transported by the -1 harmonic is absorbed by the electron current, and when $H_0 = 2H_{01}$ the power is absorbed by the -2 harmonic.

Figure 4 shows the distribution of the amplitude of the E-polarized wave along the length of the system. The two curves correspond to curve 1 of Fig. 3. Curve 2 was obtained for a focusing magnetic field $H_0 = H_{01}$ and curve 1 for $H_0 = 0.5H_{01}$. The increase in the amplitude of the E-polarized wave along the length of the system is an indication of an increasing uncoiling of the electron beam and, consequently, the absorption by the current of the power carried by the -1 harmonic of the H-polarized field.

The excitation of the fast and slow cyclotron waves by the diffracted harmonics is illustrated in Fig. 5.

The above effect can be used to control the amplitudes of harmonics radiated in multimode diffraction generators. The above situation is typical for such devices, i.e., $\theta_{-1} > 0$ and $\theta_{-2} < 0$, where the -1 diffracted harmonic extracts the electromagnetic field from the interaction region and the -2 harmonic produces positive feedback in the system. By varying the focusing magnetic field, one can control the output power of the generator optimize the feedback.

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