B R I E F   C O M M U N I C A T I O N S

MASS FORMULAS FOR CHARMED BARYONS

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The hypothesis of quark-biquark structure of ordinary and charmed baryons can be used to express unambiguously the constants in the mass formulas of the quark models with flavors u, d, s, c in terms of the F- and D-coupling constants in the baryonic symmetry SU(4). The result is generalized to SU(n) symmetry.

The mass formulas for baryons with new quantum numbers have recently attracted considerable attention. Most approaches are based on some variant of the quark model with n flavors (n = 4, 5, 6). Departures from the unitary symmetry of baryons are allowed much less frequently. This is so because, even in the SU(4) symmetry, the mass difference between the $1/2^+$ baryons in the 20-plet is so large that the analogy with SU(3) is difficult to see (see, however, [1]). This is even more so in the case of the SU(5) 45-plet and the SU(6) 70-plet containing the $1/2^+$ baryons with quantum numbers B and T.

On the other hand, it can be shown that phenomenological one-gluon potentials often lead to mass formulas for baryons (see, for example [2]) with parameters that are linearly related to the F- and D-coupling constants in baryonic SU(3) symmetry. It has been suggested that the quark-biquark structure in the interior of baryons should be examined (see, for example, [3]). The baryon masses were calculated in [4] in a potential quark-biquark model. Moreover, it has recently been shown for the octet of $1/2^+$ baryons in SU(3) that the quark-biquark flavor structure can be uniquely related to the F- and D-type tensor structures typical for the hypercharge current [5]. An analogous result is valid for the electroweak current in SU(3) and SU(4) [6]. In the quark model with three flavors, this approach leads to the Gell-Mann-Okubo mass formula. We shall therefore try to generalize the most popular mass formulas for the 20-plet of $1/2^+$ baryons in SU(4) to the case of n flavors.

We recall that the mass term of the SU(3) Hamiltonian is [7]

$$H_M^{SU(3)} = (M_0 - D_Y)\,\overline{B}{}_b^a B_a^b + (D_Y + F_Y)\,\overline{B}{}_3^a B_a^3 + (D_Y - F_Y)\,\overline{B}{}_b^3 B_3^b. \qquad (1)$$

where $B_b{}^a$ is the octet of baryons with $1/2^+$, $p = B_3{}^1$. The mass splitting operator

$$\sum_{q=u,d,s} \hat{m}_{q(B)} \hat{Y}_q + m_0 \hat{I},$$

was proposed in [5] for the quark model with flavors u, d, s, where $\hat{Y}_q$ is the hypercharge operator. The eigenvalues m'_1 and m_1 of the operator $\hat{m}_{q(B)}$ distinguish the single quark q' and the biquark (qq) in the baryon B((qq)q'), respectively, and can be uniquely expressed in terms of the F- and D-coupling constants of the hypercharge current in (1): $m_1=-F_Y$, $m_1'=D_Y-F_Y$, where $M_0+(2/3)D_Y=m_0$.

The Hamiltonian (1) can be generalized to the SU(4) 20-plet of baryons described by the tensor $b^{\alpha}_{\beta\gamma}$ ($b^{\alpha}_{\beta\gamma}=-b^{\alpha}_{\gamma\beta}$, $b^{\alpha}_{\alpha\gamma}=0$, $\alpha, \beta, \gamma=1, \ldots, 4$), by introducing into H_M the charmed baryon current

$$H_M^{SU(4)}=(M_0-D_C+F_C)\,\bar{b}_{\gamma}^{\alpha\beta}b^{\gamma}_{\alpha\beta}+(1/2)\,(D_Y+F_Y)\,\bar{b}_3^{\alpha\beta}b^3_{\alpha\beta}+ \\ +(D_Y-F_Y)\,\bar{b}_{\gamma}^{\alpha3}b^{\gamma}_{\alpha3}+(1/2)\,(D_C+F_C)\,\bar{b}_4^{\alpha\beta}b^4_{\alpha\beta}+(D_C-F_C)\,\bar{b}_{\gamma}^{\alpha4}b^{\gamma}_{\alpha4}, \tag{2}$$

where $b^a_{b4}=B^a_b$, $a, b=1, 2, 3$.

We shall introduce the mass splitting into the four-flavor quark model via the hypercharge operator $\hat{Y}_q$ and the charm operator $\hat{C}_q$. The eigenvalues of $\hat{C}_q$ are zero for q = u, d, s and + 1 for q = c. Since the mass of the c-quark is much greater than that of u, d, s quarks, we shall determine for it separately the eigenvalues m_4' and m_4 of the operator $m_{q(B)}$ that correspond to the single quark c and the c-quark in the biquark, respectively, where $m_4' \neq m_1'$ and $m_4 \neq n_1$. The mass splitting operator in the quark model can then be written in the form

$$\sum_{k=1}^{4}\widehat{m}_{q^k(B)}[\widehat{Y}_{q^k}(1-\delta_{k4})+\delta_{k4}\widehat{C}_{q^k}]+m_0\widehat{I}, \quad q^{1,2,3,4}=u,\ d,\ s,\ c. \tag{3}$$

In numerical calculations, we use m_0 = 1152 MeV, $F_Y + D_Y$ = 84 MeV, $D_Y - F_Y$ = -44 MeV, $D_C + F_C$ = -1425 MeV, $D_C - F_C$ = 1227 MeV, and indicate in parentheses the mass values predicted by the multiparameter bag model [3]. The masses of the charmed isomultiplets are then given by

$$\begin{aligned} &M_{\Sigma_c((qq')c)}=m_0+\frac{2}{3}m_1+m_4'=2450\ (2430)\ \text{MeV},\\ &M_{\Omega_c((ss)c)}=m_0-\frac{4}{3}m_1+m_4'=2830\ (2730)\ \text{MeV},\\ &M_{\Xi_c((sq)c)}=m_0-\frac{1}{3}m_1+m_4'=2640\ (2550)\ \text{MeV},\\ &M_{\Xi_{cc}((cc)q)}=m_0+\frac{1}{3}m_1'+2m_4=3720\ (3680)\ \text{MeV},\\ &M_{\Omega_{cc}((cc)s)}=m_0-\frac{2}{3}m_1'+2m_4=3970\ (3870)\ \text{MeV},\\ &q,\ q'=u,\ d. \end{aligned} \tag{4}$$

These formulas correspond exactly to the mass formulas that follow from (2) when $m_1 = -F_Y$, $m_4 = F_C$, $m_1' = D_Y - F_Y$, $m_4' = D_C - F_C$. We then have $m_0=M_0+(2/3)D_Y-D_C$. This result shows the validity of the hypothesis that there is a relationship between the quark-biquark flavor structure of baryons and the tensor structure of the baryon current.

We draw attention to the formula for the masses of the Ξ_c particles. The correspondence with the SU(4) baryonic symmetry is achieved only when the biquark is taken in the form (sq), q = u, d. The form (cq) or (cs) is in conflict with (2).

The mass formulas for the charmed hyperons $\Xi_c'^{+}(=A^{+}(\tilde{c}su))$, $\Xi_c'^{0}(=A^{0}(\tilde{c}sd))$ and $\Lambda_c'^{+}(=C_0^{+}(\tilde{c}du))$ which have the same wave-function structure as the Λ-hyperon, are as follows:

$$M_{\Xi_c'} = m_0 - \frac{1}{9} m_1 - \frac{2}{9} m_1' + \frac{4}{3} m_4 - \frac{1}{3} m_4' = 2520\ (2490)\ \text{MeV},$$
$$M_{\Lambda_c'^{+}} = m_0 + \frac{2}{9} m_1 + \frac{4}{9} m_1' + \frac{4}{3} m_4 - \frac{1}{3} m_4' = 2290\ (2300)\ \text{MeV} \qquad (5)$$

and were obtained from the following quark-biquark structure of the wave functions of these hyperons:

$$\sqrt{2}\Lambda_c'(\tilde{c}du)_{\uparrow} = \sqrt{\frac{2}{3}}(\tilde{c}_{\uparrow}d_{\uparrow})u_{\downarrow} + \sqrt{\frac{1}{3}}(\tilde{c}_{\uparrow}u_{\downarrow})d_{\uparrow} -$$
$$-\sqrt{\frac{2}{3}}(\tilde{c}_{\uparrow}u_{\uparrow})d_{\downarrow} - \sqrt{\frac{1}{3}}(\tilde{c}_{\uparrow}d_{\downarrow})u_{\uparrow}.$$

and similarly for the Ξ_c' and Λ hyperons. The arrows ↑ and ↓ represent the quark or hyperon spin components +1/2 and -1/2 and the tilde labels the heavy quark (c-quark in the present case). The biquark is marked by the parentheses. The masses of only the following four charmed isomultiplets are known at present: Λ_c+ (2281.2 ± 3.0 MeV), Σ_c (2430 ± 5 MeV), Ω_c^0 (2740 ± 20 MeV), Ξ_c' (2460 ± 25 MeV) [8], which means that no final conclusion can be made as to the validity of the formulas given by (4) and (5).

Equations (2) and (3) can readily be generalized to an arbitrary number of flavors. In the SU(n) baryonic symmetry, the mass term in the hamiltonian for the $1/2^+$ baryon, described by the tensor $B^{\alpha}{}_{[\beta_1,\ldots,\beta_{n-2}]}$ that is antisymmetric in the lower indices and is traceless (20-plet in SU(4), 45-plet in SU(5), and 70-plet in SU(6)), will be written in the form

$$H_M^{SU(n)} = \frac{1}{(n-2)!}\bar{B}_{\alpha}{}^{[\beta_1\ldots\beta_{n-2}]}B^{\alpha}{}_{[\beta_1\ldots\beta_{n-2}]}\Big(M_0 - \sum_{k=4}^{n}(D_k - F_k)\Big) +$$
$$+\frac{1}{(n-3)!}\Big[\sum_{k=3}^{n}\Big[(D_k - F_k)\bar{B}_{\alpha}{}^{[k\beta_1\ldots\beta_{n-2}]}B^{\alpha}{}_{[k\beta_1\ldots\beta_{n-2}]} + \frac{1}{(n-2)}(D_k + F_k)\times \qquad (6)$$
$$\times\bar{B}_k{}^{[\beta_1\ldots\beta_{n-2}]}B^k{}_{[\beta_1\ldots\beta_{n-2}]}\Big],\quad D_{3,4,5,6\ldots} = D_{Y,C,B,T,\ldots},\ \text{the same for } F_k,$$

which corresponds in the quark model with n flavors to the mass splitting operator

$$m_0\hat{I} + \sum_{k=1}^{n}\hat{m}_{q^k(B)}\Big[\hat{Y}_{q^k}\Big(1 - \sum_{l=4}^{n}\delta_{kl}\Big) + \sum_{l=4}^{n}\delta_{kl}\hat{Y}_l\Big],\quad \hat{Y}_{q^{4,5,6\ldots}} = \hat{C}, \hat{B}, \hat{T}, \ldots,$$

and q^k = u, d, s, c, b, t, ... for k = 1, 2, 3, 4, 5, 6, ..., respectively. The eigenvalues m'_k, m_k, k = 1, ..., n of the operator $m_{q(B)}$ are related to the F- and D-coupling constants from (6) by $m_s = -F_Y$, $m_s' = D_Y - F_Y$, $m_k = -F_k$, $m'_k = D_k - F_k$, $s = 1, 2, 3$; $k \geq 4$. Formula (6) generalizes the eightfold-wave formula (1) [7] to SU(n) baryonic symmetry. The baryon mass splitting is specified by the series of F- and D-coupling constants that have a clear group-theoretic significance. They are uniquely related to the parameters m'_k, m_k that characterize the quark-biquark structure of baryons. It may be considered that the series of F- and D-coupling constants reflects the presence of a series of mass scales that are related to the freezing out of the quark flavors and is usually taken into account in QCD calculations of gluon corrections.

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2 February 1987 NIIYaF