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BRIEF COMMUNICATIONS

A PROBLEM OF COOLING CONTROL IN HARDENING OF STEEL

O. V. Skidanova, N. I. Kulik, V. B. Glasko, Yu. K. Evseev, and S. A. Yurasov

The paper is devoted to the statement and computer solution of the inverse problem on the determination of the parameter controlling the regime of specimen cooling during hardening by providing the required cooling rate at a given depth. The investigation results can serve as a basis for elaborating automatic control systems for the technological process of thermal treatment of machine elements.

1. Irrespective of the way the hardened specimens are heated, their cooling is usually achieved [1] by a liquid coolant flowing over the specimen surface. The rate of specimen cooling decreases quite rapidly with increasing depth. To obtain the martensite structure in the near-surface layer of a given thickness it is necessary to provide a sufficiently high cooling rate at the lower layer boundary, and the problem consists in finding the values of the parameters controlling the cooling process which allow this purpose to be attained.

In the present paper the control problem is considered within a mathematical model that was earlier identified with the real process [1]. The model is rather simple, so the calculation of the control parameter value at a given depth inside the near-surface martensite layer in a steel specimen admits of complete computer-aided automation.

The possible solution instability in this problem as an inverse problem, which is the main difficulty in the solution of this class of problems and requires the application of some regularizing operators [2], is overcome by using a priori information about the range of the single unknown parameter.

In addition, we used the following initial data to provide an automatic solution of the problem. First, the dependence of thermophysical parameters of the material, such as the thermal conductivity λ, the heat capacity c, and the density ρ on the temperature T [2] (these quantities are set by tables and are supplemented with a linear interpolation subprogram). Second, the microstructure diagram [3], i.e., the dependence of the critical cooling rate v_{cr} (which assures the martensite formation upon hardening) on the carbon content in the given sort of steel, which may be alloyed (from the tabular representation of this curve only a single value is taken that corresponds to the given percentage of carbon).

Because the processes under study refer to a near-surface layer that is thin compared to the specimen size, the problem can be solved with no regard for the actual shape of the specimen and assuming it to be an infinite cylinder of radius R.

2. Within the framework of the adopted geometrical model the heat-conduction-cooling process is described by the following boundary value problem:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda(T)\frac{\partial T}{\partial r}\right) = c(T)\rho(T)\frac{\partial T}{\partial t},$$
$$T|_{t=0} = T_0, \qquad \left.\frac{\partial T}{\partial r}\right|_{r=0} = 0, \tag{1}$$
$$-\lambda(T)\left.\frac{\partial T}{\partial r}\right|_{r=R} = H(T-\tilde{T})|_{r=R}.$$

Here the condition at the surface $r = R$ corresponds to convective heat exchange during cooling by a liquid coolant of temperature $\tilde{T}$ flowing over the specimen surface, which is in agreement with the real process [4]. In the case under consideration (heating in a furnace), the initial condition at $t = 0$ corresponds to the through heating of the specimen to temperature T_0, but it can be replaced by a more general condition.

As a single parameter controlling the surface cooling, it is reasonable to take the heat exchange coefficient H. As a characteristic of the temperature field $T(r, t)$ to be compared with v_{cr} at a given depth h, we take the average cooling rate in the given temperature interval:

$$v_{av} = v_{av}(h) = (T_1 - T_2)/\Delta t.$$

Here the quantities T_s $(s = 1, 2)$ bound the critical interval that includes a minimum on the continuous cooling transformation diagram [3] $T = T(t)$, and we have $\Delta t = |t_1 - t_2|$, where t_s is determined by the algorithmic solution, relative to t, of the equations $T(R - h, t) = T_s$ whose left-hand sides are determined for each h by the solution procedure for problem (1) supplemented with a linear interpolation subprogram.

It is clear that for a fixed h the quantity v_{av} is a function of the control parameter H, and the latter can be found from the algorithmic representation of the equation

$$F(h, H) = v_{av}(h, H) - v_{cr} = 0 \tag{2}$$

as a function of h: $H = H(h)$, which is the central element of the program meant for solving the control problem.

3. Since the limits of variation of H can be set beforehand and $H(h)$ is a monotonically increasing function, the problem in question turns out to be properly posed. Equation (2) is solved automatically by the "fork" method [5], which under the indicated conditions satisfies the regularization principle [6].

The necessary information about the temperature field is obtained by means of a finite-difference two-layer implicit iteration-free scheme on a four-point pattern [5] applied for solving the heat-conduction problem (1). If y is a network function corresponding to the continuous temperature T, m is the index of the time layer in which its values have already been determined, and n is the index of the node with respect to r, then the values of $\hat{y}$ in the current $(m+1)$th layer (with step τ) at the internal nodes r_n $(n = 1, \ldots, N-1)$ are determined by the following formulas:

$$\begin{aligned}
&\frac{1}{\Delta r^2}\left[\varkappa_{n+\frac{1}{2}}(\hat{y}_{n+1} - \hat{y}_n) - \varkappa_{n-\frac{1}{2}}(\hat{y}_n - \hat{y}_{n-1})\right] + \frac{1}{r_n}\lambda(y_n)\frac{\hat{y}_{n+1} - \hat{y}_n}{2\Delta r} \\
&= \tilde{c}_n \frac{1}{\tau}(\hat{y}_n - y_n); \\
&\varkappa_{n+\frac{1}{2}} = \frac{1}{2}[\lambda(y_n) + \lambda(y_{n+1})], \quad \varkappa_{n-\frac{1}{2}} = \frac{1}{2}[\lambda(y_n) + \lambda(y_{n-1})], \\
&\tilde{c}_n = c(y_n)\rho(y_n) = \tilde{c}(y_n) \quad (r_n = n\Delta r,\ \Delta r = R/N).
\end{aligned} \tag{3}$$

To ensure an accuracy of the second order with respect to Δr the boundary conditions in problem (1) are approximated in the following way.

For $r = 0$ we have $\hat{y}_0' = \frac{1}{2\Delta r}(-3\hat{y}_0 + 4\hat{y}_1 - \hat{y}_2)$, and, by virtue of the boundary condition, we obtain $\hat{y}_2 = 4\hat{y}_1 - 3\hat{y}_0$. Substituting this expression into (3) for $n = 1$ we arrive at the condition

$$\begin{aligned}
&\hat{y}_0 = \varkappa_1 \hat{y}_1 + \mu_1, \qquad \varkappa_1 = \left[3\varkappa_{1/2} - \varkappa_{1/2} - \tilde{c}(y_1)\frac{\Delta r^2}{\tau}\right] \Big/ (3\varkappa_{3/2} - \varkappa_{1/2}), \\
&\mu_1 = \left[\tilde{c}(y_1)\frac{\Delta r^2}{\tau} y_1\right] \Big/ (3\varkappa_{3/2} - \varkappa_{1/2}).
\end{aligned}$$

Similarly, setting $\hat{y}_N' = (1/2\Delta r)(\hat{y}_{N-2} - 4\hat{y}_{N-1} + 3\hat{y}_N)$ for $r = R$ and using the boundary condition we exclude $\hat{y}_{N-2}$; then the substitution of this quantity into (3) for $n = N - 1$ results in the expression

$$\begin{aligned}
\hat{y}_N &= \varkappa_2 \hat{y}_{N-1} + \mu_2, \quad \text{where } \varkappa_2 = [\tilde{c}(y_{N-1})(\Delta r^2/\tau) + \varkappa_{N-1/2} - 3\varkappa_{N-3/2}]/[\varkappa_{N-1/2} \\
&\quad - \varkappa_{N-3/2}(H \cdot 2\Delta r/\lambda(y_N) + 3)], \\
\mu_2 &= [-2H\overline{T}\Delta r \varkappa_{N-3/2}/\lambda(y_N) - \tilde{c}(y_{N-1})y_{N-1}\Delta r^2/\tau]/[\varkappa_{N-1/2} \\
&\quad - \varkappa_{N-3/2}(2H\Delta r/\lambda(y_N) + 3)].
\end{aligned}$$

The adopted finite-difference scheme with approximation order $O(\Delta r^2+\tau)$ is solved by the factorization method [5].

The parameters of the finite-difference scheme $\Delta r = 0.01R$ mm and $\tau = 0.1$ s chosen as a result of a methodical experiment in the required interval of values of H ($H \in [5\,000,\ 16\,000]$ kcal m^{-2} h^{-1} deg^{-1}) provide, according to the Runge estimate [5], an approximation accuracy for the temperature field (in the operation region) about 2-3%. This gives a 4-5% error in the determination of v_{av}.

The program admits of a linear version of the problem, in which the values of the parameters λ, c, and ρ are "frozen" at the initial level T_0, which corresponds to very fast cooling [1].

4. The elaborated program can give the required values of the control parameter when being included in the software for a small computer, and this will help improve the quality of the products.

At the same time, the results of the mathematical experiment presented below give an indication of the dependence of the desired quantity on the geometrical parameters of the specimen. The calculations were carried out for carbon steel (Steel-20) at $T_1 = 705°C$, $T_2 = 500°C$, $v_{\text{cr}} = 200°C/s$, $T_0 = 840°C$, and $\overline{T} = 20°C$.

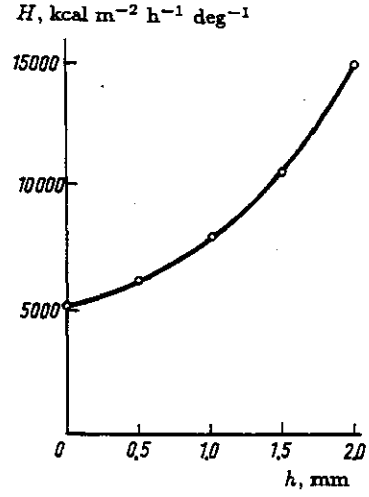

Fig. 1

Figure 1 demonstrates the dependence of H on the hardening depth, which reflects the natural increase in the surface cooling rate with increasing h. In all cases the deviation of v_{av} from $v_{\text{cr}} = 200$ deg/s at the corresponding depth did not exceed 1 deg/s.

The estimate of the $v_{\text{av}}(h_0)$ value, obtained using the data of a physical experiment at $h_0 = 2$ mm for the indicated sort of steel, $R = 24$ mm and the value $H = 15\,000$ kcal m^{-2} h^{-1} deg^{-1} calculated by the elaborated program ($v_{\text{av}}(0) = 1500$ deg/s) is about 215 deg/s. For the total error of the experiment and processing of the experimental temperature curves of ± 25 deg/s ($\sim 10\%$), the agreement with the theoretical value of $v_{\text{av}}(h_0) = 200$ deg/s can be considered satisfactory.

In the range of component dimensions we are interested in, the dependence of H on R was found to be weak. For example, at the surface ($h = 0$) we have $H = 4968$, 5001, and 5034 kcal m^{-2} h^{-1} deg^{-1} for $R = 40$, 50, and 70 mm, respectively.

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Department of Mathematics