Moscow University
Physics Bulletin
Vol. 47, No. 5, pp. 22-27, 1992

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Universiteta. Fizika
UDC 535.241.13

POLARIZATION EFFECTS IN LIGHT PROPAGATION THROUGH AN ACOUSTIC FIELD IN AN ISOTROPIC MEDIUM

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The variation of light polarization in the process of Bragg's acoustooptic interaction has been investigated theoretically. It is shown that in the propagation of linearly polarized light through an acoustic field excited in an isotropic medium the light radiation becomes elliptically polarized. By varying the ultrasound intensity or frequency it is possible to control the polarization state. The effect of polarization nonreciprocity has also been studied for two light waves propagating through an acoustic field in the opposite directions.

INTRODUCTION

In practical application of acoustooptic interaction the question of the incident and diffracted light polarization naturally arises because even an optically isotropic medium becomes anisotropic under the action of sound. The studies described in [1, 2] showed that during the diffraction of a linearly polarized plane light wave the radiation at all diffraction maxima remains linearly polarized but there can occur a turn of the polarization plane by an angle depending on the power of the acoustic wave. This conclusion is true for the Raman–Nath diffraction regime whereas in the Bragg diffraction regime or in an intermediate regime the situation becomes more complicated. The asymmetry of the diffraction pattern results in an additional phase shift of diffracted waves [3–6], owing to which, in the general case, the light polarization at the exit from the acoustooptic interaction region turns out to be elliptic. A detailed consideration of this effect is the subject of the present paper.

DIFFRACTION BY A LONGITUDINAL ACOUSTIC WAVE

Assume that in a solid isotropic dielectric bounded by parallel planes $x = 0$ and $x = l$ a longitudinal monochromatic acoustic wave propagates along the z axis:

$$a(z,t) = a_0 \sin(Kz - \Omega t), \tag{1}$$

where a_0 is the amplitude, K is the wave number, and Ω is the ultrasound frequency. In the acoustooptic interaction region, owing to the photoelastic effect, this wave changes the optical indicatrix so that its cross section by the yz plane can be represented in the form

$$\left(\frac{1}{n^2} + p_{12}a\right) y^2 + \left(\frac{1}{n^2} + p_{11}a\right) z^2 = 1, \tag{2}$$

where n is the refractive index of the unperturbed medium and p_{11} and p_{12} are the photoelasticity coefficients [7]. Relation (2) implies that for light propagating along the x axis the principal axes of the sound-induced anisotropy coincide with the y and z axes. In the medium there appear two phase diffraction gratings moving along the z axis with sound velocity v: a grating with the amplitude of variation of the refractive index $\Delta n_{\parallel} = (1/2)n^3 p_{11} a_0$ for a light wave polarized in the xz acoustooptic interaction plane (the $\|$-polarization) and a grating with $\Delta n_{\perp} = (1/2)n^3 p_{12} a_0$ for a light wave polarized along the y axis (the $\perp$-polarization). The distinction in the values of the constants p_{11} and p_{12} results in different acoustooptic interaction efficiency for these orthogonally polarized modes.

In the Bragg diffraction regime the complex wave amplitudes at the zeroth and first diffraction maxima for each type of polarization are determined by the expressions [7]

$$C_0 = C^*_{\parallel,\perp}\left[\cos\left(\frac{l}{2}\sqrt{q^2_{\parallel,\perp}+\eta^2}\right)+j\frac{\eta}{\sqrt{q^2_{\parallel,\perp}+\eta^2}}\sin\left(\frac{l}{2}\sqrt{q^2_{\parallel,\perp}+\eta^2}\right)\right]\exp\left\{-j\frac{\eta l}{2}\right\}, \quad (3)$$

$$C_1 = C^*_{\parallel,\perp}\frac{q_{\parallel,\perp}}{\sqrt{q^2_{\parallel,\perp}+\eta^2}}\sin\left(\frac{l}{2}\sqrt{q^2_{\parallel,\perp}+\eta^2}\right)\exp\left\{j\frac{\eta l}{2}\right\}, \quad (4)$$

where $C^*_{\parallel,\perp}$ is the amplitude of the incident light wave with the corresponding polarization. The energy exchange efficiency between the maxima depends on two parameters. The parameter $q_{\parallel,\perp} = (2\pi/\lambda)\Delta n_{\parallel,\perp}$, where λ is the light wavelength, is determined by the acoustic wave amplitude and, via the photoelasticity coefficients, by the incident light polarization. The parameter η characterizes the mismatch between the incidence angle θ_0 and the Bragg angle θ_B; for small angles $\eta = K(\theta_0 - \theta_B)$. In an isotropic medium η does not depend on light polarization.

The presence of complex quantities in relations (3) and (4) indicates that light propagation through the acoustic field is accompanied by changes of both the amplitudes and the phases of the interacting waves. Hence, by varying the ultrasound intensity or the angle of incidence of light one can control the amplitude and the phase of each of the natural modes of the perturbed medium and, in the case of arbitrary incident light polarization, the light polarization at diffraction maxima as well. Let us first see how this effect manifests itself in the zeroth-order diffraction.

We bring (3) to the form

$$C_0 = C^*_{\parallel,\perp}A_{\parallel,\perp}\exp\{j\varphi_{\parallel,\perp}\}. \quad (5)$$

Then for the relative amplitudes $A_{\parallel,\perp}$ and phases $\varphi_{\parallel,\perp}$ of the natural modes we obtain

$$A_{\parallel,\perp} = \sqrt{1-\frac{q^2_{\parallel,\perp}}{q^2_{\parallel,\perp}+\eta^2}\sin^2\left(\frac{l}{2}\sqrt{q^2_{\parallel,\perp}+\eta^2}\right)}, \quad (6)$$

$$\varphi_{\parallel,\perp} = \tan^{-1}\left[\frac{\eta}{\sqrt{q^2_{\parallel,\perp}+\eta^2}}\tan\left(\frac{l}{2}\sqrt{q^2_{\parallel,\perp}+\eta^2}\right)\right]-\frac{\eta l}{2}+F\left(\frac{l}{2}\sqrt{q^2_{\parallel,\perp}+\eta^2}\right), \quad (7)$$

where $F(\xi) = m\pi$ for $(m-1/2)\pi < \xi < (m+1/2)\pi$ $(m = 0, 1, 2, \ldots)$.

Consider the case when the incident light is linearly polarized and the polarization plane forms an angle α with the y axis (Fig. 1). On entering the region of acoustooptic interaction the light wave with amplitude C^* breaks up into two components with amplitudes $C^*_{\parallel} = C^*\sin\alpha$ and $C^*_{\perp} = C^*\cos\alpha$. These components diffract in the acoustic field independently of each other, and their amplitudes and phases vary in accordance with formulas (6) and (7). At the exit from the interaction region they add together, which results in an elliptically polarized wave. The orientation of the ellipse axes is determined by the relationship between the amplitudes of the constituent waves, i.e., by the expression $r = (A_{\perp}/A_{\parallel})|\cot\alpha|$, and the degree of ellipticity depends on the phase difference $\Delta\varphi = \varphi_{\perp} - \varphi_{\parallel}$.

The dependence of r and $\Delta\varphi$ on $q_{\perp}l$ (in fact, on the acoustic wave amplitude) for $\alpha = 45°$ and different values of the normalized angle difference ηl is demonstrated in Fig. 2 a. The solid and dashed lines represent the $r(q_{\perp}l)$ and $\Delta\varphi(q_{\perp}l)$ curves, respectively. The calculations were performed for fused quartz—a material frequently used in acoustooptic devices. For quartz $p_{11} = 0.121$ and $p_{12} = 0.270$, and, consequently, $q_{\parallel}/q_{\perp} = \varkappa = 0.448$. As follows from formula (6), for light incident at Bragg's angle ($\eta = 0$) the expression $A(ql)$ is described by the function $|\cos x|$, owing to which the value of r attains zero for $q_{\perp}l = \pi$, when the amplitude of the $\perp$ component vanishes, and tends to infinity at the point $q_{\perp}l = \pi/\varkappa = 7.01$, when the amplitude of the $\parallel$ component vanishes. In this case the $\Delta\varphi(q_{\perp}l)$ curve has a step-like shape: at the indicated points the phase difference gains an increment of π in a jump-like manner. When the light incidence angle or the ultrasound frequency change and the phase synchronism of the acoustooptic interaction is disturbed, the amplitudes of the components no longer attain zero, and the $r(q_{\perp}l)$ and $\Delta\varphi(q_{\perp}l)$ curves become smoother.

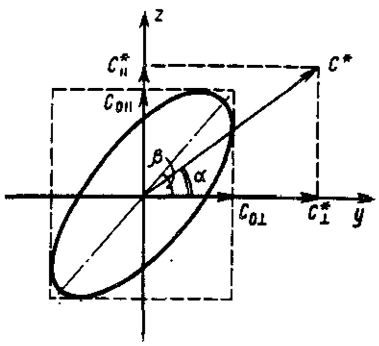

Fig. 1

Variation of light polarization in the zeroth-order diffraction.

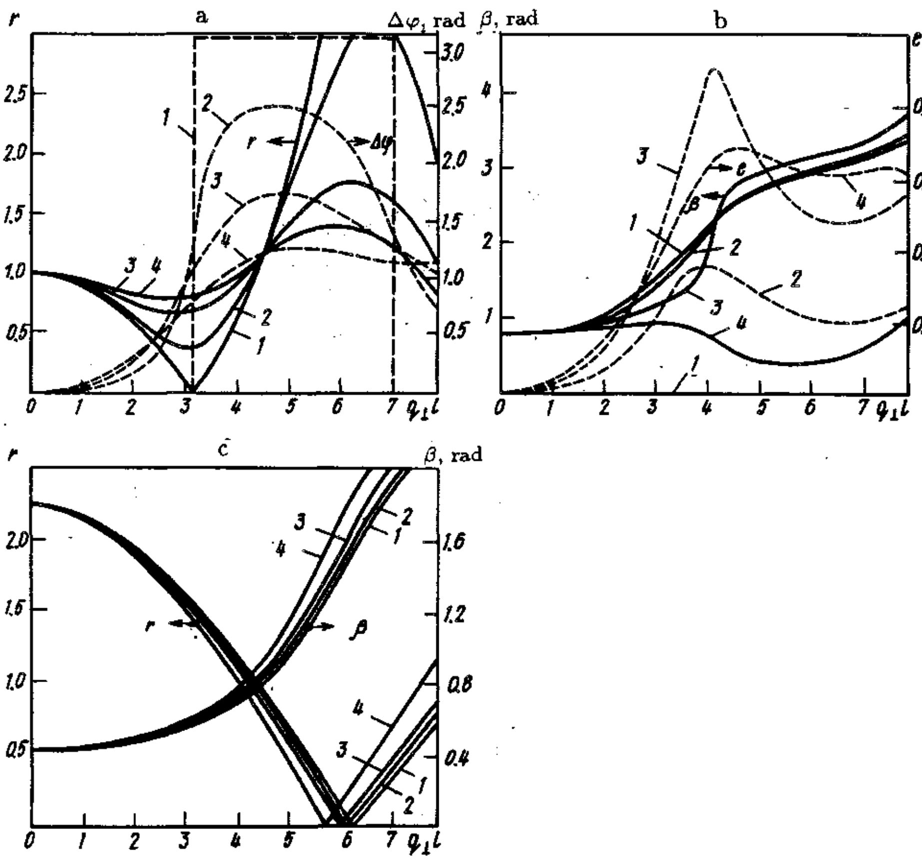

Fig. 2

The amplitude ratio r (solid curves), the phase difference $\Delta\varphi$ (dashed curves), the angle of rotation of the ellipse β (solid curves), and the ellipticity e (dashed curves) as functions of the normalized amplitude of the acoustic wave for the zeroth- (a, b) and first-order (c) diffraction: $\eta l = 0$ (1), $\pi/5$ (2), $\pi/2$ (3), and $4\pi/5$ (4).

The parameters of the polarization ellipse at the exit of the interaction region, i.e., the angle β between the major axes of the ellipses and the y axis and the ellipticity e (the ratio of the lengths of the ellipse semiaxes), can be expressed in terms of r and $\Delta\varphi$:

$$\tan 2\beta = \frac{2r\cos\Delta\varphi}{r^2-1}, \tag{8}$$

$$e^2 = \frac{r^2+1-\sqrt{(r^2-1)^2+4r^2\cos^2\Delta\varphi}}{r^2+1+\sqrt{(r^2-1)^2+4r^2\cos^2\Delta\varphi}} = \frac{r^2+1-(r^2-1)/\cos 2\beta}{r^2+1+(r^2-1)/\cos 2\beta}. \tag{9}$$

Figure 2 b demonstrates the curves of the functions $\beta(q_\perp l)$ (solid curves) and $e(q_\perp l)$ (dashed curves). The calculations were performed for the same values of the angle mismatch as in Fig. 2 a. One can see that for a low angle mismatch, as the amplitude of the acoustic wave increases, the ellipse axis turns smoothly from the initial position $\alpha = \beta = \pi/4$, and at $q_\perp l \approx 4.2$ the angle of rotation attains 90°. In this case the ellipticity changes insignificantly. When the angles coincide ($\eta = 0$) the exit light polarization remains linear irrespective of the values of $q_\perp l$. For large angle differences (of the order of $\pi/2$) the $\beta(q_\perp l)$ curves display a section of fast variation of the angle β. This takes place in the region $r \sim 1$. Here the ellipticity can attain unity, and then the output radiation will have circular polarization.

In the first-order diffraction the situation is simpler than in the case of zeroth-order diffraction. As follows from expression (4), the dependence of $\Delta\varphi$ on the amplitude of the acoustic wave has a trivial character here:

$$\Delta\varphi = F\left(\frac{l}{2}\sqrt{q_\perp^2+\eta^2}-\frac{\pi}{2}\right) - F\left(\frac{l}{2}\sqrt{\varkappa^2 q_\perp^2+\eta^2}-\frac{\pi}{2}\right). \tag{10}$$

The phase shift can assume only two values: 0 or π. Therefore the diffracted radiation is always linearly polarized. Changes of $q_\perp$ or η cause the polarization plane to rotate. The ratio of the amplitudes of the components is determined by the expression

$$r = |\cot\alpha|\cdot\frac{1}{\varkappa}\sqrt{\frac{\varkappa^2q_\perp^2+\eta^2}{q_\perp^2+\eta^2}}\cdot\left|\frac{\sin\left((l/2)\sqrt{q_\perp^2+\eta^2}\right)}{\sin\left((l/2)\sqrt{\varkappa^2q_\perp^2+\eta^2}\right)}\right| = |\cot\beta|. \tag{11}$$

The functions $r(q_\perp l)$ and $\beta(q_\perp l)$ are shown in Fig. 2 c by the solid and dashed curves, respectively. One notes that these curves have quite a different shape compared to the case of zeroth-order diffraction.

DIFFRACTION BY A SHEAR ACOUSTIC WAVE

If a shear acoustic wave is excited along the z axis in an isotropic medium, the diffraction can take place only when there is a nonzero component of the vector of displacement along the y axis. The intersection of the indicatrix by the plane $x = 0$ in the acoustooptic interaction region has the form

$$\frac{1}{n^2}(y^2+z^2)+2p_{44}ayz = 1. \tag{12}$$

The photoelasticity coefficient p_{44} in the isotropic medium is $(p_{11}-p_{12})/2$ [7]. It follows from (12) that the principal axes y' and z' of the perturbed indicatrix are turned through 45° relative to the y and z axes. In the system of principal axes Eq. (12) takes the form

$$\left(\frac{1}{n^2}+p_{44}a\right)y'^2+\left(\frac{4}{n^2}-p_{44}a\right)z'^2 = 1. \tag{13}$$

Thus, for each of the natural light modes polarized along the y' and z' axes there exists its own diffraction grating produced by the shear wave. The distinction from the case of longitudinal wave is as follows: (a) these gratings are displaced relative to each other by half the acoustic wavelength and (b) they have the same amplitude of variation of the refractive index: $\Delta n = (1/2)n^3p_{44}a_0$. The grating displacement produces no effect in the zeroth-order diffraction, but in the first-order diffraction it results in a phase shift by π between the natural modes. It follows that if a light wave with arbitrary polarization is incident on an acoustic column, at the exit, irrespective of the values of q and η, the light in the zeroth-order diffraction will have the same polarization, whereas in the first-order diffraction the polarization plane will be turned mirror-like through an angle 2α relative to the xz' plane. For example, if the incident light is polarized along the z axis, the radiation in the first-order diffraction is orthogonally polarized (i.e., along y).

POLARIZATION NONRECIPROCITY

The above results suggest the conclusion that during the acoustooptic interaction there must exist polarization nonreciprocity, that is to say, the polarization states of the light waves propagating through the acoustic field in the opposite directions turn out to be different. The physical factor causing the nonreciprocal effect is the fact that these light waves are scattered in diffraction orders of different signs, owing to which the Doppler shifts of light frequency occur in different directions: $\omega_{+1} = \omega_0 + \Omega$ in the +1st order diffraction and $\omega_{-1} = \omega_0 - \Omega$ in the −1st order diffraction (here ω_0 is the incident light frequency). The variation in the lengths of the light wave vectors due to the frequency shift results in distinctions in the Bragg angles for waves propagating in the opposite directions:

$$\sin\theta_B^{\pm} = \frac{k_{\pm 1}^2 - k_0^2 - K^2}{2k_0 K} \approx -\frac{K}{2k_0} \pm \frac{nv}{c}, \tag{14}$$

where c is the velocity of light in vacuum, $k_0 = \omega_0 n/c$, and $k_{\pm 1} = \omega_{\pm 1} n/c$. Consequently, for these waves there exist differences in the angle mismatch ηl:

$$(\eta_{-1} - \eta_{+1})l = \frac{2n\Omega l}{c\cos\theta_B}, \tag{15}$$

which produces the polarization nonreciprocity.

Expression (15) permits a different interpretation of the effect under consideration. Note that $l/\cos\vartheta_B$ is the path length of the light beam in the acoustic field and $ln/c\cos\theta_B$ is the time of light propagation through the acoustic column. Hence, the quantity $(\eta_{-1} - \eta_{+1})l$ is equal to the doubled change of the acoustic wave phase during the time of light propagation through it.

As follows from formula (15), the strongest nonreciprocity effect manifests itself at high ultrasound frequencies and large acoustooptic interaction lengths. This corresponds to the Bragg diffraction regime considered above.

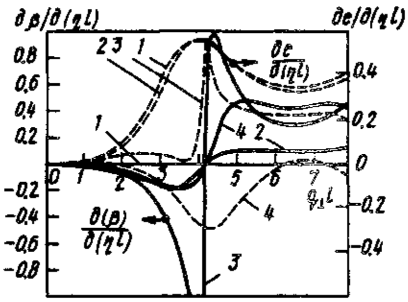

Fig. 3

Polarization nonreciprocity in the zeroth-order diffraction: $\eta l = 0$ (1), $\pi/5$ (2), $\pi/2$ (3), and $4\pi/5$ (4).

In view of the small magnitude of the effect, to calculate the nonreciprocity with respect to the angle β and the ellipticity e we can use the expressions

$$\beta_{-1} - \beta_{+1} = \frac{\partial\beta}{\partial(\eta l)}(\eta_{-1} - \eta_{+1})l = \frac{\partial\beta}{\partial(\eta l)} \cdot \frac{2n\Omega l}{c\cos\theta_B}, \tag{16}$$

$$e_{-1} - e_{+1} = \frac{\partial e}{\partial(\eta l)} \cdot \frac{2n\Omega l}{c\cos\theta_B}. \tag{17}$$

The curves $\frac{\partial\beta}{\partial(\eta l)}(q_\perp l)$ and $\frac{\partial e}{\partial(\eta l)}(q_\perp l)$ presented in Fig. 3 make it possible to determine the ranges of q and η values where the polarization nonreciprocity is maximum. For instance, for fused quartz at frequency

$f = 300$ MHz and $l = 1$ cm the difference in the angle mismatch is equal to $(\eta_{-1} - \eta_{+1})l = 0.19$. Consequently, in the region of $q_{\perp}l \approx 4.2$, where extremal values of the derivatives are attained, the nonreciprocity with respect to the angle β and the ellipticity e can attain $10°$ and 0.15, respectively, i.e., values that can easily be measured experimentally.

CONCLUSION

The investigations described show that variation of the light polarization in the acoustooptic interaction has a complicated character. The isotropic medium with an excited acoustic wave behaves as an anisotropic crystal with optic activity: in the transmitted light there appear elliptic polarization and a turn of the ellipse axes. The magnitude of the effect depends on the parameters of the acoustic wave, which opens up possibilities for creating a new type of light modulation devices. In the Bragg diffraction regime the complicated character of light polarization variation manifests itself to the full extent only in the zeroth-order diffraction. However, in the intermediate diffraction regime with which one usually deals in acoustooptic devices [7] this effect must take place at diffraction maxima of other orders as well.

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