QUANTUM ELECTRODYNAMICS WITH AN ADDITIONAL COMPACT COORDINATE

D. A. Slavnov

Vestnik Moskovskogo Universiteta. Fizika,
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For the example of quantum electrodynamics, a procedure for the elimination of UV divergences conserving locality and relativistic and BRS invariance in all stages is formulated.

A Euclidean formulation of field theory is adopted, and one more measurement is added to four-dimensional space. The points of space are characterized by Euclidean vectors

$$X \equiv (x^1, x^2, x^3, x^4, x^5) \equiv (x, x^5) \equiv (x, \varkappa),$$

and the points of the conjugate momentum space by the vectors

$$P \equiv (p^1, p^2, p^3, p^4, p^5) \equiv (p, p^5) \equiv (p, \omega).$$

Below, the indices j, k take the values 1, 2, 3, 4, and the index ν the values 1, 2, 3, 4, 5.

Let $\phi(X)$ be the general notation for all the field appearing in the theory. It is required that $\phi(X)$ be a periodic function of κ which may be expanded in Fourier series:

$$\varphi(x, \varkappa) = \sum_{\omega} \exp\{-i\omega\varkappa\} \tilde{\varphi}(x, \omega), \quad \omega = \frac{\pi n}{l}, \quad n = 0, \pm 1, \ldots . \tag{1}$$

In addition, the field averaged over κ is introduced

$$\varphi_0(x) = \frac{1}{2l} \int_{-l}^{l} d\varkappa \varphi(x, \varkappa).$$

Then, so as to be specific, consider a specific model - spinor electrodynamics. The action is specified by the formula

$$W = \int dx\, \mathscr{L}_0(x) \equiv \int dx \frac{1}{2l} \int_{-l} d\varkappa \mathscr{L}(x, \varkappa),$$

where

$$\mathcal{L}(X)=\bar{\psi}(X)(\hat{\partial}+m+ieA_0^j(x)\gamma^j)\psi(X)+(1/4)F^{jk}F^{jk}(X)+$$
$$+(1/2)\partial^5 A^j\partial^5 A^j(X)+(\lambda/2)(\partial^j A^j(X))^2+\partial^\nu\bar{C}\partial^\nu C(X).$$

Here C(X) is the wind scalar fermion field; $\hat{\partial}=\gamma^\nu\partial^\nu$ (another variant is $\hat{\partial}=\gamma^j\partial^j+i\partial^5$). In $\mathcal{L}(X)$, the term describing the interaction $\mathcal{L}_{int}$, is nonlocal with respect to κ (local with respect to x).

The action W is invariant relative to BRS transformation [1], local with respect to x, and global with respect to κ:

$$\delta\psi(X)=ie\varepsilon C_0(x)\psi(X),\quad \delta A^j(X)=-\varepsilon\partial^j C_0(x),$$
$$\delta\bar{C}(X)=\lambda\varepsilon\partial^j A_0^j(x),\quad \delta C(X)=0.$$

Here ε is a constant scalar fermion.

It is simple to establish that the model here proposed transforms to the usual model as $l \to 0$. It is sufficient here to introduce the cutoff momentum Λ in the expansion in Eq. (1), imposing an additional condition on ϕ(X)

$$\tilde{\varphi}(x,\omega)=0 \text{ when } |\omega|>\Lambda.$$

Then when $l < \pi\Lambda^{-1}$, all the fields cease to depend on κ and $\mathcal{L}_0(x)$ transforms to the effective Lagrangian of ordinary electrodynamics.

On the basis of the action W, a formal expression Z(j) for the Green's quantum functions generating the functional may readily be obtained by standard methods, for example, using a continuum integral (the expression for λ = 1 is given):

$$Z(j)=N^{-1}[\exp\{\Delta\}\cdot\exp\{-I(j,\varphi)\}\exp\{-W_{int}(\varphi)\}]_{\varphi=0}, \qquad (2)$$

where

$$\Delta=\frac{1}{(2\pi)^4}\sum_\omega\int d^4p\left[-\frac{1}{2}\frac{\delta^{jk}}{P^2}\frac{\delta^2}{\delta A^j(P)\,\delta A^k(-P)}+\right.$$
$$\left.+\frac{\delta}{\delta\psi(P)}\frac{i\hat{P}+m}{P^2+m^2}\frac{\delta}{\delta\bar{\psi}(P)}\right],$$
$$W_{int}(\varphi)=e(2\pi)^4\sum_{\omega_1}\int d^4p_1\sum_{\omega_2}\delta_{\omega_1\omega_2}\int d^4p_2\sum_{\omega_3}\delta_{\omega_3 0}\int d^4p_3\delta(p_2-p_1-p_3)\times$$
$$\times\bar{\psi}(P_2)\gamma^j A^j(P_3)\psi(P_1),$$
$$I(j,\varphi)=(2\pi)^4\sum_\omega\int d^4p[\bar{j}(P)\psi(P)+\bar{\psi}(P)j(P)+A^j(P)j^j(P)].$$

Here, by definition, it is assumed that all the summation and integration in I and W_{int} occurs after the action of all Δ on their integrands and the operation ϕ = 0.

Equation (2) is formal, since UV divergence is present here. It may be eliminated if all the integrals $\int d^4p$, appearing in W_{int} are replaced by renormalized integrals $\int d\mu(P)$, determined for two types of integrand functions F(P), which are only encountered in calculating Z(j). The first type is: $F(P) \sim \delta(p + \ldots)$; then $\int d\mu(P)=\int d^4p$. The second type is: $F(P) = F_1(P) + F_2(P)$, where $(P^2)^2F_1(P)$ is a polynomial with respect to $p^{\nu}k_{\alpha}^{\nu}$ and $\ln(p^{\nu}k_{\alpha}^{\nu})$ (here K_{α} are constant five-vectors), and F_2 decreases more rapidly than $(P^2)^{-2}$, so that the integral

$$\int d^4pF_2(p,\ \omega) = \Phi(\omega)$$

converges; the function $\Phi(\omega)$ may be written in the form

$$\Phi(\omega) = \sum_{\tau>0} a_\tau \ln^\tau(\omega/\mu) + \tilde{\Phi}(\omega,\ \mu).$$

Here a_τ does not depend on ω, and $\tilde{\Phi}(\omega,\ \mu)$ is a continuous function of ω. The dedimensionalizing parameter μ is fixed arbitrarily (analog of the subtraction point). For this type

$$\int d\mu(P)F(P) \equiv \tilde{\Phi}(\omega;\ \mu).$$

In calculating $Z(j)$, multiple renormalized integrals must be employed. Generally speaking, the result depends on their order. To conserve BRS invariance, some symmetric combination must be used. One permissible combination is as follows: first, all integration (with subsequent summation) with respect to momentum corresponding to the field A_k (variables of type p_3, ω_3) is undertaken; all this integration is completely symmetrized. Then, the same is done for the fields ψ, and then for $\bar{\psi}$. All the integrals which may be taken using δ functions are calculated first here. The definition of the renormalized integral introduced here is similar to that proposed in [2,3], but not equivalent to it.

After introducing the renormalized integral, neither UV nor IR divergences are encountered in calculating $Z(j)$. In passing to the limit as $l \to 0$, the functional of renormalized Green's functions of ordinary four-dimensional electrodynamics is obtained.

It is noteworthy that, in this approach, the set of fields is the same as in four-dimensional theory. This is especially important when using such a procedure in supersymmetric models.

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Department of Quantum Theory
and High-Energy Physics