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DIFFRACTION BY DIELECTRIC DISCONTINUITIES IN A PARTIALLY DIELECTRIC-FILLED BELOW-CUTOFF WAVEGUIDE

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Diffraction of the wave H_{10} by the below-cutoff section of a partially dielectric-filled waveguide has been considered. The problem is solved in an exact electrodynamic statement. Mathematical modeling is done by the generalized scattering matrix method. Propagation constants are obtained by differentiation with respect to a parameter. The dependence of numerical modeling results on the number of modes is studied. The results of the numerical analysis and physical experiments are compared.

Problems of electromagnetic wave diffraction by discontinuities in regular waveguides have been extensively studied [1]; methods are available for two- and three-dimensional analysis with consideration of loss in the waveguide walls and at various discontinuities: dielectric, metallic, semiconductor, ferrite, or plasma inclusions of almost any shape. By contrast, no studies have yet been made on diffraction processes caused by discontinuities in below-cutoff waveguides, except for analysis of resonance phenomena in below-cutoff filters with the waveguide section fully filled with dielectric layers [2]. At the same time, effective compact filters and microwave control elements operating in the most promising millimetric wave band can be developed on the basis of dielectric and semiconductor inserts introduced in below-cutoff sections of a waveguide. Particularly interesting are optoelectronic modulators in a below-cutoff waveguide with semiconductor inserts partially filling the waveguide section [3].

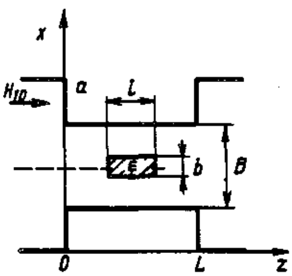

Fig. 1

Model structure: $a = 11$ mm, $B = 6.7$ mm, $b = 2$ mm, $L = 30$ mm, $l = 11$ mm, $\varepsilon = 9.83 + (i \times 0.13)$.

To study diffraction in such devices, we have decided to analyze the frequency response of a nonideal dielectric plate placed in a constriction of a rectangular waveguide parallel to the E plane (Fig. 1). The use of a single-mode approach is limited due to strong diffraction at the waveguide section discontinuities and scattering at the dielectric, therefore, consideration of higher-order modes is required.

Mathematical modeling was performed by the generalized scattering matrix method [4]. The device was represented by a combination of the following waveguide discontinuities: part of a waveguide with a step-like variation of the cross section in the H plane, a regular waveguide section, and part of a waveguide partially filled with a nonideal dielectric. A common approach was employed to obtaining scattering matrices for each of these discontinuities. In this case, the problem is plane (the plate fills the entire waveguide in the E plane, there are no cross section discontinuities in this direction) and, therefore, one can restrict oneself to setting only one magnetic Hertz vector Π_h directed parallel to the x axis along the wider waveguide wall and obeying the Helmholtz condition

$$\Delta \Pi_h + \gamma \Pi_h = 0 \tag{1}$$

in each domain. Naturally, the solution must also meet the boundary conditions (i) at the surface of the metal ($\mathbf{E}_t = 0$), (ii) at the interface of two media ($\mathbf{H}_t^1 = \mathbf{H}_t^2$, $\mathbf{E}_t^1 = \mathbf{E}_t^2$), and (iii) the radiation conditions. The electric and magnetic fields are defined thus:

$$\mathbf{E} = -i\omega \operatorname{rot} \Pi_h, \qquad \mathbf{H} = \operatorname{rot}\operatorname{rot} \Pi_h. \tag{2}$$

The Hertz vector in each domain can be represented as an expansion in terms of eigenfunctions $\Psi_{\nu m}(x)$:

$$\Pi_h^{\nu\pm} = \sum_{m=1}^{\infty} A_m^{\nu\pm} T_m^{\nu} \Psi_{\nu m}(x) \exp\{\mp i\gamma_m z\}. \tag{3}$$

Here, the superscript ν denotes the number of the domain in a given discontinuity, m is the number of the eigenfunction, and the signs $\pm$ correspond to waves traveling in a positive (+) or a negative (−) fundamental direction along the z axis. The coefficient $A_m^{\nu\pm}$ is the amplitude of the m-th fundamental wave in the ν-th domain, T_m^{ν} is a multiplier obtained from the incident power normalization condition. When the input and the output planes of the system have the same cross section, T_m^{ν} can be put equal to unity without impairing the determination of scattering matrix elements.

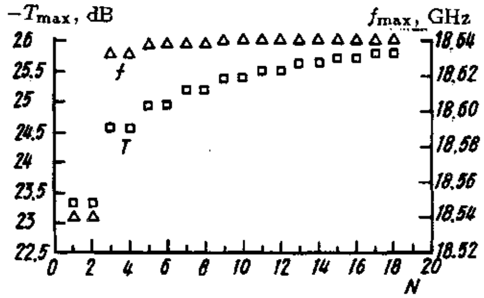

Fig. 2

Transmission at resonance (T_{max}) and resonance frequency (f_{max}) versus number of modes considered.

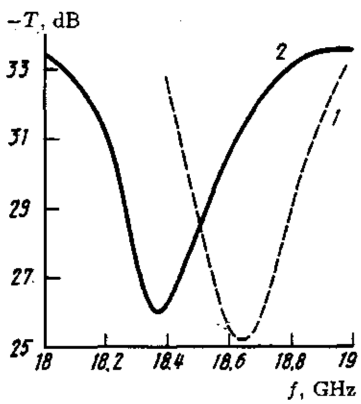

Fig. 3

Comparison of mathematical modeling (1) and experimental (2) results.

Substitution of series (3) into (2), and then into the boundary conditions on a plane normal to the waveguide axis, gives a system of linear equations relative to $A_m^{\nu\pm}$. The boundary conditions on a plane parallel to the waveguide axis (discontinuity in the form of a dielectric plate) are used for obtaining a dispersion equation and eigenfunctions of a given discontinuity. Where the boundaries coincide with the coordinate axes (this is our case) a scattering matrix S can be obtained analytically as an expression for infinite-dimensional matrices [5] the truncation of which occurs as early as at the computation stage rather than in the course of modeling.

To find a scattering matrix of a partial-filling domain, it is required to calculate the propagation constants of the fundamental waves, which are the LE_{n0} waves for the plate in the E plane. We shall use differentiation with respect to a parameter, which allows one to take exactly into account the complex permittivity of the filling [6]. The dispersion equation of LE waves for a dielectric plate in the E plane of a rectangular waveguide is known [7]. We write this equation as

$$F(\chi, t) = 0, \tag{4}$$

where $\chi = \gamma^2$ is the squared propagation constant. The parameter t will be the thickness of the dielectric plate. Considering (4) as an equation for the implicit function $\chi(t)$, we find the derivative $d\chi/dt \equiv f(\chi, t)$. As a result, we obtain an ordinary differential equation with the initial condition

$$\begin{cases} d\chi/dt = f(\chi, t), \\ \chi|_{t=0} = \chi_0, \end{cases} \tag{5}$$

where $\chi_0 = k_0^2\varepsilon - (m\pi/a)^2$ is the squared propagation constant of a uniformly filled waveguide without a plate. Cauchy's problem (5) was solved numerically by the Runge-Kutta method. No more than 50 points of integration with respect to t ($M = 50$) were required to provide convergence of solutions in the worst case of a plate filling almost entirely the cross section of the waveguide. The typical number of points that ensured an accuracy not worse than 0.1% was $M = 10$.

To verify the correctness of the numerical result, the convergence of the algorithm was studied on a model structure (Fig. 1). The dependence of the position of the resonance peak $f_{\max}$ and transmission at the resonance $T_{\max}$ on the number of modes considered was investigated (Fig. 2). It can be seen that the single-mode approximation is applicable to estimating the resonance frequency only, but it should not be used to calculate transmission (the difference in $T_{\max}$ for $N = 1$ and $N = 20$ was 10%). It is sufficient to take into account 10 to 15 modes in the field expansion to obtain a correct result.

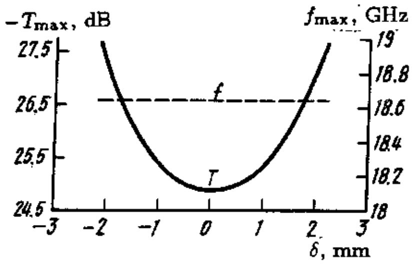

Fig. 4

Transmission at resonance ($T_{\max}$) and resonance frequency ($f_{\max}$) versus shift of the below-cutoff region with respect to the waveguide axis.

The results of the numerical analysis were compared against those of a physical experiment. Experimental data were obtained using a panoramic VSWR meter in a 8-mm band. Nine modes were considered in the field expansion. The results of the numerical modeling and the physical experiment are given in Fig. 3. It can be seen that the shape of the curve is basically correct. A comparatively small error (about 1.5%) in the determination of the center frequency is due to an inaccurate measurement of the real part of the permittivity ε' (±2%) used in the calculations, and a frequency measurement error in the experiment (±0.5%).

The partial fill of the waveguide in the experiment was a silicon insert of high absorptivity ($\varepsilon'' = 0.13$). Obviously, numerical modeling yields a correct result despite high loss in the dielectric. For practical applications, a higher transmission at maximum resonance is required. This can be achieved by using lower-absorptivity semiconductors.

We have also studied the relationship of resonance parameters versus the geometric dimensions of the filter. An interesting example is provided by the dependence of $T_{\max}$ and $f_{\max}$ on the shift δ of the below-cutoff region relative to the axis of a regular waveguide (Fig. 4). As δ increases, the resonance transmission drops due to weakening of the electrodynamic coupling between the below-cutoff region and the regular waveguide. The resonance frequency is independent of δ. This can be explained by the fact that the phase φ of electromagnetic wave reflection from the interface of the below-cutoff and the regular waveguides defined by the formula

$$\tan\varphi = 2(\operatorname{Re} Z_0 \operatorname{Im} Z_H - \operatorname{Re} Z_H \operatorname{Im} Z_0)/(|Z_H^2| - |Z_0^2|)$$

depends on the impedance of the below-cutoff section (Z_0) and that of the regular waveguide (Z_H) but does not depend on the relative positions of the waveguides.

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