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BRIEF COMMUNICATIONS

MODELING AND ANALYSIS OF THE ENERGY SPECTRA OF δ ELECTRONS GENERATED BY ULTRARELATIVISTIC HADRONS

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The authors present the results of mathematical modeling of an experiment designed to determine the parameters of the electromagnetic structure of hadrons by analyzing the energy spectra of δ electrons generated by the hadrons.

In [1] the authors describe a method for studying the electromagnetic structure of hadrons by analyzing the energy spectra of δ electrons generated by ultrarelativistic hadrons. The method allows determination of not only the rms charge radius r_0* but also a dimensionless factor M which defines the density of electric charge distribution in a hadron. The purpose of the analysis is to find the average energy $\overline{Q}$ and the average squared energy $\overline{Q^2}$ for the measured energy spectrum of δ electrons**.

If Lorentz factors γ of hadrons obey the condition $\gamma \gg 10^3$, the following relations can be obtained [1]:

$$\begin{aligned} &S_1 \sim \overline{Q}, \qquad S_2 \sim (\overline{Q^2})^{1/2}, \\ &S_1 = A_1(M) - \ln r_0, \\ &S_2 = A_2(M)/r_0. \end{aligned} \tag{1}$$

Here, the coefficients A_1 and A_2 do not depend on r_0 and are a linear function of M. Thus r_0 and M can be found with the help of (1) from the experimental values of $\overline{Q}$ and $\overline{Q^2}$.

Given below are the results of computations made to determine the required measurement accuracy for $\overline{Q}$ and $\overline{Q^2}$. These computations help relate the relative measurement errors of $\overline{Q}$, $\overline{Q^2}$, r_0, and M to the statistical data used for experiment support and to errors in measuring the energy Q.

The energy spectra of δ electrons were simulated by the Monte Carlo method. The differential section of δ electron generation was described by the expression valid for spinless relativistic hadrons:

$$d\sigma/dQ = C[Q^{-2} - Q^{-1}{Q^0_{\max}}^{-1}]G^2_E(Q), \tag{2}$$

where $G_E(Q)$ is the electrical form factor of a hadron, $Q^0_{\max}(\gamma)$ is the kinematic limit energy Q transferred to a δ electron, and C is a Q-independent factor. Calculations were made for $r_0 = 0.7$, three values of the Lorentz factor γ (10^5, 10^4, and 10^3), and several values of $Q_{\min}$, which is the minimum energy of δ electrons in a spectrum. The model factor M was selected equal to 0.38, which corresponded to the exponential distribution of an electric charge in a hadron [1]. The measurement accuracy level in the computations, α, varied from 0 to 4×10^{-2}. Two alternatives were considered. First, a systematic measurement error that yielded a negative or positive relative error α was assumed to occur. Second, a random perturbation was introduced in Q simulated according to (2). This perturbation was computed by the Gaussian distribution where α was relative fluctuation. In the mathematical experiment, arrays of values of $\overline{Q}$ and $\overline{Q^2}$ were created,

* In the text below, r_0 should be viewed as a dimensionless parameter which is numerically equal to the rms charge radius of a hadron expressed in fermis.

** Dimensionless energy Q can be conveniently introduced through normalizing to the double rest energy of an electron.

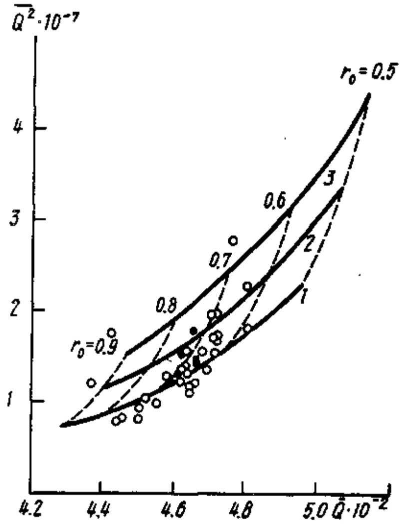

Fig. 1

Results of calculation by the Monte Carlo method for $\gamma = 10^5$, $r_0 = 0.7$, and $Q_{\min} = 60$; $N = 10^5$ (open circles) and 5×10^5 (filled circles). Curves 1, 2, and 3 describe different charge distributions in a hadron (see the text). The dashed lines show curves of $r_0 = \text{const}$.

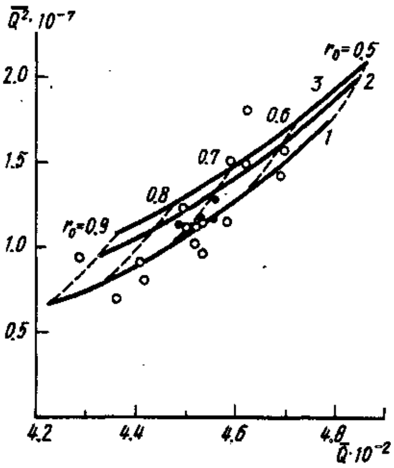

Fig. 2

Same as in Fig. 1 for $\gamma = 10^4$.

in which each pair of the above values resulted from averaging N independent simulations of Q. N was equal to 10^4, 10^5, and 5×10^5.

Figures 1 and 2 show elements of such arrays in the $\overline{Q}$ and $\overline{Q^2}$ coordinates. The open circles denote $N = 10^5$, and the filled circles stand for $N = 5 \times 10^5$. It can be seen that the scatter of the "experimental" points decreases with an increase in the size N of the statistical sample. The figures also show a curvilinear system of coordinates r_0, M, the value of r_0 ranging from 0.5 to 0.9. Curves 1, 2, and 3 represent, respectively, a uniform, an exponential, and a Yukawa-type [2] distribution of the electric charge.

It is evident from the figures that when $N \geqslant 10^5$ the computation points fall within the range of physically admissible values of r_0 and M. With $N \leqslant 10^4$ the scatter of the points is substantially larger than the "physical" range, therefore, the statistical sample size of $\sim 10^4$ is obviously insufficient for this experiment.

Note that while the distribution of $\overline{Q}$ is Gaussian (i.e., symmetric with respect to $\langle\overline{Q}\rangle$), the distribution of $\overline{Q^2}$ is noticeably asymmetric and has a tail toward larger values. For this reason, there may be considerable deviations of $\overline{Q^2}$ from the average value $\langle\overline{Q^2}\rangle$ in the experiment.

Table 1

γ	$\Delta\overline{Q}/\overline{Q}$	$\Delta\overline{Q^2}/\overline{Q^2}$	$\Delta r_0/r_0$	$\Delta M/M$
10^5	0.030	0.24	0.11	0.18
10^4	0.026	0.26	0.10	0.32
10^3	0.016	0.08	0.19	>1

Table 2

α	$\Delta\overline{Q}/\overline{Q}$	$\Delta\overline{Q^2}/\overline{Q^2}$	$\Delta r_0/r_0$	$\Delta M/M$
-2×10^{-2}	-0.02	-0.04	0.12	0.13
-4×10^{-2}	-0.04	-0.08	0.23	0.21

Relative errors in the determination of $\overline{Q}$, $\overline{Q^2}$, r_0, and M for $N = 10^5$ and $Q_{\min} = 60$ are presented in Table 1. The "physical" range of the parameters r_0 and M is reduced considerably with a decrease of the Lorentz factor γ. This, however, reduces the scatter of the "experimental" points. As a result, transition from $\gamma = 10^5$ to $\gamma = 10^3$ reveals the trend toward a decrease in the relative errors of $\overline{Q}$ and $\overline{Q^2}$ if the sample size N is fixed. This is accounted for by an increase in $Q^0_{\max}(\gamma)$ and, hence, in the role of the tail of differential section (2) with a growth of γ. However, the measurement errors for r_0 and M increase as γ decreases. Although each of these errors is a function of the errors of both $\overline{Q}$ and $\overline{Q^2}$, it might be assumed to a first approximation that

$$(\Delta r_0/r_0) = (\varkappa_{1r}r_0)^{-1}(\Delta\overline{Q}/\overline{Q}), \qquad (\Delta M/M) = (\varkappa_{2M}r_0)^{-1}(\Delta\overline{Q^2}/\overline{Q^2}), \tag{3}$$

where $\varkappa_{1r} = \partial\ln\overline{Q}/\partial r_0$ and $\varkappa_{2M} = \partial\ln\overline{Q^2}/\partial M$ [1].

It follows from [1] that $(\varkappa_{1r}r_0)^{-1} \approx 4$–$5$ and $(\varkappa_{2M}r_0)^{-1} \approx 0.8$ for $\gamma = 10^5$–10^4. Accordingly, for $\gamma = 10^3$ we have $(\varkappa_{1r}r_0)^{-1} \approx 15$ and $(\varkappa_{2M}r_0)^{-1} > 10$. As a result, the estimates of $\Delta r_0/r_0$ and $\Delta M/M$ obtained using formulas (3) are in satisfactory agreement with the results of Monte Carlo calculations (see Table 1). Thus, with N fixed, the determination accuracy of r_0 and M depends on the factors $\varkappa_{1r}(\gamma)$ and $\varkappa_{2M}(\gamma)$ that characterize the sensitivity of the method for a given γ. If $\gamma \leqslant 10^3$, as can be seen from Table 1, quantitative information on the parameter M cannot apparently be obtained.

Estimates of the measurement errors of $\overline{Q}$, $\overline{Q^2}$, r_0 and M for other sample sizes are obtainable by the rule $\sim (N)^{-1/2}$. The above results refer to unperturbed measurements of Q ($\alpha = 0$). Calculations show that random errors in measuring Q (of the Gaussian type) are insignificant. Thus, with $N = 10^5$ the value $(\Delta\overline{Q}/\overline{Q})$ is about 20 times as small as α, and $(\Delta\overline{Q^2}/\overline{Q^2}) \approx 0.2\alpha$. The systematic errors are

$$(\Delta\overline{Q}/\overline{Q})_s = \alpha, \qquad (\Delta\overline{Q^2}/\overline{Q^2})_s = 2\alpha,$$

where α may be either positive or negative (depending on the sign of ΔQ). Table 2 lists measurement errors of $\overline{Q}$, $\overline{Q^2}$, r_0, and M for the case of a systematic underestimation of the energy Q (α and ΔQ are negative). The calculation parameters were as follows: $r_0 = 0.7$, $Q_{\min} = 60$, $\gamma = 10^5$, and $N = 10^5$. Using relations (1) and considering the signs of the errors one can derive the following approximate relations:

$$(\Delta r_0/r_0) \approx -5(\Delta\overline{Q}/\overline{Q}),$$
$$(\Delta M/M) \approx -2(1-M)^{-1}(\Delta\overline{Q^2}/\overline{Q^2}). \quad (4)$$

The estimates obtained by (4) agree with the results of the calculations by the Monte Carlo method. It can be seen from (4) that a systematic underestimation (overestimation) of Q leads to an increase (decrease) in r_0 and M as it is tantamount to a faster (slower) reduction in the electrical form factor $G_E(Q)$ with increasing trasferred energy Q.

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