Moscow University Physics Bulletin
Vol. 46, No. 6, pp. 46–52, 1991

Vestnik Moskovskogo Universiteta. Fizika
UDC 539.172.3

THE USE OF THE REDUCTION METHOD TO INCREASE THE RESOLUTION OF PHOTONUCLEAR EXPERIMENTS

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The use of the reduction method without a priori information, invariant relative to nuclear models, helps improve the energy resolution of experimental data up to a value equal to one or two energy steps at which these data are measured. This energy resolution, which can be several times higher than the experimentally attainable one, corresponds to the full width at half maximum of a new apparatus function synthesized in the process of solving the problem and having a shape close to Gaussian.

1. INTRODUCTION

In the majority of nuclear spectrometric experiments based on the accepted model of interaction of radiations with matter, the directly measured quantity is a result of action of the linear integral operator on the cross section or the intensity of the process f of interest to the researcher. This experimental value, which is often called the experimental yield, hereinafter referred to as ξ, has a statistical scatter (or noise) ν. In other words,

$$\xi = Af + \nu, \tag{1}$$

where A is the linear integral operator.

Photonuclear experiments, which are typical representatives of the linear scheme of measurements (1), are used to obtain the energy dependence of the reaction cross section $f(k)$ on the photon energy k initiating the reaction, while the directly measured yield, or the vector ξ (ξ_j, $j = 1, \ldots, m$), is the product of the experimental photon spectrum $W(E_j, k)$ and the reaction cross section $f(k)$ integrated over the energy. The yield point obtained at the energy E_j is determined by the relation

$$\xi_j = \xi(E_j) = \alpha \int_D W(E_j, k) f(k)\, dk + \nu_j, \quad j = 1, \ldots, m. \tag{2}$$

The dependence of $W(E_j, k)$ on the gamma-quantum energy k is a bremsstrahlung spectrum in the case when a photon originates from deceleration of an electron in the target nucleus field [1]; in the case of a quasi-monoenergetic experiment (either with in-flight annihilation of accelerated positrons with target electrons or with tracer bremsstrahlung photons) this dependence has a shape of an asymmetric peak [2-5]. In the course of the experiment the energy E_j varies. In the case of a bremsstrahlung experiment, E_j is the upper limit of the bremsstrahlung spectrum, and in a quasi-monoenergetic experiment E_j is the energy of the photon spectrum maximum. D is the photon energy region above the reaction threshold, where the spectrum $W(E_j, k)$ is non-zero; α is the normalization factor; ν_j is the noise realization corresponding to the point ξ_j. Therefore, the operator A describes the integration over the energy of the product of the photon spectrum $W(E_j, k)$ and the cross section $f(k)$ and, in particular, includes a preset energy interval E_j, $j = 1, \ldots, m$ over which the experimental yield is measured. In the case of a large number of counts, the statistical scatter ν_j has a normal distribution and its covariation matrix Σ has only a main diagonal whose elements are equal to the dispersions of the yield points, since each yield point ξ_j is measured independently of another.

Various approaches to the problem of getting information on the energy dependence of the cross section $f(k)$ are known. The first, instrumental approach, is associated with the use of experimental equipment having a monoenergetic apparatus function (photon spectrum). This analysis is based on the fact that, the

closer $W(E_j, k)$ to the delta function $\delta(k-E_j)$, the stronger are the reasons for interpreting the experimental yield $\xi(E_j)$ as a cross section $f(E_j)$. When the apparatus function has a shape of a sufficiently narrow peak, e. g., is a Gaussian one, this function is usually called a line. There are restrictions that do not allow the line of the experimental photon spectrum to be made arbitrarily narrow. For example, the best resolution of the quasi-monoenergetic experiments obtained lies in the range of 80-100 keV, while the resolution providing an acceptable counting speed is 200-400 keV. In the case of the bremsstrahlung experiment, the photon spectrum has nothing in common with a line, and the experimental yield $\xi(E_j)$ of the bremsstrahlung experiment cannot be interpreted as the cross section $f(E_j)$. However, at the time when first photonuclear yields were obtained in the field of the giant dipole resonance, the researchers realized that, in constructing a certain linear combination of the yield points (e. g., $\hat{f}(E_i) = a\xi(E_j) + b\xi(E_{j+1})$), the corresponding linear combination of the photon spectra $\widehat{W}(E_i, k) = aW(E_j, k) + bW(E_{j+1}, k)$ can have a shape resembling a line. Then the value of $\hat{f}(E_i)$ should be interpreted as the point of a new experimental yield obtained with a new photon spectrum $\widehat{W}(E_i, k)$, where E_i is the energy of the maximum of this spectrum. This was, briefly, the photon difference method used in the 1950s and its more up-to-date three-point modification [6].

Another approach is based on the assumption that the cross section is a superposition of some well-known functions, e. g., Gaussian or Lorentzian functions, dependent on the free parameters. These parameters are found by least squares minimization of the difference between the vectors Af and ξ. This method is justified only if one has sufficient a priori information on the shape of the cross section $f(k)$ (at least the number of resonances in the cross section is known, as in the case of a discrete spectrum) and only has to introduce refinements to some parameters of the cross section.

At the same time, in some practical cases it happens that a priori information on the cross section is inadequate, and there is no sound physical grounds for a description of the cross section shape by some or other set of known functions. This is a typical picture of the current state of the study of photoneutron cross sections in the region of the giant dipole resonance, where the cross section can consist of both a continuous spectrum and a large number of resonances whose shape is not known beforehand and it is possible, in particular, that it cannot be described by a Gaussian or a Lorentzian function. In this case it is reasonable to make as few assumptions as possible about the cross section, even if these assumptions do not contradict the experimental data. Of course, such a priori information as a non-negative value of the cross section and it being equal to zero below the reaction threshold is not questionable and should be used, while the assumption on the cross section smoothness not following from either theory or experiment may lead the researcher astray (because data obtained with poor resolution cannot be evidence of the absence of resonances with a width less than that of the apparatus function).

The method presented in this paper is invariant relative to nuclear models and assumptions, such as cross section smoothness or the presence or absence of peculiarities in the cross sections. This method makes no use of such information.

2. THE METHOD OF MEASUREMENT REDUCTION IN THE [A, Σ] MODEL OF THE PHOTONUCLEAR EXPERIMENT

In the reduction method [7] the experimental yield (1), (2) is transformed into the vector

$$\hat{\mathbf{f}} = U\mathbf{f} + \hat{\nu} \qquad (\hat{f}_i = \hat{f}(E_i), \quad i = 1, \ldots, n), \tag{3}$$

which should be interpreted as a result of the experiment in which the apparatus function (or the photon spectrum) $\widehat{W}(E_i, k)$ is described by the operator U and its properties are set by the researcher. For example, as mentioned above, of interest might be a result obtained with an apparatus function as monochromatic as possible. If the experimental apparatus function described by the operator A has a form of a line, but this line is not narrow enough, the problem is stated as one aimed at increasing the energy resolution of the experimental result, for which U describes a narrower line.

The initial data are the experimental yield ξ, the covariant matrix Σ which characterizes the error ν of the yield ξ, and the experimental operator A describing the photon spectrum (or the apparatus function $W(E_j, k)$. The data processing must give the vector $\hat{\mathbf{f}}$ (3) and the covariant matrix G describing the error $\hat{\nu}$ of the vector $\hat{\mathbf{f}}$. To solve the problem it is necessary to find such a linear operator R that the vector $\hat{\mathbf{f}} = R\xi$

could be interpreted as the best mean-square evaluation of $U\mathbf{f}$. To avoid the use of a priori information, one should introduce the condition of problem solvability

$$RA = U, \tag{4}$$

because only in this case the reduction vector

$$\hat{\mathbf{f}} = R\xi = U\mathbf{f} + (RA - U)\mathbf{f} + R\nu \tag{5}$$

is independent of the unknown $\mathbf{f}$, and the error can be estimated through the covariant matrix Σ of the noise of the experimental yield and the operators A and U. Under this assumption, the value of the error $M\|R\nu\|^2$ in Eq. (5) is minimum, if

$$R = U(\Sigma^{-1/2}A)^{-}\Sigma^{-1/2},$$

where the dash stands for the operator pseudo-inversion [7], $\|\cdot\|$ is the operator norm, M is the mathematical expectation, and the covariant matrix characterizing the error $\hat{\nu} = R\nu$ of the vector $\hat{\mathbf{f}}$ is equal to

$$G = R\Sigma R^{*}.$$

It is clear that not any of the preset operators U will meet the solvability condition (4), just as not any operator U enabling the problem to be solved will describe an apparatus function whose shape would satisfy the experimenter. Therefore, the next step will be synthesis of such an operator U which, first, would meet condition (4) of the solution existence in a problem without a priori information and, second, would correspond as much as possible to the experimenter's ideas of a monochromatic instrument. One of the most widespread line shapes generally considered monochromatic is the Gaussian shape. Let us assume that an operator U_0 is used to describe the desired apparatus function having a Gaussian shape, and find an operator U, close to U_0, in the form of a product KA of an arbitrary operator K and an operator A (to satisfy condition (4)) by minimizing by K the difference of U from U_0 in the sense that

$$\|U - U_0\|_2^2 = \mathrm{tr}((U - U_0)(U - U_0)^{*}).$$

In this case, the operator U is found from the expression

$$U = U_0A - A$$

and is determined solely by the operator U_0 and the experimental operator A. The difference of U from U_0 is characterized by the operator mismatch $\|U - U_0\|_2$.

3. SELECTION OF OPTIMUM ENERGY RESOLUTION

Some methods of improving the resolution of experimental yields are given in [8-10]. A reduction problem with restriction of the noise level $M\|R\nu\|^2$ was tackled there. Operators that had various preset properties were synthesized: (a) an operator closest to the Lorentzian, (b) an operator of an arbitrary symmetric shape in the region from $E_i - \Delta/2$ to $E_i + \Delta/2$ and equal to zero outside this region, (c) an operator close in shape to the delta function and localized as in the case (b). In all these papers the operator A depended only on the difference of the arguments $(k - E_j)$, so that the vector $A\mathbf{f}$ was a convolution. The question of whether a still better energy resolution could be obtained with the help of the procedures described was not discussed in the above papers. In the present paper we determine the maximum possible resolution obtainable by means of the above-described procedure (this, of course, does not mean that a still better resolution cannot be obtained by some other method). The energy resolution is being increased as long as it can be referred to as corresponding to the full width at half maximum of the apparatus function. Instead of restricting the noise level $M\|R\nu\|^2$, we tried to minimize this noise in order to study the effect on the result of the operator mismatch alone, without considering the effect of the estimation error due to the noise level restriction.

In the model calculations the experimental apparatus function $W(E_j, k)$ corresponded to the quasi-monoenergetic experiment [4], so that the vector Af was not a convolution as is the case in many spectroscopic experiments. The "true" cross section f was defined by two Gaussian curves with a width of 10 keV separated by 200 keV. The yield ξ (1), (2) was calculated with an energy interval of 100 keV. The statistical scatter ν was modeled to be 100 times smaller compared to the actual scatter in such experiments, in order to eliminate the effect of the noise ν on the reduction vector $\hat{\mathbf{f}}$.

In order to find a minimal width of the apparatus function line at which the synthesized apparatus function has a satisfactory shape, let us consider the dependence of the operator mismatch $\|U - U_0\|_2$ divided by the norm of the operator U_0 on the width of the apparatus function line δ_0, described by the operator U_0:

$$g(\delta_0) = \|U - U_0\|_2 / \|U_0\|_2. \tag{6}$$

Dependence (6) is shown in Fig. 1 by a continuous line. One can see that for the δ_0 values exceeding the measurement step by a factor of 1.1-2 the slope of the curve is much smaller than for low δ_0. This implies that the procedure used to make U and U_0 closer to each other is the most efficient in the region of low δ_0. For higher δ_0 there is little sense in sacrificing the energy resolution for the sake of converging the desired and the synthesized apparatus functions, and, in principle, a decrease of the slope of the relative mismatch $g(\delta_0)$ can determine the optimum synthesis region.

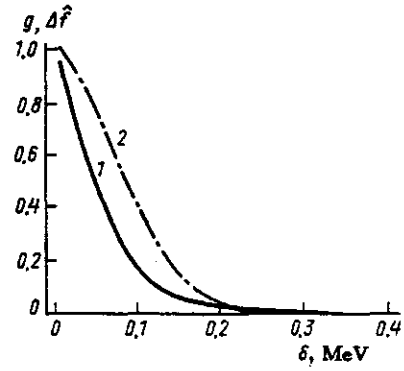

Fig. 1

Operation characteristic $g(\delta_0)$ (1) and standard deviation of the reduction vector $\Delta\hat{\mathbf{f}}$ (2).

However, it should be noted that researchers, while paying special attention to the full width at half maximum for the line, admit of various forms of this line: rectangular, triangular, or Gaussian. It is also desirable that the apparatus function differ as little as possible from a narrow peak. In spite of the fact that the results obtained by means of a Gaussian or a rectangular apparatus function differ, they have a common property which characterizes the resolution: if two resonances in the cross section $f(k)$ are spaced at an interval exceeding the width of the Gaussian or the rectangular function, they will be undistinguishable in such an experiment. Therefore, the quality of the synthesis may also be considered satisfactory when the mismatch between U and U_0 does not exceed the difference norm of a Gaussian and a rectangular function of the same width ($\|U_g - U_r\|_2 / \|U_g\|_2 = 0.37$). This value is located precisely in the region of decrease of the curve $g(\delta_0)$ slope in Fig. 1.

In the region of low δ_0 (smaller than the step ΔE_j of measurement of the experimental yield) the difference between the synthesized U and the desired U_0 is too large, and the synthesized apparatus function has a complex shape making it difficult to correctly interpret the result (Fig. 2, curve 1, $\delta_0 = 20$ keV). For such a shape of the apparatus function there is no generally accepted definition of the energy resolution. The use of such an apparatus function can lead to the appearance of a "false structure" (Fig. 3, curve 1). In the region where the curve $g(\delta_0)$ changes its slope the synthesized apparatus function $\widehat{W}(E_i, k)$ described

by the operator U, assumes a shape close to the Gaussian one (Fig. 2, curve 2, $\delta_0 = 150$ keV), and its energy resolution is characterized by the full width at half maximum. Figure 2 also shows an experimental apparatus function (curve 3) described by the operator A. One can see that the new (synthesized) apparatus function $\widehat{W}(E_i, k)$ has a much smaller width (140 keV) than the experimental one (330 keV); furthermore, it has no asymmetry, which is a feature of the experimental quasi-monoenergetic photon spectrum.

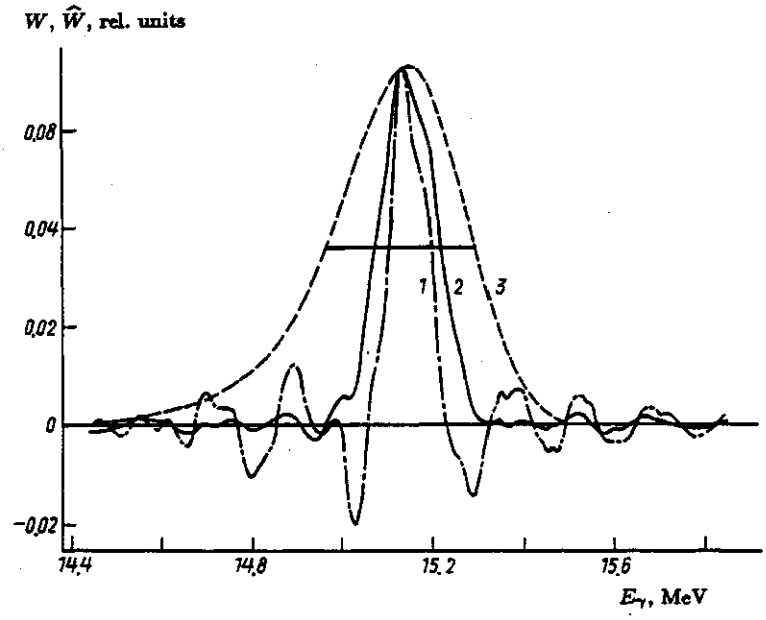

Fig. 2

Synthesized apparatus function corresponding to the operator U at $\delta_0 = 20$ keV (1) and 150 keV (2); experimental apparatus function corresponding to the operator A (3). The graphs are normalized so as to coincide in maximum.

Of course, in each particular experiment it is for the researcher to decide whether the synthesized apparatus function is convenient for him. However, in the case of a quasi-monoenergetic experiment we may recommend a width of the operator U_0 1.5 times the energy step of the yield measurement, which ensures a satisfactory synthesis of the apparatus function.

The experimental conditions may differ for different regions of the energies E_j. For example, the energy step used to measure the experimental yield can be varied. Moreover, the line width at which the desired operator U_0 is in agreement with the synthesized U can be different for the two adjacent energies E_i at which the cross section is sought. Therefore, the energy resolution enabling a qualitative synthesis of the operator U can vary from point to point. Such a situation is very common in practice with graphic data presentation, when the researcher presents the resultant points which have the highest possible energy resolution for this particular experiment. For example, such is the case in the Penfold-Leiss method with a variable analysis step [11]; the yields themselves of quasi-monoenergetic experiments, which are often interpreted as the cross sections of reactions, have a variable energy resolution. It should be remembered, however, that a too rapid change of the energy resolution $\delta_{i+1} - \delta_i$ (by a value exceeding the difference between the adjacent energies $E_{i+1} - E_i$) can give rise to a "false structure" in $\widehat{\mathbf{f}}$.

Figure 3 compares the reduction vectors $\widehat{\mathbf{f}}$ obtained using the apparatus functions closest (curve 1) to the very narrow function ($\delta_0 = 20$ keV) and (curve 2) to the Gaussian apparatus function upon varying the slope of the curve $g(\delta_0)$ ($\delta_0 = 140$ keV). The "false structure" present in curve 1 is strongly suppressed in curve 2, and there is good agreement between curve 2 and the vector $U_0\mathbf{f}$ (curve 3). The two peaks on Fig. 3, present in the model cross section and undistinguishable in the yield $\boldsymbol{\xi}$ (points), are clearly resolved in the reduction vector $\widehat{\mathbf{f}}$ (curve 2). This indicates that the application of the reduction method indeed enables one

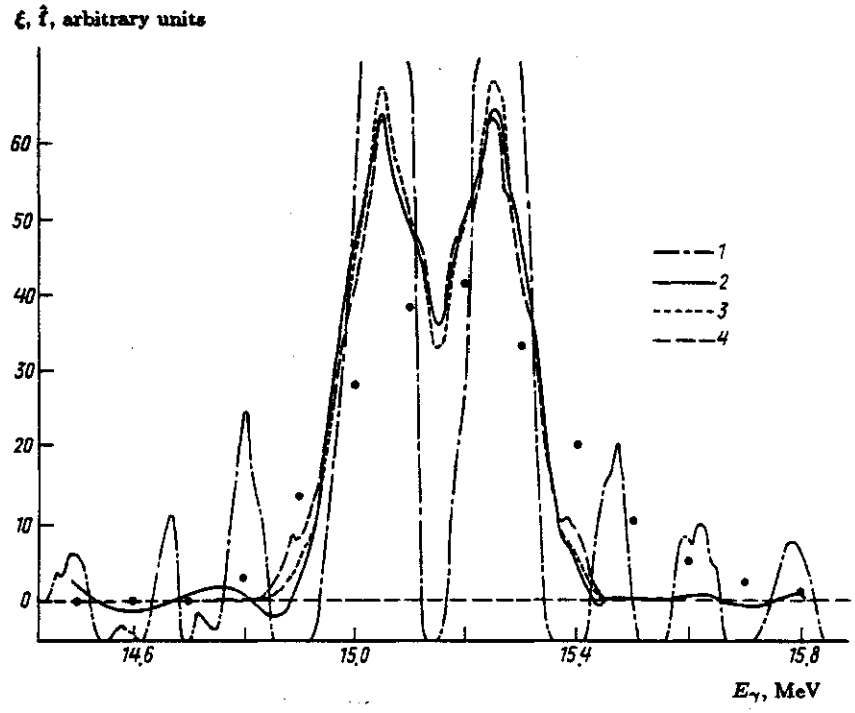

Fig. 3

Experimental yield $\xi = Af + \nu$ (dots); reduction vector $\hat{\mathbf{f}}$ at $\delta_0 = 20$ keV (1); reduction vector when the slope of the relative mismatch curve varies (2); vector $U_0\mathbf{f}$ when the slope of the relative mismatch curve varies (3); vector $\Gamma\mathbf{f}$, where Γ has Gaussian shape with the same full width at half maximum as U (4).

to increase the energy resolution of the data. Here the energy resolution of the reduction result is determined (in terms generally accepted in the experimental physics) as the full width at half maximum of a Gaussian-like apparatus function. For every point of the reduction vector the apparatus function $\widehat{W}(E_i, k)$ is calculated in the process of solving the problem and is a characteristic of the instrument. Of considerable importance is the fact that the energy resolution for which a satisfactory shape of the apparatus function is synthesized, is determined by the value of the energy step ΔE_j with which the experimental yield is measured. The width of the synthesized apparatus function lies in the range from ΔE_j to $2\Delta E_j$ and is independent of the width of the apparatus function of the quasi-monoenergetic experiment in a wide range of variations of this function (from ΔE_j to $10\Delta E_j$). During the treatment this makes it possible to obtain, upon processing of the data, an energy resolution several times better than the one attainable experimentally.

However, the reduction of the data to the maximum possible resolution is not always justified. A large value of the statistical scatter of ν in the yield ξ can give rise to considerable errors of $\hat{\nu}$ in the reduction vector $\hat{\mathbf{f}}$. Therefore one should look for a compromise choice between the energy resolution and the statistical scatter $\hat{\nu}$ of the vector $\hat{\mathbf{f}}$. One can fix the required value of the error ε and then start lowering the energy resolution of the result till the reduction error becomes lower than ε. This procedure encounters no difficulties if one bears in mind that as the width of the synthesized apparatus function increases, the relative mismatch (6) decreases and so does the difference between the synthesized apparatus function and the Gaussian function. A typical dependence of the reduction error $\Delta\hat{f}$ on δ_0 (line width of the operator U_0) is shown in Fig. 1 by curve 2. This dependence is normalized to the maximum value, because it differs only by a constant factor for various standard deviations of the yield points. For a standard deviation $\Delta\xi = 0.02$, the maximum value of the standard deviation $\Delta\hat{f}$ is 1.

4. CONCLUSIONS

The present paper has studied the possibilities of employing the reduction method in the [A, Σ] model (without a priori information) to increase the energy resolution of experimental nuclear spectrometric data. A photoneutron quasi-monoenergetic experiment has been used as an example. It has been shown that, knowing the apparatus function with which the experimental data have been obtained, one can enhance the energy resolution to a value corresponding to one or two energy steps used to measure the experimental yield. Then the reduction results are interpreted in terms generally accepted in the spectrometry as a result of an experiment yielding a higher resolution. Its apparatus function is synthesized in the process of solving the problem, has a shape close to the Gaussian one and is known for each point of the reduction result. The Gaussian-like shape of this apparatus function makes it possible to unambiguously determine the energy resolution of the reduction result by the full width at half maximum of the synthesized apparatus function.

The method also enables one to eliminate asymmetry typical of experimental apparatus functions.

The method is totally independent of theoretical models which describe the subject of the study, i.e., the cross section of this process.

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