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# INVESTIGATIONS OF VELOCITY FIELD IN FLOWS OF COMPLEX STRUCTURE

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**The hydrogen bubble method is used to study the structure of a flow of dense liquid moving in a quiescent liquid of a lower density. It is shown that in the mixing layer with stratified density distribution the velocity profile can be calculated by the Prandtl formula provided the effect of the density field on the velocity is taken into account.**

The study of processes occurring at the interface between two flows of differing density is a major hydrophysical problem. Interactions of such flows are not only of considerable theoretical importance but also have wide practical applications in view of ever increasing disposal of industrial waste to water basins. To successfully solve the arising pollution problems one must have knowledge of such characteristics as distribution of the velocity and density fields in the contact zone between flows of differing density and velocity. As shown in a number of studies by Yu. G. Pyrkin *et al.* performed at the Department of Marine and Inland Water Physics (see, e. g., [1–3]), the velocity field in this case has an intricate temporal-spatial pattern. The latter circumstance poses some difficulties for conducting investigations under natural conditions. At the same time, many questions arising in studies of mixing layers in the contact zones between two or several streams can be answered in laboratory-scale modeling experiments.

The objective of this work is to study the structure of a stratified flow moving in a quiescent liquid of lower density. A rectangular channel made of plexiglass was used in the experiments. The channel width was 15 cm and height 30 cm. The channel consisted of a left and a right chamber divided by a partition that could be lifted. Water from an elevated tank was fed to the right chamber 0.5 m long and the water level was kept constant. From the right chamber the liquid flew to the left chamber through a honeycomb mounted near the bottom. The flowing liquid layer thickness was controlled by varying the height of partition lift. Measurements were performed in the left chamber of the channel 1.1 m in length. Preliminary experiments revealed that three zones could be identified in the test section of the setup: zone 1, where the inlet parameters affected the flow; zone 2, the test section proper, where the measurements were carried out; and zone 3, where the flow was affected by the exit orifice. Zones 1 and 3 were not considered in this study.

Water of differing salinity $s$ was used as working liquids. The temperature of the flows studied was the same and below room one, so that only salinity should be known to determine the density field. The initial salinity value in the liquids studied was measured by weighing. The vertical density distribution in the mixing zone was determined from salinity measurements by the electric conductivity method. The density was calculated from the electric conductivity of water. The electric conductivity was measured using a technique described in [4]. The sensor size was 100 $\mu$m.

The velocity field was determined by visualizing it with the aid of hydrogen bubbles [5]. The method is as follows. A vertical cathode in the form of a thin, strong, and corrosion-resistant wire and a plate of an arbitrary shape serving as anode were introduced into the flow. The anode was mounted on the side wall of the channel. The pulse current passing through the test liquid electrolyzed it, the $H^+$ ions produced in electrolysis were attracted to the thin cathode wire, became neutralized and generated a multitude of fine molecular hydrogen bubbles which stuck to the cathode surface. The pulse duration (1 ms) is small as compared to the time between pulses (100 ms). The pulse current action resembles shaking which forces the bubbles stuck to the cathode to break off. The bubbles are entrained by the flowing liquid and their string outlines the velocity profile. The next electric discharge pulse follows and the cycle repeats again. The cathode wire diameter was 40 $\mu$m and the hydrogen bubble diameter in the experiments described was 30 $\mu$m. Floating up of bubbles of this size due to the buoyancy force did not perturb the velocity field studied [5].

The cathode wire length exceeded the flow height, which enabled us to visualize the instantaneous velocity profile throughout the flow section, including the near-bottom boundary layer and the mixing layer. The cathode was positioned in the central flow section. Examples of vertical velocity distributions obtained by the hydrogen bubble method that cover both the bottom boundary layer and the mixing layer are shown in Fig. 1.

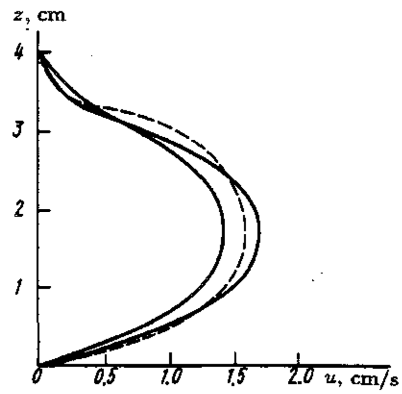


**Fig. 1**

Examples of vertical velocity distribution in a flow of dense liquid moving in a stagnant liquid of lower density.

The experimental procedure was as follows. First, a two-layer liquid was set up in the test section. The lower layer 3 cm thick was colored and has a higher salinity than the upper one. The upper liquid layer was 5 cm thick. A simultaneous opening of the exit orifice and the honeycomb produced a slip flow of the lower layer with respect to the upper one. The flow velocity of the lower layer could be varied. A mixing layer was formed at the interface between the flows of different salinity and a bottom boundary layer appeared at the channel bottom. The salinities of the upper and lower liquid layers differed by 10‰.

The Reynolds numbers as calculated from the maximal flow velocity ($u_{max}$) and flow height ranged from 500 to 800. Such Reynolds numbers suggested that in the bottom boundary layer a flow regime would be established whose velocity profile would comply with the Poiseuille equation [6, p. 80]

$$u = \frac{1}{2\mu}\frac{dp}{dx}z^2 + az + b. \tag{1}$$

The boundary conditions for the boundary layer considered will be as follows:

$$\begin{aligned} u &= 0 \qquad &\text{at } z = 0, \\ du/dz &= 0 \qquad &\text{at } z = h_{max}. \end{aligned} \tag{2}$$

The Poiseuille equation (1) with due regard for the above boundary conditions is transformed into

$$u = \frac{1}{2\mu}\frac{dp}{dx}z^2 - \frac{1}{\mu}\frac{dp}{dx}h_{max}z; \tag{3}$$

here $p$ is the pressure and $\mu$ is the dynamic viscosity of water. We assume that

$$u^+ = \frac{u}{u_{max}}, \qquad z^+ = \frac{z}{h_{max}}, \qquad x^+ = \frac{x}{h_{max}}, \qquad p^+ = \frac{p h_{max}}{\mu u_{max}}, \tag{4}$$

where $h_{max}$ is the horizontal section at which the velocity reaches its maximum $u_{max}$, and write the Poiseuille equations (3) in the dimensionless form

$$u^+ = \frac{1}{2}\frac{dp^+}{dx^+}(z^+)^2 - \frac{dp^+}{dx^+}z^+. \tag{5}$$

The $dp^+/dx^+$ value can be determined from the condition that $u^+ = 1$ at $z^+ = 1$. It was found to be $-2$. Then Eq. (5) assumes the following form

$$u^+ = z^+(2 - z^+). \tag{6}$$

Processing of the experimental data obtained showed that, as could well be expected, with Reynolds numbers below 600 the velocity profiles are well described by Eq. (5).

By differentiating Eq. (5) and returning to dimensional coordinates one can determine the parameters of the viscous flow at the channel bottom. According to calculations the friction shear stress at the bottom ranged between $5.5 \times 10^{-3}$ and $8.0 \times 10^{-3}$ g cm$^{-1}$ s$^{-2}$ in the experiments considered. Thus, when the velocity profile in the bottom boundary layer satisfies Eq. (5), from known $h_{max}$ and $u_{max}$ we can calculate the dynamic flow characteristics such as pressure gradient along the flow and viscous friction at the channel bottom.

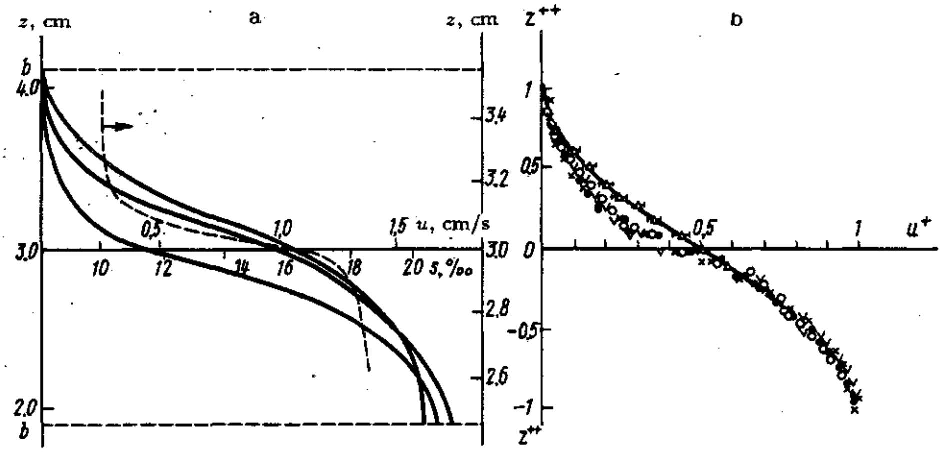


**Fig. 2**

Measured vertical profiles of velocity (solid lines) and salinity (dashed lines) in the mixing layer (a) and dimensionless velocity profiles in the mixing layer (b). The solid line in Fig. 2 b is calculated by formula (8). Triangles denote the velocities corrected for the density field effect by formula (9).

Now let us examine the mixing layer. Figure 2 a shows typical vertical profiles of averaged velocity in the mixing zone between the two streams of differing salinity, one of which is moving while the other is stagnant. Figure 2 also presents a vertical salinity profile. We denote the mixing layer thickness by $2b$. For convenience of the further analysis we fix the origin of the frame of reference at the mixing layer center. This layer spreads from the horizontal section, where the average velocity equals zero, to the section where $u = u_{max}$.

We nondimensionalize the vertical velocity profiles within the mixing layer using the quantities $b$ and $u_{max}$ and write them in terms of $u^+ = u/u_{max}$ and $z^{++} = z/b$. It is seen from Fig. 2 b that the average velocity profiles plotted in the above coordinates are similar and obey a common dependence. It is also obvious that the generalized vertical profile of the averaged velocity in the mixing layer is asymmetric, which is specific to stratified mixing layers. The density stratification in the mixing layer will be characterized by the Richardson number, which, as applied to the above experiments, can be written as

$$\mathrm{Ri} = g\beta \frac{ds/dz}{(du/dz)^2}, \tag{7}$$

where $\beta = \dfrac{1}{\rho}\dfrac{\partial \rho}{\partial s}[‰]^{-1}$ is the salinity compaction coefficient of water.

The $du/dz$ gradient was evaluated from the measured vertical distributions $u(z)$. The salinity gradients $ds/dz$ were found assuming a linear salinity variation between the measurement points. The measurement discreteness near the interface between the flows was 1 mm.

Calculations revealed that the Ri number is nonzero only within a narrow range of heights $z^{++}$ from $-0.4$ to $+0.8$. It is in this range of dimensionless heights that the effect of the density stratification on the vertical velocity distribution in the mixing layer should be expected. Above and below this layer the velocity profile must be similar to that in a nonstratified mixing layer.

The velocity distribution in a turbulent mixing layer with a nonstratified density between two liquid flows moving at velocities $u_1$ and $u_2$ and contacting along the $z = 0$ straight line was derived by Prandtl [7]. When the velocity $u_2 = 0$, i.e., for the conditions used in our experiments, the Prandtl formula reads [7, p. 656]

$$u^+ = \frac{u}{u_{\max}} = \frac{1}{2}[1 + 1.5z^{++} - 0.5(z^{++})^3]. \tag{8}$$

The velocity profile calculated by this formula is displayed in Fig. 2 b by the solid line. As has been expected, deviations of the measured velocity profiles, indicated in Fig. 2 by various symbols, from the Prandtl profile calculated for a nonstratified turbulent mixing layer are observed only in the range of heights where the Richardson number is nonzero.

To take into account the density field effect on the velocity field in the mixing layer we use the results obtained in [8], where the expression was derived relating the vertical profile of the averaged velocity with the Richardson number:

$$\widetilde{u}(z) = u^+(z)\sqrt[4]{1 + 0.4\mathrm{Ri}(z)}. \tag{9}$$

Here $\widetilde{u}(z)$ and $u^+(z)$ are the velocities at the horizontal section $z$ in mixing layers with stratified and nonstratified densities, respectively. According to the experimental data at the depth $z = -b$ the Richardson number $\mathrm{Ri}(-b) = 0$. Hence, at $z = -b$

$$\sqrt[4]{1 + 0.4\mathrm{Ri}} = 1 \quad \text{and} \quad \widetilde{u}_{\max} = u^+_{z=b} = u^+_{\max}.$$

The calculated velocity profiles allowing for the density field effect are also presented in Fig. 2 b by triangles with indication of the root mean square values specifying the scatter of the measured velocities at each horizontal section. Figure 2 b demonstrates that the vertical velocity profile plotted in the $\widetilde{u}$–$z^{++}$ coordinates, i.e., corrected for the density field effect, agrees well with the profile calculated by the Prandtl formula.

Thus, we have shown that:

1) when the velocity profile obeys the Poiseuille equation the dimensionless pressure gradient in the flow $\dfrac{dp^+}{dx^+} = \dfrac{h^2_{\max}}{\mu u_{\max}}\dfrac{dp}{dx} = -2$, which allows determining the longitudinal pressure gradient $dp/dx$ from the known $h_{\max}$ and $u_{\max}$;

2) when the density field effect on the velocity field is taken into account by the method suggested in our paper, the vertical flow velocity distribution in a mixing layer with the stratified density can be calculated by the Prandtl formula.

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