

SECOND VISCOSITY IN A VIBRATIONALLY NONEQUILIBRIUM GAS

A. I. Osipov and A. V. Uvarov

Vestnik Moskovskogo Universiteta. Fizika,
Vol. 42, No. 6, pp. 52-56, 1987

UDC 533.6

The relevance of the concept of second viscosity in hydrodynamic processes in a nonequilibrium gas is discussed with allowance for the presence of a pump and heat transfer. It is shown that the second viscosity coefficient may be negative under certain particular conditions but this does not affect the overall positive sign of the total production of entropy in the system.

It is well known that second viscosity is a macroscopic manifestation of a departure from equilibrium in the internal degrees of freedom [1-4]. It appears only in a special class of solutions of the equations of relaxation hydrodynamics. These solutions are distinguished by the fact that the relatively fast relaxation process is considered complete in the zero-order approximation and the local departure from equilibrium at each instant of time is wholly determined by the hydrodynamic fields at that time [3]. In the case of the propagation of sound, this requirement is equivalent to the condition $\omega\tau \ll 1$, where ω is the frequency of sound and τ is the relaxation time. Anomalous absorption of sound at low frequencies can therefore be looked upon as a manifestation of the second viscosity effect. At frequencies satisfying the condition $\omega\tau \geq 1$, it is meaningless to speak of second viscosity because the deviation of the stress tensor from the equilibrium tensor cannot be described by the simple relation $\pi_{ik} \sim \xi \delta_{ik} \text{div } \mathbf{v}$, where ξ is the second viscosity and δ_{ik} is the Kronecker symbol.

The amplification of sound is possible during the propagation of acoustic waves in a vibrationally nonequilibrium gas with a surplus of vibrational energy [5-7]. It was noted in [6] that the amplification of sound can then be described in terms of a negative second viscosity coefficient. We know that this approach must be treated with caution because second viscosity is meaningful only at low frequencies and it is precisely in this frequency range that the results of [6] become inaccurate because of restrictions on the vibrational nonequilibrium.

The aim of this paper is to investigate the relevance of second viscosity in the description of hydrodynamic motion in an initially nonequilibrium gas, using the solutions of the equations of relaxation hydrodynamics. Our approach is based on the results reported in [3].

Consider a monotonic gas in a nonequilibrium stationary state, maintained by a pump supplying power I to internal (vibrational) degrees of freedom (e.g., optical radiation or gas discharge) and heat transfer at the rate Q from translational degrees of freedom (I and Q refer to a unit mass). We assume that I and Q depend on the thermodynamic parameters of the gas. For example, in the

case of a laser pump and fast V-V exchange, we can consider that $I = I(\epsilon)$; the rate of heat transfer in the simplest case is given by $Q = \alpha(T - T^*) / (R\rho)$ where ϵ is the mean vibrational energy per unit mass, ρ is the gas density, α is the heat transfer coefficient, T^* is the wall temperature, T is the gas temperature, and R is a characteristic linear dimension of the system. The equations of continuity, motion, and energy, and also the relaxation equation for ϵ , will be written in the form

$$\frac{dp}{dt} + \rho \operatorname{div} \mathbf{v} = 0, \quad (1)$$

$$\rho \frac{d\mathbf{v}}{dt} = -\operatorname{grad} p, \quad (2)$$

$$\frac{dE}{dt} = -\rho \frac{d}{dt} \left(\frac{1}{\rho} \right) + I - Q, \quad (3)$$

$$\frac{d\epsilon}{dt} = \frac{\epsilon_0 - \epsilon}{\tau} + I \quad (4)$$

where p and \mathbf{v} are, respectively, the pressure and velocity of the gas and $\epsilon_0 = \epsilon(T)$. Equations (1)-(4) do not take into account effects associated with ordinary viscosity and thermal conductivity that are due to the deviation of the distribution functions for the translational and internal degrees of freedom from the equilibrium functions corresponding to temperatures T and T_1 ($T \neq T_1$). We shall take the independent variables to be p , \mathbf{v} , ϵ and one of the thermodynamic functions, i.e., the entropy s , the total energy E , or some other characteristic function denoted by z . The specific choice of z depends on the particular problem. The quantities E , ϵ , s and z are referred to unit mass.

When we investigate hydrodynamic motion described by (1)-(4), it is convenient to use the small parameter $\mu = v_s \tau / L$ instead of the acoustic parameter $\omega \tau$, where v_s is the velocity of sound and L is a critical macroscopic length. We shall seek the solution of (1)-(4) by assuming that relaxation is complete in the zero-order approximation and a quasistationary two-temperature distribution ($T \neq T_1$) has evolved and is determined by the hydrodynamic field and the instantaneous values of the pump and heat transfer. The local deviation from quasistationary values (indicated by the superscript 0) is taken into account in the next approximation and is a function now of the same quantities as the zero-order approximation. We thus have

$$\epsilon(\mathbf{r}, \rho, z, \mathbf{v}) = \epsilon^{(0)}(\mathbf{r}, \rho, z, \mathbf{v}) + \mu \epsilon^{(1)}(\mathbf{r}, \rho, z, \mathbf{v}), \quad (5)$$

where

$$\epsilon^{(0)} = \epsilon_0(T^{(0)}) + I(\epsilon^{(0)}, \rho, z) - \tau(\epsilon^{(0)}, \rho, z); \quad I(\rho, z, \epsilon^{(0)}) = Q(\rho, z, \epsilon^{(0)}). \quad (6)$$

Analogous expansions can be written for the derivatives of the hydrodynamic variables with respect to time:

$$\begin{aligned} \partial \rho / \partial t &= \mu N^{(1)}(\mathbf{r}, \rho, z, \mathbf{v}) + \mu^2 N^{(2)}(\mathbf{r}, \rho, z, \mathbf{v}) + \dots \\ \partial \mathbf{v} / \partial t &= \mu U^{(1)}(\mathbf{r}, \rho, z, \mathbf{v}) + \mu^2 U^{(2)}(\mathbf{r}, \rho, z, \mathbf{v}) + \dots \\ \partial z / \partial t &= \mu Z^{(1)}(\mathbf{r}, \rho, z, \mathbf{v}) + \mu^2 Z^{(2)}(\mathbf{r}, \rho, z, \mathbf{v}) + \dots \end{aligned} \quad (7)$$

The explicit form of the coefficients in (7) is determined by comparing them with (1)-(4). Thus, since $\operatorname{grad} p$ and $\operatorname{div} \mathbf{v}$ are of the order of μ , we have

$$\mu N^{(1)} = -(\mathbf{v}, \text{grad } \rho) - \rho \text{ div } \mathbf{v}; \quad N^{(2)} = N^{(3)} = \dots = 0.$$

The expansion for p is

$$p = p^{(0)} + \mu p_e \varepsilon^{(1)}, \quad (8)$$

so that

$$\mu \mathbf{U}^{(1)} = -\text{grad } p^{(0)}/\rho; \quad \mu \mathbf{U}^{(2)} = -\text{grad } (p_e \varepsilon^{(1)})/\rho$$

where $p^{(0)}$ is the pressure in the stationary state for $\mu = 0$ and $p_e = \left(\frac{\partial p}{\partial \varepsilon} \right)_{\rho, z, \varepsilon = \varepsilon^{(0)}}$. We can now use (4) to determine $\varepsilon^{(1)}$. To within terms of the first order in μ , we have

$$\frac{d\varepsilon^{(0)}}{dt} = \mu \varepsilon^{(1)} \frac{c_0 T_e - 1 + (I\tau)_e}{\tau^{(0)}} \quad (9)$$

and

$$\frac{d\varepsilon^{(0)}}{dt} = \mu \varepsilon_p^{(0)} N^{(1)} + \mu \varepsilon_z^{(0)} Z^{(1)} + \varepsilon_p^{(0)} (\mathbf{v}, \text{grad } \rho) + \varepsilon_z^{(0)} (\mathbf{v}, \text{grad } z) \quad (10)$$

where $c_0 = \partial \varepsilon_0 / \partial T$, $T_e = \partial T / \partial \varepsilon$.

The variable z must be chosen so that (7) and (5) are valid. When hydrodynamic perturbations propagate through a gas that is initially in equilibrium, it is convenient to take $z = s$, since the quantity

$$\frac{ds}{dt} = \left(\frac{1}{T_1} - \frac{1}{T} \right) \frac{\varepsilon_0 - \varepsilon}{\tau} + \frac{I}{T_1} - \frac{Q}{T} \quad (11)$$

is then of the second order of small quantities. When $I = Q \neq 0$ and $T \neq T_1$, we can take z to be either E or s . The condition $I = Q$, which enables us to neglect $I - Q$ in (3), is then satisfied if $\tau_t \gg \tau$, where τ_t is the smallest of the characteristic times associated with the pump and the heat transfer process. Equating (9) and (10), we obtain the relation between $\varepsilon^{(1)}$ and $\text{div } \mathbf{v}$. We next substitute $\varepsilon^{(1)}$ in (8) and the result in (2). This gives the following expression for the second viscosity:

$$\xi = \frac{(\varepsilon_p^{(0)} + \varepsilon_E^{(0)}) p^{(0)} / \rho^2 \rho \tau}{c_0 T_e - 1 + I\tau_e} p_e \quad (12)$$

Thus, the introduction of second viscosity is equivalent to an abbreviated method of description in which instead of the four equations (1)-(4) we retain only three, and this is possible when $\mu \ll 1$. Since $T = (E - \varepsilon)/c$, where c is the specific heat associated with the translational-rotational degrees of freedom, the partial derivatives in (12) are given by $p_e = -k_B \rho / (mc)$, and $T_e = -1/c$ where m is the mass of the molecule and k_B is the Boltzmann constant. The derivatives of $\varepsilon^{(0)}$ can be found from (6):

$$\varepsilon_p^{(0)} = \frac{c\tau_p}{c + c_0 - c\tau_e}; \quad \varepsilon_E^{(0)} = \frac{c_0 + c\tau_E}{c + c_0 - c\tau_e}$$

The final result is therefore

$$\xi = \frac{k_B \rho \tau}{m} \frac{p^{(0)} (c_0 + c\tau_E) / \rho^2 - c\tau_p}{(c_0 + c - c\tau_e)^2} \quad (13)$$

When $\tau = \tau(T)$, this expression becomes identical with the expression for ξ obtained in [6] by harmonic analysis, since the condition $\xi = (-\rho/m) \text{Im}(\omega^2/k^2)$ is equivalent to $z = E$. We note that our derivation is not limited by the acoustic approximation.

When

$$\rho^{(0)}(c_0 + c_1 \tau_E) / \rho^2 < c_1 \tau_e$$

the second viscosity ξ becomes negative which corresponds to, for example, the amplification of low-frequency acoustic waves.

If instead of $\tau_t \gg \tau$ we now have $\tau_t \leq \tau$, we may consider that both relaxation and heat transfer processes are complete in the zero-order approximation, so that

$$\rho = \rho^{(0)}(\rho) + \mu \rho_e E^{(1)} + \mu \rho_E E^{(1)}, \quad (14)$$

where $E = E^{(0)} + \mu E^{(1)}$. The subsequent procedure is the same as before. The first order corrections in $\varepsilon^{(1)}$ and $E^{(1)}$ are found by solving the equations

$$\frac{d\varepsilon^{(0)}}{dt} = \varepsilon_p^{(0)} \frac{d\rho}{dt} = \mu \varepsilon^{(1)} \frac{c_0 T_e - 1 + (I\tau)_e}{\tau^{(0)}} + \mu E^{(1)} \frac{c_0 T_E + (I\tau)_E}{\tau^{(0)}}, \quad (15)$$

$$\frac{dE^{(0)}}{dt} = E_p^{(0)} \frac{d\rho}{dt} = \frac{\rho^{(0)}}{\rho^2} \frac{d\rho}{dt} + \mu (I-Q)_e \varepsilon^{(1)} + \mu (I-Q)_E E^{(1)}. \quad (16)$$

Hence, using (14) and (2), we obtain

$$\begin{aligned} \frac{\xi}{\rho\tau} = & \frac{(\rho^{(0)}/(\rho^2\tau) - E_p^{(0)}/\tau) [(c_0 T_e - 1 + (I\tau)_e) \rho_e - (c_0 T_E + (I\tau)_E) \rho_E] +}{(c_0 T_e - 1 + (I\tau)_e) (I-Q)_E - (c_0 T_E + (I\tau)_E) (I-Q)_e} + \\ & + \frac{\varepsilon_p^{(0)} [(I-Q)_e \rho_e - (I-Q)_E \rho_E]}{(c_0 T_e - 1 + (I\tau)_e) (I-Q)_E - (c_0 T_E + (I\tau)_E) (I-Q)_e}. \end{aligned} \quad (17)$$

The partial derivatives in (17) are found from (6) for $z = E$:

$$\frac{\partial \varepsilon^{(0)}}{\partial \rho} = \frac{c_0 T_p + (I\tau)_p + [c_0 T_E + (I\tau)_E] (Q-I)_p / (I-Q)_E}{1 - c_0 T_e - (I\tau)_e + [c_0 T_e + (I\tau)_E] (I-Q)_e / (I-Q)_E}, \quad (18)$$

$$\frac{\partial E^{(0)}}{\partial \rho} = \frac{[c_0 T_e - 1 + (I\tau)_e] (I-Q)_p / (I-Q)_e + c_0 T_p + (I\tau)_p}{-c_0 T_E - (I\tau)_E + [c_0 T_e - 1 + (I\tau)_e] (I-Q)_E / (I-Q)_e}.$$

Equations (17)-(18) define the second viscosity. We note that, when $\tau \geq \tau_t$, the second viscosity ceases to be a characteristic of the medium because it depends on the pump and heat-transfer parameters.

Like the absorption coefficient, the second viscosity can change sign. However, this does not affect the sign of the entropy production σ . Actually, $\sigma = (1/T_1 - 1/T)(e_0 - \varepsilon) / \tau > 0$. In contrast to ordinary hydrodynamics in which $\sigma = \xi (\text{div } \mathbf{v})^2$ [8], we find that in our case $\sigma = \sigma_0 + \mathbf{A} \text{div } \mathbf{v} + \dots$. The fact that ξ may be negative does not affect the sign of σ because the additional term in the entropy production, which is proportional to $\xi \text{div } \mathbf{v}$, is small in comparison with the entropy production σ_0 under undisturbed stationary conditions.

REFERENCES

1. L. I. Mandelstam and M. A. Leontovich, Zh. Eksp. Teor. Fiz., vol. 7, pp. 438-449, 1937.
2. S. V. Vallander, E. A. Nagnibeda, and M. A. Rydalevskaya, Some Problems in the Kinetic Theory of Chemically Reactive Gas Mixtures [in Russian], Leningrad, 1977.
3. E. V. Stupochenko, S. A. Losev, and A. I. Osipov, Relaxation Processes in Shock Waves [in Russian], Moscow, 1966.
4. M. N. Kogan, Dynamics of Tenuous Gases [in Russian], Moscow, 1967.
5. H. I. Bauer and H. E. Bass, Phys. Fluids, vol. 16, no. 7, pp. 988-996, 1973.
6. E. Ya. Kogan, Doctoral Thesis for Fiz.-Mat. Science Degree [in Russian], Kuibyshev State Pedagogical Institute, 1984.
7. A. I. Osipov and A. V. Uvarov, Vestn. Mosk. Univ. Ser. 3 Fiz., Astron. [Moscow University Physics Bulletin], no. 6, pp. 74-77, 1984.
8. L. D. Landau and E. M. Lifshits, Fluid Dynamics [in Russian], Moscow, 1953.

10 June 1986

Chair of Molecular Physics
and Physical Measurement