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THE EFFECT OF TRANSVERSE INHOMOGENEITY OF THE PUMPING FIELD ON ELECTRON MOTION IN NONADIABATIC MAGNETIC UNDULATORS WITH GUIDING MAGNETIC FIELD

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Setting up optimal radiation regimes of helical relativistic electron beams requires a definite value of the electron pitch factor. The presence of a transverse inhomogeneity of the pumping field in real undulators leads to the following two main consequences: a widening of the nonlinear resonance and a drift of the guiding centers of the particles. A theory of these effects is developed for symmetric and asymmetric undulators with a homogeneous guiding magnetic field.

Different constructions of magnetic undulators [1] are extensively used for creating curvilinear relativistic electron beams applied in microwave devices with transverse interaction. The superimposition of the guiding magnetic field in the region of beam transport and the creation of a resonant pumping regime make it possible to significantly reduce the required magnitude of the undulator field [2]. The maximum transverse particle momentum is attained in nonadiabatic undulators.

To reduce the electron velocity spread it is necessary to provide minimum pumping field inhomogeneity over the beam width h, which can be achieved in undulators of symmetric (plane [1] or coaxial [3]) construction with relativistic electron beam (REB) injection at the pumping amplitude minimum with respect to the transverse coordinate. In this case the effects relating to the inhomogeneity can be neglected for sufficiently narrow beams ($kh < 0.1$; $k = 2\pi/d$, where d is the undulator period) and small pumping amplitudes (for $kr_L \ll 1$, where r_L is the Larmor radius). The electron dynamics in nonadiabatic undulators with a guiding magnetic field was considered in this approximation in [2].

The setting up of optimal REB radiation regimes in microwave devices requires a quite definite value of the electron transverse momentum $p = (p_x^2 + p_y^2)^{1/2}/m_0c$ or of the electron pitch factor (spiraling) $g = p/p_z$, where p_z is the longitudinal momentum also normalized to m_0c. For instance, gyrotrons usually require $g > 1$ and cyclotron-resonance masers or free-electron lasers require $p \approx 1$, so that disregard for the indicated inhomogeneity effects is not always possible. For adiabatic undulators these questions were considered in [4–6], and the nonadiabatic case is investigated in the present paper.

The presence of transverse inhomogeneity of the undulator field is a consequence of its longitudinal (along the beam propagation and guiding magnetic field direction coinciding with the z axis) periodicity. For example, for plane geometry the vector potential $A = A_y$ (where x is the transverse coordinate; the system is homogeneous along the y axis) satisfies the Laplace equation

$$\partial^2 A/\partial x^2 + \partial^2 A/\partial z^2 = 0, \tag{1}$$

and for symmetric systems (when $x = 0$ is the symmetry plane) it is given by the formula

$$A(x,z) = \sum_n A_n, \qquad A_n = \frac{B_n}{kn}\cosh(knx)\cdot\cos(knz), \tag{2}$$

where n is the index of the spatial harmonic and B_n is its amplitude. The components of the undulator magnetic field (without consideration for the guiding field) are

$$B_{xn} = -\partial A_n/\partial z = B_n\cosh(knx)\sin(knz), \quad B_{zn} = \partial A_n/\partial x = B_n\sinh(knx)\cos(knz). \tag{3}$$

The amplitudes B_n of the harmonics depend on the undulator geometry and usually rapidly decrease with increasing index n, and therefore in what follows we confine ourselves to only the first (resonance) harmonic ($n = 1$). For asymmetric undulators relations (1)–(3) are valid if the hyperbolic functions in them (cosh and sinh) are replaced by exponential functions of the same arguments. The pumping field proper producing the electron transverse spiraling is the component B_{xn}.

The indicated pumping field inhomogeneity and the related presence of the z component lead to the following two main effects. First, in the process of motion and increase of the Larmor radius $r_L = cp/\omega_B$ (where $\omega_B = eB/m_0$ is the nonrelativistic cyclotron frequency) of electrons the effective pumping amplitude increases along the electron trajectory. This must result in a widening of the nonlinear resonance band as compared to the case when the inhomogeneity is not taken into account. The effect also depends on the transverse coordinate X_0 of particle entrance to the undulator. Second, while by virtue of the undulator symmetry, there can be no drift of axial electrons ($X_0 = 0$ is the pumping amplitude minimum) in the direction of the x axis, the other types of motion in the undulator must be accompanied by this drift.

This process can be considered qualitatively based on the concept of ponderomotive interaction in weakly inhomogeneous fields [7]. In a reference frame that is in translatory motion with electron velocity, the transverse force acting on the electrons is determined by the gradient of the quasi-potential, which is inversely proportional to the expression $\Omega^2 - \omega_B^2/\gamma^2$, where $\Omega = kc\beta_z$ is the bounce frequency; β_z and γ are the longitudinal velocity normalized to the light velocity c and the particle relativistic factor, respectively [8]. Thus, in a weak guiding magnetic field ($\omega_B/kc \equiv \mu < p_z \equiv \beta_z\gamma$) the electrons are pushed out of the region of strong field, and the undulator focuses the beam at the axis, which is analogous to the case of periodic magnetic focusing. In the opposite case ($\mu > p_z$) the beam is defocused in the transverse direction. A change in the force direction can also occur in the process of particle spiraling because, if energy is conserved in the magnetic field, it is accompanied by a decrease in the longitudinal particle momentum. In the vicinity of the resonance in question ($\mu \approx p_0$, where p_0 is the initial longitudinal electron momentum) any quantitative estimates based on the quasi-potential are not correct, but the above qualitative considerations are valid and are confirmed below by a rigorous analysis.

EQUATIONS OF MOTION IN THE DRIFT APPROXIMATION (SYMMETRIC UNDULATORS)

Assume that at the undulator entrance ($z = 0$) we have a mono-velocity and rectilinear REB ($\beta_z = \beta_0$ and $p = 0$). The electrons differ only in the position of the injection place relative to the transverse coordinate X_0. The spatial beam charge is neglected and only the first spatial harmonic of the pumping field is taken into account. Since the particle energy is conserved in the magnetic field ($\gamma = \gamma_0$), the transverse (p) and longitudinal (p_z) momenta are related by the simple equation $p^2 + p_z^2 = p_0^2 = \gamma_0^2 - 1$, and it suffices to write down the equations only for the Cartesian transverse momentum components p_x and p_y:

$$\begin{aligned} p_x' &= -\mu\frac{p_y}{p_z}\left(1 + \frac{B_z}{B}\right), \\ p_y' &= \mu\frac{p_x}{p_z}\left(1 + \frac{B_z}{B}\right) - \alpha_0\cosh(x)\sin(\xi), \end{aligned} \tag{4}$$

where $\alpha_0 = eB_1/(m_0kc)$, $\mu = \omega_B/kc = eB/(m_0kc)$, $B_1/B = \alpha_0/\mu$, $B_z/B = \alpha_0/\mu\sinh(x)\cos(\xi)$, and the prime symbolizes the differentiation with respect to the normalized longitudinal coordinate $\xi = kz$.

Following the drift approximation, we seek the electron transverse coordinates (x, y) in the form of a sum of the coordinates (X, Y) of the guiding center and an additional term related to the rotation over the Larmor spiral of the normalized radius $\varepsilon = kr_L = p/\mu$:

$$\begin{aligned} x &= X + \varepsilon\cos\psi, \quad y = Y + \varepsilon\sin\psi, \\ \psi &= \int \mu/p_z\, d\xi + \varphi, \end{aligned} \tag{5}$$

where ψ and φ are the slow and fast cyclotron rotation phases, respectively. Differentiating (5) we obtain the following expressions for the momenta:

$$\begin{aligned} p_x &= -p\sin\psi + p_zX', \\ p_y &= p\cos\psi + p_zY'. \end{aligned} \tag{6}$$

Differentiating (6) once more, substituting (5) and (6) into (4), and discarding the nonresonant terms we derive the system of equations

$$\begin{aligned} p' &= -\langle \alpha_0 \cosh(x) \sin(\xi) \cos(\psi) \rangle, \\ p\varphi' &= \langle \alpha_0 \cosh(x) \sin(\xi) \sin(\psi) \rangle, \\ \mu X' &= \mu p/p_z \langle B_z/B \sin(\psi) \rangle + \langle \alpha_0 \cosh(x) \sin(\xi) \rangle, \\ \mu Y' &= -\mu p/p_z \langle B_z/B \cos(\psi) \rangle. \end{aligned} \tag{7}$$

Here the broken brackets $\langle \dots \rangle$ symbolize averaging with respect to the fast rotation phase ψ. We introduce a slow phase $\theta = \xi - \psi$ relative to the pumping field. Then, on substituting $\xi = \psi + \theta$ into (7), we can apply (5) to perform the averaging in (7) analytically by means of the relations

$$\begin{aligned} \frac{1}{\pi} \int_0^\pi \cosh(\varepsilon \cos\psi)\, d\psi &= I_0(\varepsilon), \\ \frac{1}{\pi} \int_0^\pi \sin^2\psi \cosh(\varepsilon \cos\psi)\, d\psi &= I_1(\varepsilon)/\varepsilon, \end{aligned} \tag{8}$$

where I_0 and I_1 are the modified Bessel functions and $\varepsilon = p/\mu = p_0/\mu(1 - p_z^2/p_0^2)^{1/2}$. As a result, we obtain

$$\begin{aligned} p' &= -\alpha_0 \cosh(X) \sin(\theta) I_1'(\varepsilon), \\ \theta' &= 1 - \frac{\mu}{p_z} - (\alpha_0 \cosh(X) \cos(\theta)/p) \frac{I_1(\varepsilon)}{\varepsilon}, \\ \mu X' &= \alpha_0 \sinh(X) \sin(\theta) \left(1 - \frac{\mu}{p_z}\right) I_1(\varepsilon), \\ \mu Y' &= -\alpha_0 \sinh(X) \cos(\theta) \frac{p}{p_z} I_1'(\varepsilon). \end{aligned} \tag{9}$$

The prime in the Bessel functions in Eqs. (9) symbolizes the derivative with respect to the argument $(d/d\varepsilon)$. The last of these equations describing the drift in the direction of the y axis is of no significant interest and, when necessary, can be considered separately because, by virtue of the homogeneity of the undulators under study in this direction, the other equations do not depend on the coordinate Y.

In view of the constancy of energy $(pp' = -p_z p_z')$, we derive the phase integral of motion from the first three equations to within the terms quadratic with respect to α_0:

$$\alpha_0 \cosh(X) p \cos(\theta) = \int_{p_z}^{p_0} \frac{(p_z - \mu)\, dp_z}{I_1'(\varepsilon)}. \tag{10}$$

The equation for the coordinate X of the guiding center can be written in the form

$$X' = \frac{1}{\mu^2} \frac{\sinh(X)}{\cosh(X)} \frac{I_1(\varepsilon)}{\varepsilon I_1'(\varepsilon)} (p_z - \mu) p_z', \tag{11}$$

whence follows the drift integral of motion:

$$\ln \frac{\sinh(X)}{\sinh(X_0)} = -\frac{1}{\mu^2} \int_{p_z}^{p_0} \frac{I_1(\varepsilon)}{\varepsilon I_1'(\varepsilon)} (p_z - \mu)\, dp_z. \tag{12}$$

Note that the right-hand sides in (10) and (12) depend only on the magnitudes of the transverse momentum p_z and on the normalized guiding magnetic field μ. As is seen from (11) and (12), for axial electrons $(X_0 = 0)$ the drift is actually absent; its direction for $\mu < p_z$ is focusing because $p_z' < 0$. It should also be noted that in the isophase regime, when the relation $\mu = p_z$ is maintained by variation of the guiding magnetic field, there is no drift either.

For subsequent discussion it is convenient to pass to relative variables:

$$\begin{gathered} a_0 = \alpha_0/(2p_0), \quad x = p_z/p_0, \quad m = \mu/p_0, \\ p/p_0 = (1-x^2)^{1/2}, \quad \varepsilon = p/\mu = p/p_0 \cdot p_0/\mu = (1-x^2)^{1/2}/m. \end{gathered} \tag{13}$$

Then integrals (10) and (12) are written

$$a_0 \cosh(X)(1-x^2)^{1/2}\cos(\theta) = Q(x,m), \qquad Q(x,m) = \int_x^1 \frac{(x-m)\,dx}{2I_1'(\varepsilon)}, \tag{14}$$

$$\frac{\sinh(X)}{\sinh(X_0)} = \exp\{-R(x,m)\}, \qquad R(x,m) = \frac{1}{m^2}\int_x^1 \frac{I_1(\varepsilon)}{\varepsilon I_1'(\varepsilon)}(x-m)\,dx. \tag{15}$$

By means of (14) system of equations (9) is reduced to a single equation:

$$xx' = 2I_1'(\varepsilon)[a_0^2\cosh^2(X)(1-x^2) - Q^2(x,m)]^{1/2}, \tag{16}$$

which, together with drift integral (15), describes the electron motion in an undulator with homogeneous guiding magnetic field.

As is seen from Eq. (16), the maximum transverse momentum we are interested in is attained under the condition that the radicand on the right-hand side ($p' \sim x' = 0$), which can be more suitably written as

$$a_0^2\cosh^2(X)(1+x) = (1-x)\widetilde{Q}^2(x,m), \qquad \widetilde{Q}(x,m) = Q(x,m)/(1-x) \tag{17}$$

vanishes. The coordinate X of the guiding center can be excluded using drift integral (15):

$$\cosh^2(X) = 1 + \sinh^2(X) = 1 + \sinh^2(X_0)\exp\{-2R(x,m)\}. \tag{18}$$

Thus, the problem is reduced to a single transcendental equation with respect to x with parameters a_0 (the pumping amplitude) and m (the magnitude of the guiding magnetic field). It should be stressed that in the normalizations used there is no explicit dependence in (17) and (18) on the energy of the beam electrons, and therefore the results below are of universal character for any accelerating voltages.

NONLINEAR RESONANCE WITH NEGLECT OF THE TRANSVERSE INHOMOGENEITY

As was already mentioned, the pumping inhomogeneity can be neglected for sufficiently narrow beams ($X_0 \ll 1$) and small pumping amplitude ($a_0 \ll 1$). In this case the electron pitch factor is also small, and we can assume that in Eqs. (15)–(18) $\varepsilon \ll 1$ and, accordingly, $I_1(\varepsilon) \approx \varepsilon/2$ and $I_1'(\varepsilon) \approx 1/2$. Then the particle drift is practically absent ($X \approx X_0 \ll 1$), and the particle motion is determined by the equation

$$\begin{gathered} xx' = [(1-x)(a_0^2(1+x) - (1-x)\widetilde{Q}^2(x,m))]^{1/2}, \\ \widetilde{Q}(x,m) = [(1+x)/2 - m]. \end{gathered} \tag{19}$$

The nonlinear resonance curve is found from Eq. (17) and can be described by the relation [2]

$$m_{1,2} = \frac{1+x}{2} \pm a\left[\frac{1+x}{1-x}\right]^{1/2}, \qquad a = a_0\cosh(X_0) \approx a_0, \tag{20}$$

where the signs ($\pm$) correspond to the two branches of the curve (see the dashed lines in Fig. 1).

In the case of small pumping amplitudes under consideration we pass to reduced variables:

$$t = (1-x)/2a^{2/3}, \quad \Delta = (1-m)/a^{2/3}. \tag{21}$$

In view of the smallness of a, in these variables Eq. (19) can be written in the form

$$\begin{aligned} dt/d\xi &= a^{2/3}[tF(t,\Delta)]^{1/2}, \\ F(t,\Delta) &= 1 - t(t-\Delta)^2. \end{aligned} \tag{22}$$

If the equation $F(t,\Delta) = 0$ is solved with respect to the detuning Δ, then the reduced nonlinear resonance curve can be represented as

$$\Delta = t \pm t^{-1/2}. \tag{23}$$

This is a cubic equation with respect to t, and for $\Delta < \Delta_0$ it possesses one real root and a pair of complex conjugate roots, whereas for $\Delta > \Delta_0$ it has three real positive roots, where $\Delta_0 = 3/(2^{2/3})$. In both cases we are interested in the minimum real root $t_1(\Delta)$ given by the formulas

$$\begin{aligned} &\Delta < \Delta_0: \\ &\qquad t_1 = (u+v) + 2\Delta/3, \qquad D = 1/4 - (\Delta/3)^3, \\ &\qquad u = (D^{1/2} + 1/2)^{2/3}, \qquad v = (D^{1/2} - 1/2)^{2/3}; \end{aligned} \tag{24}$$

$$\begin{aligned} &\Delta > \Delta_0 \ (D < 0): \\ &\qquad t_1 + 2\Delta/3[1 + \cos(\theta/3 + 2\pi/3)], \qquad \cos(\theta) = (1+4D)/(1-4D). \end{aligned} \tag{25}$$

Presented below are some particular values of the roots:

$$t_1(0) = 1, \quad t_1(\Delta_0) = t_0 = 2^{4/3}. \tag{26}$$

The value of the root $t_1(\Delta)$ determines the minimum longitudinal electron momentum in the interaction process and, accordingly, makes it possible to find the maximum value of the transverse momentum p and the pumping amplitude required to attain this maximum:

$$a = A_m(\Delta)\left[2\left[1 - \left(1 - \frac{p^2}{p_0^2}\right)^{1/2}\right]\right]^{3/2}, \quad A_m(\Delta) = \frac{1}{8t_1(\Delta)^{3/2}}. \tag{27}$$

If the required transverse momentum is sufficiently small: $p^2/p_0^2 \ll 1$, then this expression can be simplified:

$$a = A_m(\Delta)p^3/p_0^3. \tag{28}$$

The dependence of the dimensionless coefficient A_m on the reduced detuning Δ/Δ_0 is resonant. Its minimum value is attained at the singular point and is equal to $A_m(\Delta_0) = 1/32$. For the zero detuning the coefficient is four times as great: $A_m(0) = 4A_m(\Delta_0) = 1/8$.

If the electron pitch factor (spiraling) $g = p/p_z$ is taken as the preassigned parameter, then formula (27) can be represented in the form

$$a = A_m(\Delta)[2[1-(1+g^2)^{-1/2}]]^{3/2}; \quad a = A_m(\Delta)g^3 \qquad (g^2 \ll 1). \tag{29}$$

It follows that with a fixed detuning the required magnetic undulator field amplitude depends on the spiraling solely for any particle energies.

Knowing roots (24)–(26) we can determine the necessary interaction length:

$$\xi = \frac{1}{a^{2/3}}I(\Delta), \quad I(\Delta) = \int_0^{t_1} \frac{dt}{[tF(t,\Delta)]^{1/2}}. \tag{30}$$

The integral on the right-hand side is expressed in terms of the complete elliptic integral of the first kind K. For the optimal interaction length L we obtain

$$L/d = B(\Delta)p_0^2/p^2, \quad B(\Delta) = (2t_1(\Delta)/\pi)I(\Delta). \tag{31}$$

In particular, for the exact cyclotron resonance ($\Delta = 0$) we have

$$B(0) = \frac{4}{\pi}\frac{K(\sin(15^\circ))}{3^{1/4}} \approx 1.546.$$

THE EFFECT OF THE TRANSVERSE INHOMOGENEITY ON THE ELECTRON DYNAMICS

In the general case the integrals Q in (14) and R in (15) cannot be expressed in terms of special functions and must be calculated numerically. Therefore we shall derive approximate polynomial representations. Since regimes with transverse particle stopping ($p_z = 0$ and $p = p_0$) are of no significant practical interest, we note that the argument of the modified Bessel functions is $\varepsilon \ll 1$ ($0 < x < 1$ and $m \approx 1$). Therefore we can use their expansions up to the second-order terms, and the smaller the electron pitch factor in the process of electron motion in the undulator, the higher the expansion accuracy:

$$\begin{gathered} 1/(2I_1'(\varepsilon)) \approx 1 - 3\varepsilon^2/8 = (1-s) + sx^2, \qquad s = 3/(8m^2), \\ I_1(\varepsilon)/(\varepsilon I_1'(\varepsilon)) \approx (1 - 3\varepsilon^2/8)(1 + 3\varepsilon^2/8) = [(1-s) + sx^2][(1+s/3) - sx^2/3]. \end{gathered} \tag{32}$$

In this approximation the integration is performed quite easily, which results in explicit expressions for $Q(x,m)$, $\tilde{Q}(x,m)$, and $R(x,m)$ in the form of polynomials in x. It is clear that, when necessary, terms of higher order with respect to ε can also be taken into account in expansions (32), which, however, results in no fundamental effects.

The representations derived substantially facilitate the qualitative and quantitative analysis of Eq. (17). It turns out to be convenient to take x ($0 < x \leqslant 1$) as a parameter and to solve the equation with respect to m. In this case the left-hand side of (17) is practically constant in the neighborhood of $m \approx 1$ and the right-hand side regarded as a function of m resembles, qualitatively, a quadratic parabola ($\tilde{Q} \sim [m - \overline{m}(x)]^2$, where $\overline{m}(x)$ is the position of the minimum of the function). Hence, for a fixed longitudinal momentum x there exist two roots $m_{1,2}$ corresponding to the two branches of the nonlinear resonance curve. This procedure is easily performed numerically, and the values in (20) can be taken as the zeroth approximation. The calculation results are presented in Fig. 1, where branches 1 correspond to axial electrons ($X_0 = 0$) undergoing no transverse drift. Of importance is the broadening of the resonance curve and its bending "to the left" at large-angle beam spiraling, which corresponds to the presented qualitative considerations. For off-axis particles (branches 2) the deformation and broadening of the curve are even greater. In this case the solution of (17) and (18) results in a function $X(x,m)$ making it possible to investigate the particle drift. Analysis of the data shows that in regimes with moderate spiraling, which are of practical interest, the electron drift is not large and is also in agreement with the qualitative analysis: the beam is first slightly focused at the undulator axis and then, at the place of maximum spiraling, is defocused to a width somewhat exceeding the initial one.

Of practical interest is the question of maximum electron spiraling for a given pumping amplitude a_0. As in the case of the absence of transverse inhomogeneity [2], it is attained at the singular point (see (27) and (28)) although the position of this point changes. The corresponding data are shown in Fig. 2. As could well be expected, with increasing pumping amplitude the transverse inhomogeneity gives rise to an increase in the electron spiraling, and the amplitude value at which the electron stopping is possible ($p/p_0 = 1$) reduces. In this case the off-axis electrons undergo a larger-angle spiraling.

The method presented is also applicable for analyzing beam motion in asymmetric undulators. Formally, the transition is achieved by replacing the hyperbolic functions (cosh and sinh) in all the formulas by exponential functions of the same arguments. In particular, the drift integral (15) now has the form $X = X_0 - R(x,m)$. An increase of the transverse inhomogeneity results in a strengthening of the beam drift and a significant spread of the transverse momenta, which can be roughly estimated using Eq. (28) if we set $a = a_0\exp(kh)$, where h is the beam width; then we have $(\Delta p/p)_N \approx (kh)/3$. An analogous estimation for

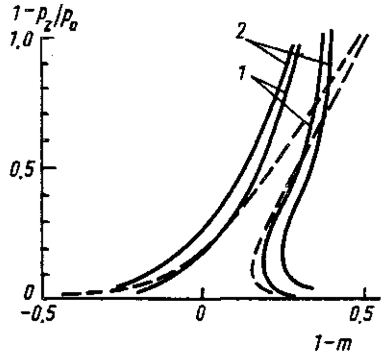

Fig. 1

Nonlinear resonance curves in nonadiabatic magnetic undulators with (solid lines) and without (dashed lines) allowance for the transverse inhomogeneity of the pumping field for $a_0 = 3 \times 10^{-2}$; $X_0 = 0$ (1) and $X_0 = 1$ (2).

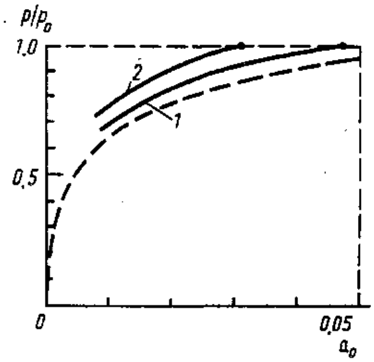

Fig. 2

Dependence of the maximum transverse electron momentum on the pumping amplitude for symmetric undulators: $X_0 = 0$ (1) and $X_0 = 1$ (2) (the dashed line corresponds to disregard of inhomogeneity).

symmetric undulators yields $(\Delta p/p)_S \approx (kh)^2/12$, whence it becomes clear that they have a smaller spread by approximately $(\Delta p/p)_S/(\Delta p/p)_N \approx (kh)/4$ times compared to asymmetric undulators. Qualitatively, the same relationship also holds for drift displacements of the beam.

Thus, the effect of the transverse inhomogeneity of the undulator field is substantial for regimes with large electron pitch factors and for beams of large width relative to the inhomogeneity scale. From the standpoint of velocity spread and transverse drift of electrons, in this case symmetric undulators have substantially better characteristics than asymmetric ones.

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