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THE THEORY OF TRANSVERSELY INHOMOGENEOUS HEAVY-CURRENT BEAM-PLASMA AMPLIFIER UNDER CONDITIONS OF COLLECTIVE CERENKOV EFFECT

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The paper is concerned with both linear and nonlinear theories of amplifying electromagnetic plasma waves in a transversely inhomogeneous beam-plasma system of a general form under conditions of the collective Cerenkov effect. The optimum length and efficiency of amplification, as well as the output electromagnetic radiation power have been calculated analytically and numerically.

In recent years, the advent of powerful plasma SHF amplifiers and electromagnetic radiation oscillators has aroused interest of researchers in the theory of transversely inhomogeneous beam-plasma systems[1–3].

Plasma electromagnetic waves are known to undergo amplification under conditions of the collective Cerenkov effect, when the beam and the plasma are well separated across a waveguide [4]. This is the situation that is considered below. A round metal waveguide with thin tubular beam and plasma inside is analyzed in detail. The nonlinear stabilization mechanism is shown to be determined by factors depending on the beam current. A general analytical solution to the problem is given (for arbitrary currents), and the most important amplifier characteristics are determined.

For generality, first consider a waveguide of an arbitrary cross section, within which there are thin electron beam and plasma fully magnetized by a longitudinal magnetic field. The corresponding unperturbed densities have the form:

$$n_{0b} = S_b\delta(\mathbf{r}_\perp - \mathbf{r}_b)n_b, \qquad n_{0p} = S_p\delta(\mathbf{r}_\perp - \mathbf{r}_b)n_p. \tag{1}$$

Here $\mathbf{r}_\perp$ is the coordinate across the waveguide, $\mathbf{r}_b$ and $\mathbf{r}_p$ determine the positions of the beam and the plasma, respectively, in the waveguide, and S_b and S_p are the beam and plasma cross section areas.

In the boundary value problem of amplification of input oscillations ($z = 0$), the nonlinear interaction of a thin beam and plasma under conditions of the collective Cerenkov effect is described by the following set of integro-differential equations [5]:

$$\begin{aligned}
&\frac{dy}{d\xi} = \eta,\\
&\frac{d\eta}{d\xi} = (1+\mu\eta)^{3/2}\Big\{\frac{i}{2}\Big[\exp\{-iy\}\Big(1 - i\mu\frac{d}{d\xi}\Big)\rho - \text{c. c.}\Big]\\
&\qquad\qquad + \frac{1}{2}\nu(\varepsilon\exp\{-iy - i\eta_0\xi\} + \text{c. c.})\Big\},\\
&\frac{d\varepsilon}{d\xi} = \nu\rho\exp\{i\eta_0\xi\},\\
&\rho = \frac{1}{\pi}\int_0^{2\pi}\exp\{iy\}\,dy_0,
\end{aligned} \tag{2}$$

where ρ is the amplitude of beam charge density perturbations, ε is the dimensionless amplitude determining the transverse component of the electric field of the plasma wave undergoing amplification, y and η are the Lagrangian coordinates of electrons in the beam,

$$\xi = \frac{\omega}{u}\frac{\mu}{2\gamma^2}z, \tag{3}$$

z is the coordinate along the waveguide axis, u is the velocity of an unperturbed beam, and $\gamma = (1 - u^2/c^2)^{-1/2}$.

Set (2) has been obtained and studied in [6] in handling the initial value problem* for model systems. It depends on three parameters, viz., (a) the detuning η_0, which characterizes the deviation of the phase velocity of an unperturbed plasma wave from the beam velocity u, (b) the heavy-current parameter μ, and (c) the quantity ν determining the beam-plasma interaction conditions. For the transversely inhomogeneous beam-plasma amplifier we are considering, these parameters have the form:

$$\eta_0 = \frac{1}{\mu}\left(1 - \frac{1}{\alpha_p}\right), \qquad \mu = (4\gamma^4\alpha_b)^{1/2}, \qquad \nu = \widetilde{\alpha}^{1/2}\mu^{-1/2}(1 - \mu\eta_0), \tag{4}$$

where for a waveguide of an arbitrary cross section

$$\begin{gathered} \alpha_b = \frac{\omega_b^2 R^2}{u^2\gamma^5}\,\frac{S_b}{S_w}R_b(x), \qquad \alpha_p = \frac{\omega_p^2 R^2}{u^2\gamma^2}\,\frac{S_p}{S_w}R_p(x), \qquad \widetilde{\alpha} = \frac{G^2}{R_b R_p}, \\ R_j(x) = \sum_{n=1}^{\infty}\frac{1}{k_{\perp n}^2 R^2 + x^2}\,\frac{\varphi_n^2(\mathbf{r}_j)}{\|\varphi_n\|^2}, \qquad j = b,\, p, \\ G(x) = \sum_{n=1}^{\infty}\frac{1}{k_{\perp n}^2 R^2 + x^2}\,\frac{\varphi_n(\mathbf{r}_b)\varphi_n(\mathbf{r}_p)}{\|\varphi_n\|^2}, \qquad x = \frac{\omega R}{u\gamma}. \end{gathered} \tag{5}$$

Here R is the characteristic waveguide cross section radius, S_w is the cross section area, $\varphi_n(\mathbf{r}_j)$ are the waveguide eigenfunctions at the points the beam or the plasma occur, $\|\varphi_n\|$ are the eigenfunction norms, and $k_{\perp n}$ are the transverse wave numbers. The dependence of the geometrical factors R_j and G on x (in fact on the frequency ω) is a consequence of the nonlinear dispersion of beam and plasma waves. Later on the equations for R_j and G will be written in an explicit form for the geometry that will be specified.

The quantity $\widetilde{\alpha}$ that determines ν is the coupling parameter and characterizes the interaction between the beam and the plasma depending on their relative positions across the waveguide. For example, it follows from Eq. [5] that when the beam and plasma coordinates coincide, we have $\widetilde{\alpha} = 1$ (strong coupling [5]), and when the beam and the plasma are well apart, and the collective Cerenkov effect takes place, then

$$\widetilde{\alpha} \ll 1. \tag{6}$$

Hereafter we assume inequality (6) to be fulfilled. The parameter ν may then be of the order of magnitude of unity.

Note that Eqs. (2) were derived for a linear plasma. This approach is valid if the displacement of plasma electrons in the longitudinal electric field E_z is small compared with the wavelength $k_z^{-1} \approx u/\omega$, that is, if $\lambda_p = (e|E_z|/mk_z u^2) \ll 1$ [5]. The criterion for plasma linearity written in dimensionless variables has the form:

$$\lambda_p = \frac{1}{2}\widetilde{\alpha}^{1/2}\frac{\mu^{3/2}}{4\gamma}|\varepsilon'|, \tag{7}$$

where $\varepsilon' = \nu\varepsilon + i(1 - i\mu d/d\xi)\rho$ is proportional to the electric field longitudinal component. This is very well seen from the second equation of set (2), which by its sense includes just the longitudinal field. The numerical values of the criterion for plasma linearity will be given below.

Set (2) has the following first integral:

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{dy_0}{\sqrt{1+\mu\eta}} + \frac{\mu^2}{8}|\rho|^2 + \frac{\mu}{8}|\varepsilon|^2 = \text{const}, \tag{8}$$

where the first term on the left-hand side is the change in the kinetic energy flux of beam electrons, and the second and third terms are the electromagnetic energy fluxes of beam and plasma waves, respectively.

* That is why there are differences in signs between set (2) and equations from [5, 6].

Considering that, at worst, the electromagnetic energy can only be extracted from plasma, let us introduce the efficiency of transformation of the kinetic energy of beam electrons into radiation energy, or amplification efficiency, in the form:

$$K = \frac{\mu}{8}(|\varepsilon|^2 - |\varepsilon_0|^2), \tag{9}$$

where $\varepsilon_0 = \varepsilon|_{\xi=0}$.

For the geometry most interesting from the practical point of view (a round metal waveguide of radius R containing thin tubular beam and plasma), we have $\varphi_n = J_0(k_{\perp n} r)$, where $k_{\perp n} = \mu_{0n}/R$ and μ_{0n} are the roots of the zero-order Bessel function. The infinite sums in the geometrical factors are calculated using the Knezer–Sommerfeld formula [7], and R_b, R_p, G, and $\widetilde{\alpha}$ can be written as

$$\begin{gathered} R_j(x) = \frac{1}{2} I_0^2(x a_j) T_j, \qquad j = b, p, \\ G(x) = \frac{1}{2} I_0(x a_b) I_0(x a_p) \begin{cases} T_b, & a_p \leqslant a_b, \\ T_p, & a_p \geqslant a_b, \end{cases} \\ \widetilde{\alpha} = \begin{cases} \frac{T_b}{T_p}, & a_p \leqslant a_b, \\ \frac{T_p}{T_b}, & a_p \geqslant a_b, \end{cases} \\ T_j = \frac{K_0(x a_j)}{I_0(x a_j)} - \frac{K_0(x)}{I_0(x)}, \end{gathered} \tag{10}$$

where K_0 and I_0 are the Bessel functions of an imaginary argument, $a_b = r_b/R$, $a_p = r_p/R$, and r_b and r_p are the thin tubular beam and plasma radii, respectively.

The heavy-current parameter μ, which depends on the beam density via α_b and which determines the mechanism of amplification of plasma waves, can be conveniently written through the beam current J_b and the limiting vacuum current J_0 [5]:

$$\mu = \left[4 \frac{J_b}{J_0} \gamma^2 \left(\frac{\gamma^{2/3} - 1}{\gamma^2 - 1} \right)^{3/2} \frac{R_b(x)}{R_b(0)} \right]^{1/2}. \tag{11}$$

In the limit of low frequencies ($x \ll 1$) and large γ values, $\mu = (4J_b/J_0)^{1/2}$. Note that if $\widetilde{\alpha} \ll 1$ and the beam and the plasma are well apart across the waveguide, the beam transport conditions are determined by the following factors. If $r_b < r_p$, the highest possible beam current corresponds to the limiting vacuum current for a waveguide with $R = r_p$. The plasma then acts as a metal surface. If $r_b > r_p$, the highest possible current for the beam transport is calculated by the model of two coaxial metal surfaces. Then $J_{\max} \sim 2 - 3J_0$. Our use of the notion of the limiting vacuum current of a thin tubular beam is to a large extent based on practice common to both vacuum and SHF electronics.

In the linear approximation when $\varepsilon \sim \varepsilon_0 \exp(i\delta\xi)$, where according to (3) $\delta = (u/\omega)(2\gamma^2/\mu)\delta k_z$ and δk_z is the dimensional amplification factor, set (2) yields the dispersion equation [4]

$$[\delta^2 - (1 + \mu\delta)](\delta + \eta_0) = -\nu^2. \tag{12}$$

Using the term in square brackets in Eq. (12), we can easily find

$$\delta_{1,2} = \frac{1}{2}\mu \left(1 \pm \sqrt{1 + \frac{4}{\mu^2}} \right), \tag{13}$$

which determine the spectra of the slow and fast beam waves. In the boundary value problem, the sign "+" corresponds to the slow wave. For low-current beams, when $J_b \ll J_0$, we have $\delta_{1,2} \approx \pm 1$, and if $J_b \gg J_0$, $\delta_1 \approx \mu$ and $\delta_2 \approx -1/\mu$ [4].

Amplification under the collective Cerenkov effect conditions is known to be a wave-wave-type interaction. It occurs when the phase velocity of an unperturbed plasma wave v_{ph} is of the order of magnitude of the phase velocity of the slow beam wave v_b or, using the variables of Eq. (4), $\eta_0 \approx -\delta_1$. Note that a strict equality corresponds to the highest amplification. Using the representation $\delta = \delta_1 + \delta'$ in Eq. (12) then yields the imaginary part of the amplification factor:

$$\delta' = -i\frac{\nu}{(4+\mu^2)^{1/4}}. \tag{14}$$

Equality (14) holds if the inequality

$$\nu \ll (4+\mu^2)^{3/4} \tag{15}$$

is satisfied. Otherwise, the collective Cerenkov effect is impossible, and we have a transition from wave-wave to wave-particle interactions.

With low-current beams ($\mu \ll 1$ or $J_b \ll J_0$) Eq. (14) is simplified, the amplification factor is written as [4]

$$\delta = 1 - i\nu/\sqrt{2}, \tag{16}$$

and inequality (15) transforms into

$$\nu \ll 2\sqrt{2}. \tag{17}$$

In the other limit when $\mu \gg 1$ we have

$$\delta = \mu - i\nu/\sqrt{\mu} \tag{18}$$

and the condition of applicability of Eq. (18) is

$$\nu \ll \mu^{3/2}. \tag{19}$$

A nonlinear stabilization of amplification under conditions of the collective Cerenkov effect is, as shown in [5], determined by three physical effects. Their competition is controlled by the parameters μ and ν (or the beam current, J_b, and the coupling parameter, $\widetilde{\alpha}$). Consider the most interesting limits, when one of the possible mechanisms predominates.

If $\nu \lesssim 1$ (this does not mean that condition (6) breaks), then, given a fairly wide range of μ variation (the important thing is preservation of inequality (15)), the amplification is stabilized as a result of capture of electrons by the field of the slow beam wave and of phase reversal [5]. Equations (2) can then only be analyzed numerically. The calculations were performed for the following fixed parameters: $\gamma = 2$; $R = 1.8$ cm; $r_p = 0.9$ cm ($a_p = 0.5$); $\omega_p = 25 \times 10^{10}$ s^{-1}; $\omega = 7.78 \times 10^{10}$ s^{-1} ($x = 2.7$), and $r_b = 1.44$ cm ($a_b = 0.8$). These parameters correspond to an actual experimental situation [8]. The beam current value was varied.

If $(J_b/J_0) = 0.1$ ($\mu = 0.31$ and $\nu = 0.91$), the optimum amplification length for an input power $P_0 \simeq 180$ kW is $z_{\max} \simeq 63$ cm, $K_{\max} \simeq 23\%$, and the output radiation power is $P_{\text{ex}} \simeq 200$ MW. For a higher current when $J_b/J_0 = 0.5$ ($\mu = 0.7$ and $\nu = 0.79$) and for the same input power P_0, we have $z_{\max} \simeq 52$ cm, $K_{\max} \simeq 31\%$, and $P_{\text{ex}} \simeq 1300$ MW. Lastly, if $(J_b/J_0) = 1$ ($\mu = 0.98$ and $\nu = 0.77$), then $z_{\max} \simeq 48$ cm, $K_{\max} \simeq 4\%$, and $P_{\text{ex}} \simeq 340$ MW for $P_0 \simeq 180$ kW. The reason for this decrease in efficiency and, therefore, in output power was the use of a fixed frequency x in this series of calculations. The maximum amplification condition ($\eta_0 = -\delta_1$) was therefore broken when the current was increased. All of the calculations were performed with $\widetilde{\alpha} \simeq 0.15$, and the criterion of plasma linearity, $\lambda_p \sim 0.1$, was satisfied.

If $\nu \ll 1$, the amplification of plasma waves is stabilized by the effect of a nonlinear frequency shift irrespective of the μ value [9]. This corresponds to including only cubic nonlinear terms in Eqs. (2) and makes it possible to obtain an analytical solution to the problem [10]. Note that, depending on the beam current J_b (or the parameter μ), cubic nonlinearities have different origins. For low-current beams ($J_b \ll J_0$), the nonlinear frequency shift is due in the first place to deceleration of the beam [5, 10, 11], whereas in the limit of high currents, the amplification stabilization is mainly determined by a change in the momentum or by the relativistic dependence of the frequency of Langmuir beam oscillations on their amplitude [6, 11]. With intermediate currents, both mechanisms contribute to the nonlinear amplification dynamics.

Equations containing cubic nonlinearities alone are obtained by expanding the initial equations in electron trajectories and momenta as suggested in [10, 11]. Let us introduce the electron momentum

$$p = (1+\mu\eta)^{-1/2}, \tag{20}$$

and write the coordinate and the momentum of the electron in the form [10, 11]:

$$\begin{aligned} y &= y_0 + w(\xi) + \widetilde{y}(\xi, y_0), \qquad |\widetilde{y}| \ll 1, \\ p &= \langle p\rangle + (1/2)\mu[A(\xi)\exp\{-iy\} + \text{c. c.}], \end{aligned} \tag{21}$$

where $w(\xi)$ and $\langle p\rangle$ are the constant shift and the mean electron momentum, respectively, and $\widetilde{y}$ and $A(\xi)$ are their oscillations. By substituting (20) and (21) in Eqs. (2), applying the theory of residues to integrate these equations with respect to y_0, and expanding the integrated equations in the wave amplitudes with an accuracy to cubic nonlinearities inclusively we derive the following set of equations:

$$\begin{aligned} \frac{dw}{d\xi} &= \frac{1}{4}\left[(\mu|\rho|^2 + |\varepsilon|^2 - |\varepsilon_0|^2) + 6\mu|A|^2\right], \\ \frac{d\rho}{d\xi} &= -2i\left[1 + \frac{3}{8}\mu(\mu|\rho|^2 + |\varepsilon|^2 - |\varepsilon_0|^2) + \frac{3}{2}\mu^2|A|^2\right]A, \\ \frac{dA}{d\xi} &= -\frac{i}{2}\exp\{-iw\}\Big(1 - i\mu\frac{d}{d\xi}\Big)\widetilde{\rho} - \frac{1}{2}\nu\varepsilon\exp\{-iw\}, \\ \frac{d\varepsilon}{d\xi} &+ i\eta_0\varepsilon = \nu\rho, \\ \widetilde{\rho} &= \rho\exp\{iw\}. \end{aligned} \tag{22}$$

After eliminating A from set (22) and passing to slowly varying amplitudes ε' and ρ' with the help of the representation

$$\varepsilon = \varepsilon'\exp\{-i\eta_0\xi - iw\}, \qquad \widetilde{\rho} = \rho'\exp\{-i\eta_0\xi - iw\} \tag{23}$$

we can rewrite Eqs. (22) in the form (primes omitted):

$$\begin{aligned} \frac{d\varepsilon}{d\xi} &= \nu\rho, \\ \frac{d\rho}{d\xi} - i\Delta\rho &= -\frac{\nu}{2\eta_0 + \mu}\varepsilon, \\ \Delta &= \frac{3}{8}\eta_0^2\frac{2(\mu\eta_0 - 4/3) - \mu\eta_0(\mu\eta_0 - 2)}{2\eta_0 + \mu}|\rho|^2. \end{aligned} \tag{24}$$

Here Δ is the nonlinear detuning caused by the nonlinear frequency shift, and $\eta_0 = -\delta_1$.

Solving set (24) is trivial; the solutions written in terms of elliptic functions are ($\rho = |\rho|$ and $\varepsilon = |\varepsilon|$):

$$\rho^2 = \rho_{\max}^2\frac{\mathrm{sn}^2(y, r)}{1 + (\varepsilon_{\max}^2/\varepsilon_0^2)\mathrm{cn}^2(y, r)}, \qquad \varepsilon^2 = \frac{\varepsilon_{\max}^2}{1 + (\varepsilon_{\max}^2/\varepsilon_0^2)\mathrm{cn}^2(y, r)}, \tag{25}$$

where

$$\begin{aligned} r &= 1 - \frac{\varepsilon_0^2}{\varepsilon_{\max}^2}, \qquad y = \frac{\nu}{(4+\mu^2)^{1/4}}\xi, \\ \rho_{\max} &= 4\sqrt{\frac{2}{3}}\frac{\nu^{1/2}(4+\mu^2)^{1/8}}{\eta_0[2(4/3 - \mu\eta_0) - \mu\eta_0(2 - \mu\eta_0)]^{1/2}}, \\ \varepsilon_{\max} &= 4\sqrt{\frac{2}{3}}\frac{\nu^{1/2}(4+\mu^2)^{3/8}}{\eta_0[2(4/3 - \mu\eta_0) - \mu\eta_0(2 - \mu\eta_0)]^{1/2}}. \end{aligned} \tag{26}$$

The distance at which the amplitudes ρ and ε reach a maximum is determined by the equation

$$\xi_0 = \frac{(4+\mu^2)^{1/4}}{\nu}\ln\Big(2\sqrt{2}\,\frac{\varepsilon_{\max}}{\varepsilon_0}\Big), \tag{27}$$

and the efficiency of the transformation of the beam kinetic energy into radiation energy and the equation for the output radiation power are

$$K = \frac{\mu}{8}\sqrt{4+\mu^2}\,\rho^2, \qquad P_{\text{ex}} = \frac{mc^2}{e}(\gamma-1)J_bK. \tag{28}$$

With low-current beams, when $\mu \ll 1$, and the principal mechanism of nonlinear stabilization is the deceleration of the beam, Eqs. (25) to (28) are simplified considerably:

$$\begin{gathered} y = \frac{\nu}{\sqrt{2}}\xi, \qquad \xi_0 = \frac{\sqrt{2}}{\nu}\ln\Big(2\sqrt{2}\,\frac{\varepsilon_{\max}}{\varepsilon_0}\Big), \\ \rho_{\max} = 2\sqrt[4]{2}\,\nu^{1/2}, \qquad \varepsilon_{\max} = 2(2\sqrt{2})^{1/2}\nu^{1/2}, \\ K_{\max} = \sqrt{2}\,\mu\nu \simeq 1.5\tilde{\alpha}^{1/2}\frac{R_b(x)}{R_b(0)}\Big(\frac{J_b}{J_0}\Big)^{1/4}. \end{gathered} \tag{29}$$

For heavy-current beams, when $\mu \gg 1$, and the nonlinear frequency shift is governed by the relativistic dependence of the frequency of Langmuir beam oscillations on the amplitude, Eqs. (25) through (28) take the form:

$$\begin{gathered} y = \frac{\nu}{\sqrt{\mu}}\xi, \qquad \xi_0 = \frac{\sqrt{\mu}}{\nu}\ln\Big(2\sqrt{2}\,\frac{\varepsilon_{\max}}{\varepsilon_0}\Big), \\ \rho_{\max} = 4\sqrt{\frac{2}{3}}\,\nu^{1/2}\mu^{-11/4}, \qquad \varepsilon_{\max} = 4\sqrt{\frac{2}{3}}\nu^{1/2}\mu^{-9/4}, \\ K_{\max} = \frac{4}{3}\nu\mu^{-7/2} \simeq \tilde{\alpha}^{1/2}\frac{R_b(0)}{R_b(x)}\frac{J_0}{J_b}. \end{gathered} \tag{30}$$

Note that the validity of the analytical approach used in this work is substantiated by a direct numerical simulation of the initial set of equations (set (2)). By way of example, we plotted the spatial dynamics of $|\rho|$ ($|\varepsilon|$ behaves similarly) for the following system parameters: $\gamma = 2$; $R = 1.8$ cm; $r_p = 0.9$ cm; $r_b = 1.44$ cm; $x = 4.5$; $\omega_p = 25 \times 10^{10}$ s^{-1}; and $J_b/J_0 = 3$ (Fig. 1). The parameters μ and ν are then equal to 1.48 and 0.64, respectively. One can see that the solution has a "soliton" character in agreement with Eqs. (25), and $|\rho_{\max}| \simeq 0.32$. It follows from general analytical expression (26) that $|\rho_{\max}| \simeq 0.34$. Numerical solutions of this type for a simpler beam-plasma system were first obtained in [6].

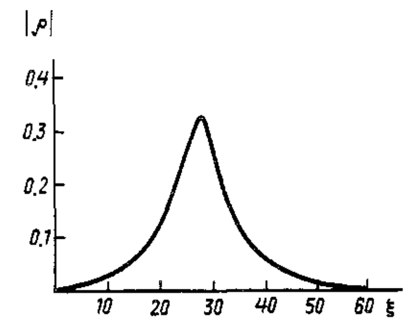

Fig. 1

Spatial dynamics of $|\rho|$ for $J_b/J_0 = 3$.

We will note in conclusion that transversely inhomogeneous beam-plasma waveguides under conditions of the collective Cerenkov effect can, as follows from the results obtained, be used to develop efficient amplification systems operating in the microwave region.

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