QUASISYNCHRONOUS EXCITATION OF PLANE ACOUSTIC WAVES OF FINITE AMPLITUDE BY A MOVING δ-LOCALIZED SOURCE

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An analytic description is obtained for the generation of nonlinear sound by moving δ-localized sources.

The development of theoretical methods for the analysis of the generation of nonlinear sound by moving point sources was stimulated by the expanding range of possible physical applications. Nonlinear acoustic effects are fundamentally important in stimulated Mandelstam-Brillouin scattering [1], the excitation of sound by a scanning light beam [2,3], and for oscillations established in acoustic resonators [4]. The effective interaction with strain waves prevents the penetration of the acoustic barrier by the electron-hole drops [5]. Experiments [6-8] indicate that an electron-hole plasma shock and a melting front can propagate in the crystal with near-sonic velocity. Quasisynchronous excitation of nonlinear sound may also play an important part under these conditions. In such cases, the moving sources of acoustic waves are δ-localized in space, i.e., on the electron-hole plasma front or the melt-crystal separation boundary.

The generation of acoustic waves of finite amplitude by extended sources moving with near-sonic velocity can be described by [2]

$$D_t - \Delta D_\xi - \varepsilon c_0 D D_\xi = f_\xi, \quad D(\xi, t=0) = 0 \tag{1}$$

where $\xi = x - vt$, v is the velocity of the sources, c_0 is the velocity of sound, $\Delta = v - c_0$ is the velocity difference ($|\Delta| \ll c_0$), ε is the nonlinear acoustic parameter, $D = U_\xi$ is the deformation of the crystal, U is the displacement in the acoustic wave, and the function $f_\xi(\xi)$ describes the distribution of sources of sound in space. The inhomogeneous quasilinear equation (1) can be studied on the phase plane [3,4]. Analytic solutions have been obtained in an implicit form [9,10] for special cases of $f(\xi)$. In either case, the results obtained are difficult to use, e.g., to calculate the reaction of acoustic waves on the source [11]. It is thus necessary to find explicit acoustic-wave profiles satisfying (1), if only for certain model source distributions. We note that this type of solution was found in [1] for a periodic saw-tooth right hand side. In this paper, we obtain an explicit description in the form of the profiles of nonlinear strain waves excited by sources that are δ-localized in space.

Substituting $f = f_0\Theta(\xi)$ on the right hand side of (1), where $\Theta(\xi)$ is the unit function, we obtain

$$D_t - \Delta D_\xi - \varepsilon c_0 D D_\xi = (\mathrm{sign}\, f_0)\, |f_0|\, \delta(\xi) \tag{2}$$

where $\delta(\xi)$ is the Dirac-delta-function. When the generation of sound by electron-hole plasma is analyzed, we can take the deformation mechanism to be $|f_0| = |d|n_{fr}/2\rho_0 c_0$, where d is the constant of the deformation potential of the electron-hole pair, n_{fr} is the plasma concentration on the front, and ρ_0 is the equilibrium density of the crystal. When the excitation of sound on the melting front is examined, we take $|f_0| = |\Delta\rho| c_0/2\rho_0$, where $\Delta\rho$ is the change in the density of the medium on melting. The sign on the right hand side of (2) depends on whether the medium is compressed or expands when the electron-hole pairs or melting is photogenerated.

The solutions of (2) must satisfy the following symmetry conditions:

$$D(\varepsilon, f_0) = -D(-\varepsilon, -f_0),$$
$$D(\xi, \Delta, f_0) = -D(-\xi, -\Delta, -f_0).$$

It is therefore sufficient to examine (2), for example, for $\varepsilon > 0$, $f_0 > 0$. In terms of the dimensionless variables $D = D/D_0$ $(D_0 = (|f_0|/\varepsilon c_0)^{1/2})$, $\Delta = \Delta/\Delta_0$ $(\Delta_0 = \varepsilon c_0 D_0)$, $\xi = \xi/\xi_0$, $t = t\Delta_0/\xi_0$, where ξ_0 is the unit of length, we can transform (2) so that it takes the form

$$D_t - \Delta D_\xi - DD_\xi = \delta(\xi). \tag{3}$$

We note that, according to (3), the following conservation law is valid for nonstationary acoustic waves [3]:

$$\int_{-\infty}^{\infty} D d\xi = t. \tag{4}$$

Acoustic waves described by the linearized Eq. (3)

$$D(\varepsilon = 0) = -\frac{1}{\Delta}[\theta(\xi) - \theta(\xi + \Delta t)],$$

grow without limit as the source velocity approaches the velocity of sound:

$$D(\varepsilon = 0, \Delta = 0) = \delta(\xi)t.$$

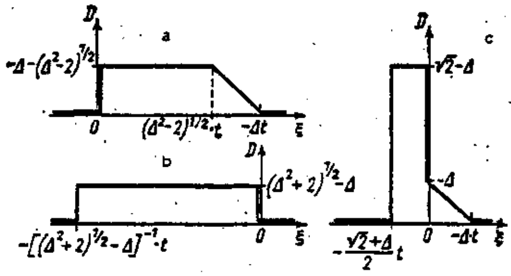
Consequently, for small departures from synchronism ($|\Delta| \leqslant 1$), the self-action of the excited acoustic waves must be correctly taken into account.

We have examined the nonlinear equation (3) on the phase plate. We succeeded in obtaining an explicit description of the profiles of the excited strain waves (see the figure).

In the subsonic sound geneation regime ($\Delta \leqslant -\sqrt{2}$, figure, a), all the acoustic disturbances run ahead of the sources, which are localized at the origin. Since in the case that we are considering $f_0 > 0$, finite-amplitude rarefaction waves are excited ($D > 0$), and propagate with subsonic velocities for $\varepsilon > 0$, we can see why the leading edge of the acoustic pulse spreads out in proportion to $-\Delta - \sqrt{\Delta^2 - 2})t$ (see figure, a).

In the subsonic sound generation regime ($\Delta \geq 0$, figure, b), all the excited acoustic waves lag behind the sources.

In the mixed regime ($-\sqrt{2} \leqslant \Delta < 0$, figure, c), some of the sound waves run ahead of the sources and some are amplified so much that their propagation velocity is less than the source velocity. We note that, in the last two regimes, the position of the trailing edge, which is formed by the nonlinear steepening of the acoustic pulse [3], was determined from the integral relation (4).



Typical shape of acoustic pulses excited by a moving δ-localized source [a) subsonic generation of sound: $\Delta \leq -\sqrt{2}$; b) ultrasonic regime: $\Delta \geq 0$; c) mixed regime: $-\sqrt{2} \leq \Delta \leq 0$].

The above analytic solutions are simple enough to suggest that they might be effectively applied to the investigation of the interaction between nonlinear sound and rapidly moving compressible plasma. There is alsoundoubted interest in the generalization of these solutions to quasi-one-dimensional problems in nonlinear acoustics [12].

REFERENCES

1. A. A. Karabutov, E. A. Lapshin, and O. V. Rudenko, Zh. Eksp. Teor. Fiz., vol. 71, no. 1(7), pp. 111-121, 1976.
2. A. A. Karabutov and O. V. Rudenko, Akust. Zh., vol. 25, pp. 536-542, 1979.
3. V. E. Gusev and A. A. Karabutov, Ibid., vol. 27, pp. 213-219, 1981.
4. V. E. Gusev, Ibid., vol. 30, pp. 204-212, 1984.
5. S. G. Tikhodeev, Usp. Fiz. Nauk, vol. 145, pp. 3-50, 1985.
6. A. Forchel et al., J. Lumin., vol. 30, pp. 67-81, 1985.
7. J. P. Wolfe, Ibid., vol. 30, pp. 82-113, 1985.
8. C. Hirlimann, J. Phys. (Paris), vol. 44, Conf. C5, pp. 99-105, 1983.
9. V. E. Gusev and O. V. Rudenko, Vestn. Mosk. Univ. Ser. 3 Fiz. Astron. [Moscow University Physics Bulletin], no. 4, pp. 117-119, 1978.
10. V. E. Gusev, Vestn. Mosk. Univ. Ser. 3 Fiz. Astron. [Moscow University Physics Bulletin], no. 6, pp. 7-12, 1981.
11. M. I. D'yakonov and A. V. Subashiev, Zh. Eksp. Teor. Fiz., vol. 75, no. 5(11), pp. 1943-1951, 1978.
12. S. A. Akhmanov, V. E. Gusev, A. A. Karabutov, and O. V. Rudenko, in: Proc. Fifth All-Union Conf. on Nonresonant Interactions of Optical Radiation with Matter [in Russian], pp. 371-372, Leningrad, 1981.

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