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Vestnik Moskovskogo
Universiteta. Fizika
UDC 530.12:550.831

THEORY OF GRAVITATIONAL FIELD

A. A. Logunov

Within the framework of special relativity theory, based on the geometrization principle, fundamental equations are derived for gravitational field, which necessarily involve the graviton mass. According to these equations, a homogeneous and isotropic Universe develops cyclically and can only be "planar." The existence of a large "latent" mass of substance in this Universe is predicted. The theory excludes the existence of "black holes."

INTRODUCTION

Einstein's general relativity theory (GRT), whose fundamental equations were constructed by Einstein and Hilbert in 1915, opened up a new stage in the study of gravitation phenomena. However, almost from the very beginning, along with achievements, this theory has also encountered conceptual difficulties in the determination of physical characteristics of the gravitational field and, as a consequence, in the statement of energy-momentum conservation laws.

As will be seen below, it is possible to combine Poincaré's idea of gravitational field [1] (as a physical field of the Faraday–Maxwell type) and Einstein's idea of Riemann geometry of space-time. This can be done within the framework of special relativity theory, which describes phenomena in both inertial and noninertial reference frames, if one uses the geometrization principle reflecting the universality of the field-substance gravitational interaction and introduces a graviton mass. This approach led to relativistic gravitation theory (RGT) [2] possessing all conservation laws as do all the other physical theories.

1. GENERAL PRINCIPLES OF RGT

On passing to the construction of the theory of gravitational field we shall proceed from the following basic principles.

1. RGT is based on special relativity theory, and therefore the Minkowski space (the pseudo-Euclidean geometry of space-time) is the fundamental space for all physical fields, including the gravitational field. This principle is necessary and sufficient for both the energy-momentum and angular momentum conservation laws to hold for substance and gravitational field taken together. In other words, the Minkowski space reflects the dynamic properties common to all forms of matter, and, consequently, there exist unified physical characteristics that make it possible to describe quantitatively the transformation of forms of matter into one another. The Minkowski space cannot be regarded as a priori existing because it reflects properties of matter and hence is inseparable from it. This space has a profound physical significance because it determines the universal properties of matter, such as energy, momentum, and angular momentum.

Gravitational field is described by a symmetric tensor $\Phi^{\mu\nu}$ of the second rank and is a real physical field having energy-momentum density, rest mass m, and polarization states corresponding to spins 2 and 0 by virtue of the equations

$$D_\mu \Phi^{\mu\nu} = 0, \tag{1}$$

where D_μ is the covariant derivative in the Minkowski space.

Besides the exclusion of nonphysical field states, Eq. (1) introduces the metric tensor $\gamma_{\mu\nu}$ of the Minkowski space in the theory, which permits separation of inertia forces from forces due to the action of gravitational field. The choice of the diagonal metric $\gamma_{\mu\nu}$ makes it possible to completely exclude the action of inertia forces. The metric of the Minkowski space allows one to introduce the notion of length and time standards in the absence of gravitational field. As will be seen later, the interaction between the tensor gravitational field and the substance can be introduced in such a way as to sort of deform the Minkowski space, changing the metric properties without violating the causality principle.

2. Because the gravitational field is described by the symmetric second-rank tensor $\Phi^{\mu\nu}$ and its interaction with other fields can be considered universal, there arises a unique possibility to "connect" this field directly to the tensor $\gamma^{\mu\nu}$ in the substance Lagrangian density according to the rule

$$L_M(\tilde{\gamma}^{\mu\nu}, \Phi_A) \rightarrow L_M(\tilde{g}^{\mu\nu}, \Phi_A), \tag{2}$$

where

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\Phi}^{\mu\nu}, \quad \tilde{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}, \quad \tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\, \gamma^{\mu\nu}, \quad \tilde{\Phi}^{\mu\nu} = \sqrt{-\gamma}\, \Phi^{\mu\nu}, \tag{3}$$

Φ_A are the substance fields, $g = \det g_{\mu\nu}$, $\gamma = \det \gamma_{\mu\nu}$, and $\tilde{g}^{\mu\nu}\tilde{g}_{\nu\sigma} = \delta^{\mu}_{\sigma}$. The raising and lowering of indices for the tensor $\Phi^{\mu\nu}$ is achieved with the aid of $\gamma_{\mu\nu}$, and for the tensor $g^{\mu\nu}$ this is done using the metric tensor of the Riemann space. By substance we mean all forms of matter with the exception of the gravitational field.

This kind of interaction between the gravitational field and the substance leads to the concept of an effective Riemann space, where motion of substance takes place, and to the geometrization principle. According to this principle, the substance motion under the action of the gravitational field $\Phi^{\mu\nu}$ in the Minkowski space with the metric tensor $\gamma_{\mu\nu}$ is identical to its motion in the effective Riemann space with the metric $g_{\mu\nu}$. In this approach, the effective Riemann space is of field origin due to the presence of the gravitational field $\Phi^{\mu\nu}$. Because the metric properties are determined by the tensor of the effective Riemann space in the presence of gravitational field and by the tensor $\gamma_{\mu\nu}$ of the Minkowski space in its absence, the RGT can give an answer to the question of how the dimensions of the body and the clock rate vary under the action of the gravitational field. A theory where the tensor $\gamma_{\mu\nu}$ is not involved in the field equations cannot, in principle, answer such questions.

In GRT the gravitational field is characterized by the metric tensor $g_{\mu\nu}$, whereas in our theory it is determined by the tensor quantity $\Phi^{\mu\nu}$, and the effective Riemann space is constructed using the field $\Phi^{\mu\nu}$ and also the metric tensor $\gamma^{\mu\nu}$ of the Minkowski space that fixes a definite choice of the coordinate system.

In our theory there exist Galilean (inertial) coordinate systems, and therefore acceleration is of absolute character. A test body moves in the effective Riemann space along a geodesic line of the space, but the motion is not free because it is caused by gravitational field. If the test body were charged, it would irradiate electromagnetic waves because, in the general case, its motion in the field would have a variable acceleration. Since the effective Riemann space is created by the gravitational field $\Phi^{\mu\nu}$ located in the Minkowski space, it can always be set (and this is very important) in a single coordinate system. This means that we shall deal only with Riemann spaces that are defined on a single chart. From this point of view, Riemann spaces with complex topology are completely excluded because they are not of field origin.

It should be noted that since the substance motion occurs in the effective Riemann space, the equations of this motion do not involve the metric tensor $\gamma_{\mu\nu}$ of the Minkowski space. The Minkowski space affects the substance motion only via the metric tensor $g_{\mu\nu}$ of the Riemann space, which is determined from equations that contain the metric tensor $\gamma_{\mu\nu}$.

2. GAUGE TRANSFORMATION GROUP

Since the substance Lagrangian density has the form

$$L_M(\tilde{g}^{\mu\nu}, \Phi_A), \tag{4}$$

it is easy to find a group of gauge transformations under which the variation of the substance Lagrangian density can only be a 4-divergence expression. To this end we use the invariance of action:

$$S_M = \int L_M(\tilde{g}^{\mu\nu}, \Phi_A)\, d^4x \tag{5}$$

under an arbitrary infinitesimal variation of the coordinates

$$x'^{\alpha} = x^{\alpha} + \xi^{\alpha}(x), \tag{6}$$

where ξ^α is an infinitesimal displacement 4-vector.

Under these coordinate transformations the field functions $\tilde{g}^{\mu\nu}$ and Φ_A change in the following way:

$$\begin{aligned}\tilde{g}'^{\mu\nu}(x') &= \tilde{g}^{\mu\nu}(x) + \delta_\xi \tilde{g}^{\mu\nu}(x) + \xi^\alpha(x) D_\alpha \tilde{g}^{\mu\nu}(x),\\ \Phi'_A(x') &= \Phi_A(x) + \delta_\xi \Phi_A(x) + \xi^\alpha(x) D_\alpha \Phi_A(x),\end{aligned} \tag{7}$$

where the expressions

$$\begin{aligned}\delta_\xi \tilde{g}^{\mu\nu}(x) &= \tilde{g}^{\mu\alpha} D_\alpha \xi^\nu(x) + \tilde{g}^{\nu\alpha} D_\alpha \xi^\mu(x) - D_\alpha(\xi^\alpha \tilde{g}^{\mu\nu})\\ \delta_\xi \Phi_A(x) &= -\xi^\alpha(x) D_\alpha \Phi_A(x) + F^{B;\alpha}_{A;\beta} \Phi_B(x) D_\alpha \xi^\beta(x)\end{aligned} \tag{8}$$

are Lie variations.

The operators δ_ξ satisfy the conditions of Lie algebra, i. e., the commutation relation

$$[\delta_{\xi_1}, \delta_{\xi_2}](\cdot) = \delta_{\xi_3}(\cdot), \tag{9}$$

and the Jacobi identity

$$[\delta_{\xi_1}, [\delta_{\xi_2}, \delta_{\xi_3}]] + [\delta_{\xi_3}, [\delta_{\xi_1}, \delta_{\xi_2}]] + [\delta_{\xi_2}, [\delta_{\xi_3}, \delta_{\xi_1}]] = 0,$$

where

$$\xi_3^\nu = \xi_1^\mu D_\mu \xi_2^\nu - \xi_2^\mu D_\mu \xi_1^\nu = \xi_1^\mu \partial_\mu \xi_2^\nu - \xi_2^\mu \partial_\mu \xi_1^\nu. \tag{10}$$

For relation (9) to hold it is necessary that the following conditions be fulfilled:

$$F^{B;\mu}_{A;\nu} F^{C;\alpha}_{B;\beta} - F^{B;\alpha}_{A;\beta} F^{C;\mu}_{B;\nu} = f^{\mu\alpha;\tau}_{\nu\beta;\sigma} F^{C;\sigma}_{A;\tau}, \tag{11}$$

where the structural constants f are given by the formula

$$f^{\mu\alpha;\tau}_{\nu\beta;\sigma} = \delta^\mu_\beta \delta^\alpha_\sigma \delta^\tau_\nu - \delta^\alpha_\nu \delta^\mu_\sigma \delta^\tau_\beta. \tag{12}$$

It can easily be shown that they satisfy the Jacobi identity:

$$f^{\alpha\nu;\sigma}_{\beta\mu;\tau} f^{\tau\rho;\omega}_{\sigma\epsilon;s} + f^{\nu\rho;\sigma}_{\mu\epsilon;\tau} f^{\tau\alpha;\omega}_{\sigma\beta;s} + f^{\rho\alpha;\sigma}_{\epsilon\beta;\tau} f^{\tau\nu;\omega}_{\sigma\mu;s} = 0 \tag{13}$$

and possess the property of antisymmetry:

$$f^{\alpha\nu;\rho}_{\beta\mu;\sigma} = -f^{\nu\alpha;\rho}_{\mu\beta;\sigma}.$$

Under the coordinate transformation (6) the variation of action is equal to zero:

$$\delta_C S_M = \int\limits_{\Omega'} L'_M(x')\, d^4x' - \int\limits_{\Omega} L_M(x)\, d^4x = 0. \tag{14}$$

The first integral in (14) can be written as

$$\int\limits_{\Omega'} L'_M(x')\, d^4x' = \int\limits_{\Omega} J L'_M(x')\, d^4x,$$

where

$$J = \det\left(\frac{\partial x'^{\alpha}}{\partial x^{\beta}}\right).$$

In the first order with respect to ξ^{α} the determinant J has the form

$$J = 1 + \partial_{\alpha}\xi^{\alpha}(x). \tag{15}$$

In view of the expansion

$$L'_M(x') = L'_M(x) + \xi^{\alpha}(x)\frac{\partial I_M}{\partial x^{\alpha}},$$

and expression (15), the variation can be represented in the form

$$\delta_C S_M = \int\limits_{\Omega} [\delta L_M(x) + \partial_{\alpha}(\xi^{\alpha} L_M(x))] d^4x = 0.$$

Since the volume of the region of integration Ω is arbitrary, we have the identity

$$\delta L_M(x) = -\partial_{\alpha}(\xi^{\alpha}(x) L_M(x)), \tag{16}$$

where the Lie variation δL_M is expressed as

$$\delta L_M(x) = \frac{\partial L_M}{\partial \tilde{g}^{\mu\nu}}\delta\tilde{g}^{\mu\nu} + \frac{\partial L_M}{\partial(\partial_{\alpha}\tilde{g}^{\mu\nu})}\delta(\partial_{\alpha}\tilde{g}^{\mu\nu}) + \frac{\partial L_M}{\partial \Phi_A}\delta\Phi_A + \frac{\partial L_M}{\partial(\partial_{\alpha}\Phi_A)}\delta(\partial_{\alpha}\Phi_A). \tag{17}$$

In particular, it follows that if the scalar density depends on $\tilde{g}^{\mu\nu}$ and the derivatives $\partial_{\alpha}\tilde{g}^{\mu\nu}$ solely, then after transformation (8) its variation can only be a 4-divergence expression:

$$\delta L(\tilde{g}^{\mu\nu}(x)) = -\partial_{\alpha}\big(\xi^{\alpha}(x) L(\tilde{g}^{\mu\nu}(x))\big), \tag{16a}$$

where the Lie variation δL is given by formula

$$\delta L(\tilde{g}^{\mu\nu}(x)) = \frac{\partial L}{\partial \tilde{g}^{\mu\nu}}\delta\tilde{g}^{\mu\nu} + \frac{\partial L}{\partial(\partial_{\alpha}\tilde{g}^{\mu\nu})}\delta(\partial_{\alpha}\tilde{g}^{\mu\nu}). \tag{17a}$$

Lie variations (8) were found as a consequence of coordinate transformations (6). However, this can also be considered from another viewpoint when (8) can be regarded as gauge transformations. In this case the arbitrary infinitesimal 4-vector $\xi^{\alpha}(x)$ is a gauge vector and not a coordinate displacement vector. In what follows, to stress the distinction between the gauge group and the coordinate transformation group we shall denote the group parameter as $\varepsilon^{\alpha}(x)$, and the transformation of field functions

$$\begin{aligned} &\tilde{g}^{\mu\nu}(x) \to \tilde{g}^{\mu\nu}(x) + \delta\tilde{g}^{\mu\nu}(x), \\ &\Phi_A(x) \to \Phi_A(x) + \delta\Phi_A(x) \end{aligned} \tag{18}$$

with increments

$$\begin{aligned} &\delta_{\varepsilon}\tilde{g}^{\mu\nu}(x) = \tilde{g}^{\mu\alpha} D_{\alpha}\varepsilon^{\nu}(x) + \tilde{g}^{\nu\alpha} D_{\alpha}\varepsilon^{\mu}(x) - D_{\alpha}(\varepsilon^{\alpha}\tilde{g}^{\mu\nu}), \\ &\delta_{\varepsilon}\Phi_A(x) = -\varepsilon^{\alpha}(x) D_{\alpha}\Phi_A(x) + F^{B;\alpha}_{A;\beta}\Phi_B(x) D_{\alpha}\varepsilon^{\beta}(x) \end{aligned} \tag{19}$$

will be called gauge transformations.

Fully in accordance with formulas (9) and (10), the operators satisfy the conditions of the same Lie algebra, i. e., the commutation relation

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}](\cdot) = \delta_{\varepsilon_3}(\cdot), \tag{20}$$

and the Jacobi identity

$$[\delta_{\varepsilon_1}, [\delta_{\varepsilon_2}, \delta_{\varepsilon_3}]] + [\delta_{\varepsilon_3}, [\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]] + [\delta_{\varepsilon_2}, [\delta_{\varepsilon_3}, \delta_{\varepsilon_1}]] = 0. \tag{21}$$

Here, by analogy with (10), we have

$$\varepsilon_3^\nu = \varepsilon_1^\mu D_\mu \varepsilon_2^\nu - \varepsilon_2^\mu D_\mu \varepsilon_1^\nu = \varepsilon_1^\mu \partial_\mu \varepsilon_2^\nu - \varepsilon_2^\mu \partial_\mu \varepsilon_1^\nu.$$

The gauge group has arisen from the geometrized structure of the scalar Lagrangian density $L_M(\tilde{g}^{\mu\nu}, \Phi_A)$ of the substance, and, by virtue of identity (16), its variation under gauge transformations (19) can only be a divergence expression.

Thus, the geometrization principle which determined the universal character of interaction between substance and gravitational field enabled us to form the non-commutative infinite-dimensional gauge group (19).

The essential distinction between the gauge and coordinate transformations will manifest itself at the key point of the theory in the construction of scalar density for the Lagrangian of the gravitational field proper. The distinction is due to the fact that under a gauge transformation the metric tensor $\gamma_{\mu\nu}$ does not change, and consequently, by virtue of (3), we have

$$\delta_\varepsilon \tilde{g}^{\mu\nu}(x) = \delta_\varepsilon \tilde{\Phi}^{\mu\nu}(x).$$

Based on (19), the transformation for the field is derived:

$$\delta_\varepsilon \tilde{\Phi}^{\mu\nu}(x) = \tilde{g}^{\mu\alpha} D_\alpha \varepsilon^\nu(x) + \tilde{g}^{\nu\alpha} D_\alpha \varepsilon^\mu(x) - D_\alpha(\varepsilon^\alpha \tilde{g}^{\mu\nu}),$$

but it substantially differs from the field transformation under coordinate displacement:

$$\delta_\xi \tilde{\Phi}^{\mu\nu}(x) = \tilde{\Phi}^{\mu\alpha} D_\alpha \xi^\nu(x) + \tilde{\Phi}^{\nu\alpha} D_\alpha \xi^\mu(x) - D_\alpha(\xi^\alpha \tilde{\Phi}^{\mu\nu}).$$

Under gauge transformations (19) the equations of substance motion do not change because for any such transformation the variation of the substance Lagrangian density is a divergence expression.

3. THE LAGRANGIAN DENSITY AND EQUATIONS OF MOTION FOR THE GRAVITATIONAL FIELD PROPER

As is known, using the tensor $g_{\mu\nu}$ alone it is impossible to construct the scalar Lagrangian density of the gravitational field proper relative to arbitrary coordinate transformations as a quadratic form in derivatives of the order no higher than first. Therefore, along with the metric $g_{\mu\nu}$, this Lagrangian density must also necessarily include the metric $\gamma_{\mu\nu}$. However, under gauge transformation (19) the metric $\gamma_{\mu\nu}$ does not change. Consequently, for the variation of the Lagrangian density of the gravitational field proper under this transformation to be a divergence expression some strong constraints must be imposed on its structure. It is this point where a fundamental distinction between the gauge and the coordinate transformations arises. Whereas the coordinate transformations impose practically no constraints on the structure of the scalar Lagrangian density of the gravitational field proper, the gauge transformations make it possible to find the Lagrangian density. A direct general method of constructing the Lagrangian is described in [2].

Here we select a simpler method for constructing the Lagrangian. Based on (16a), we conclude that the simplest scalar densities $\sqrt{-g}$ and $\tilde{R} = \sqrt{-g}R$, where R is the scalar curvature of the effective Riemann space, change under gauge transformation (19) in the following way:

$$\sqrt{-g} \to \sqrt{-g} - D_\nu(\varepsilon^\nu \sqrt{-g}), \tag{22}$$

$$\tilde{R} \to \tilde{R} - D_\nu(\varepsilon^\nu \tilde{R}). \tag{23}$$

The scalar density $\tilde{R}$ is expressed in terms of the Christoffel symbols

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \tag{24}$$

as

$$\tilde{R} = -\tilde{g}^{\mu\nu}(\Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\lambda\sigma} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\lambda}) - \partial_{\nu}(\tilde{g}^{\mu\nu}\Gamma^{\sigma}_{\mu\sigma} - \tilde{g}^{\mu\sigma}\Gamma^{\nu}_{\mu\sigma}). \tag{25}$$

Since the Christoffel symbols are not tensor quantities, each term in (25) is not a scalar density. However, the introduction of the tensor quantity $G^{\lambda}_{\mu\nu}$

$$G^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}(D_{\mu}g_{\sigma\nu} + D_{\nu}g_{\sigma\mu} - D_{\sigma}g_{\mu\nu}) \tag{26}$$

makes it possible to write the scalar density in the form

$$\tilde{R} = -\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda}) - D_{\nu}(\tilde{g}^{\mu\nu}G^{\sigma}_{\mu\sigma} - \tilde{g}^{\mu\sigma}G^{\nu}_{\mu\sigma}). \tag{27}$$

We note that in (27) each individual term behaves under arbitrary coordinate transformations as a scalar density. In view of (22) and (23) we conclude that the variation of the expression

$$\lambda_1(\tilde{R} + D_{\nu}Q^{\nu}) + \lambda_2\sqrt{-g} \tag{28}$$

under arbitrary gauge transformations can only be a divergence expression. Taking the vector density Q^{ν} in the form

$$Q^{\nu} = \tilde{g}^{\mu\nu}G^{\sigma}_{\mu\sigma} - \tilde{g}^{\mu\sigma}G^{\nu}_{\mu\sigma},$$

we eliminate in (28) the terms with higher-than-first-order derivatives and obtain the following Lagrangian density:

$$-\lambda_1\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\lambda\nu}) + \lambda_2\sqrt{-g}. \tag{29}$$

It is thus seen that the requirement that the variation of the Lagrangian density of the gravitational field proper under gauge transformation (19) be a divergence expression uniquely determines the structure of Lagrangian density (29). However, if we confine ourselves only to this density, then the equations of the gravitational field will be gauge invariant, and the metric $\gamma_{\mu\nu}$ of the Minkowski space will no longer be contained in the system of equations determined by Lagrangian density (29). Because this approach involves no metric $\gamma_{\mu\nu}$ of the Minkowski space, it becomes impossible to represent the gravitational field (as a physical field of the Faraday–Maxwell type) in the Minkowski space. For Lagrangian density (29) the introduction of the metric $\gamma_{\mu\nu}$ by means of Eqs. (1) provides no way out because the physical quantities — the interval and the curvature tensor of the Riemann space and also the tensor $t^{\mu\nu}_{g}$ of the gravitational field will depend on the choice of the gauge, which is physically inadmissible.

To preserve the concept of field in the Minkowski space and to exclude the ambiguity it is necessary to supplement the Lagrangian density of gravitational field with a term violating the gauge group. At first glance it may seem that there is considerable arbitrariness in the choice of the Lagrangian density of the gravitational filed because the group can be violated in many diverse ways. However, this is not so because, as a result of the physical requirement imposed by Eqs. (1) on the polarization properties of the gravitational filed as a field with spins 2 and 0, the term that violates group (19) must be chosen so that Eqs. (1) become consequences of the system of equations for the gravitational and substance fields, since only in this case we will not have an overdetermined system of differential equations.

To this end we introduce the term

$$\gamma_{\mu\nu}\tilde{g}^{\mu\nu} \tag{30}$$

in the scalar Lagrangian density of the gravitational field; its variation under conditions (1) and transformations (19) is also a divergence expression but only on the class of vectors satisfying the condition

$$g^{\mu\nu}D_{\mu}D_{\nu}\varepsilon^{\sigma}(x) = 0. \tag{31}$$

An almost similar situation takes place in electrodynamics with a nonzero photon rest mass. By virtue of (28)–(30), we can write the expression

$$L_g = -\lambda_1\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda}) + \lambda_2\sqrt{-g} + \lambda_3\gamma_{\mu\nu}\tilde{g}^{\mu\nu} + \lambda_4\sqrt{-\gamma} \tag{32}$$

for the general scalar Lagrangian density. The last constant term in (32) has been introduced to make the Lagrangian density vanish in the absence of the gravitational field. The narrowing of the class of gauge vectors due to the introduction of term (30) automatically makes Eqs. (1) consequences of gravitational field equations. This will be shown directly in what follows.

According to the principle of least action, the equations for the gravitational field proper have the form

$$\frac{\delta L_g}{\delta \tilde{g}^{\mu\nu}} = \lambda_1 R_{\mu\nu} + \frac{1}{2}\lambda_2 g_{\mu\nu} + \lambda_3 \gamma_{\mu\nu} = 0, \tag{33}$$

where $R_{\mu\nu}$ is the Ricci tensor, which we write in the form

$$R_{\mu\nu} = D_\lambda G^\lambda_{\mu\nu} - D_\mu G^\lambda_{\nu\lambda} + G^\sigma_{\mu\nu} G^\lambda_{\sigma\lambda} - G^\sigma_{\mu\lambda} G^\lambda_{\nu\sigma}. \tag{34}$$

Because in the absence of the gravitational field Eqs. (33) must be satisfied identically, we have

$$\lambda_2 = -2\lambda_3. \tag{35}$$

We now find the density of the energy-momentum tensor of the gravitational field in the Minkowski space:

$$t_g^{\mu\nu} = -2\frac{\delta L_g}{\delta \gamma_{\mu\nu}} = 2\sqrt{-\gamma}\Big(\gamma^{\mu\alpha}\gamma^{\nu\beta} - \frac{1}{2}\gamma^{\mu\nu}\gamma^{\alpha\beta}\Big)\frac{\delta L_g}{\delta \tilde{g}^{\alpha\beta}} + \lambda_1 J^{\mu\nu} - 2\lambda_3 \tilde{g}^{\mu\nu} - \lambda_4 \tilde{\gamma}^{\mu\nu}, \tag{36}$$

where

$$J^{\mu\nu} = D_\alpha D_\beta(\gamma^{\alpha\mu}\tilde{g}^{\beta\nu} + \gamma^{\nu\alpha}\tilde{g}^{\beta\mu} - \gamma^{\alpha\beta}\tilde{g}^{\mu\nu} - \gamma^{\mu\nu}\tilde{g}^{\alpha\beta}). \tag{37}$$

If the dynamic equations (33) are taken into account in expression (36), then we obtain the equations for the gravitational field proper in the form

$$\lambda_1 J^{\mu\nu} - 2\lambda_3 \tilde{g}^{\mu\nu} - \lambda_4 \tilde{\gamma}^{\mu\nu} = t_g^{\mu\nu}. \tag{38}$$

For Eq. (38) to be satisfied identically in the absence of gravitational field it is necessary to set

$$\lambda_4 = -2\lambda_3. \tag{39}$$

Since for the gravitational field proper the relation

$$D_\mu t_g^{\mu\nu} = 0 \tag{40}$$

always holds, Eq. (38) implies that

$$D_\mu \tilde{g}^{\mu\nu} = 0. \tag{41}$$

Thus, Eqs. (1) determining the polarization states of the field directly follow from Eqs. (38). In view of Eqs. (41), field equations (38) can be written as

$$\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\Phi}^{\mu\nu} - \frac{\lambda_4}{\lambda_1}\tilde{\Phi}^{\mu\nu} = -\frac{1}{\lambda_1} t_g^{\mu\nu}. \tag{42}$$

In the Galilean coordinates this equation has a simple form:

$$\Box \tilde{\Phi}^{\mu\nu} - \frac{\lambda_4}{\lambda_1}\tilde{\Phi}^{\mu\nu} = -\frac{1}{\lambda_1} t_g^{\mu\nu}. \tag{43}$$

It appears natural to interpret the factor $-\lambda_4/\lambda_1$ as the square of the graviton mass, m^2, and, according to the principle of correspondence, $-1/\lambda_1$ must be equated to 16π. Hence, all the unknown constants entering the Lagrangian density are determined:

$$\lambda_1 = -\frac{1}{16\pi}, \quad \lambda_2 = \lambda_4 = -2\lambda_3 = \frac{m^2}{16\pi}. \tag{44}$$

The constructed scalar Lagrangian density of the gravitational field proper has the form

$$L_g = \frac{1}{16\pi}\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda}) - \frac{m^2}{16\pi}\Big(\frac{1}{2}\gamma_{\mu\nu}\tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma}\Big). \tag{45}$$

The corresponding dynamic equations for the gravitational field proper can be written as

$$J^{\mu\nu} - m^2\tilde{\Phi}^{\mu\nu} = -16\pi t^{\mu\nu}_g \tag{46}$$

or

$$R^{\mu\nu} - \frac{m^2}{2}(g^{\mu\nu} - g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}) = 0. \tag{47}$$

These equations substantially restrict the class of gauge transformations and retain only the trivial ones that satisfy the Killing conditions. Such transformations are consequences of the Lorentz invariance and are present in any theory.

The above Lagrangian density leads to Eqs. (47), which imply Eqs. (41), and therefore outside the substance we have ten equations for ten unknown field functions. Using Eqs. (41), the unknown field functions $\Phi^{0\alpha}$can be readily expressed in terms of the field functions Φ^{ik}, where the indices i and k run over the values 1, 2, and 3. Thus, in the Lagrangian density of the gravitational field proper the structure of the mass term violating the gauge group is uniquely determined by the polarization properties of the gravitational field.

4. EQUATIONS OF MOTION FOR GRAVITATIONAL FIELD AND SUBSTANCE

The total Lagrangian density of substance and gravitational field is expressed as

$$L = L_g + L_M(\tilde{g}^{\mu\nu}, \Phi_A), \tag{48}$$

where L_g is defined by (45).

Based on (48), applying the principle of least action, we derive a complete system of equations for substance and gravitational field:

$$\frac{\delta L}{\delta \tilde{g}^{\mu\nu}} = 0, \tag{49}$$

$$\frac{\delta L_M}{\delta \Phi_A} = 0. \tag{50}$$

Since with an arbitrary infinitesimal change of the coordinates the variation δS_M of action is zero, we have

$$\delta S_M = \delta \int L_M(\tilde{g}^{\mu\nu}, \Phi_A)\, d^4x = 0,$$

whence an identity is obtained in the form [2]

$$g_{\mu\nu}\nabla_\lambda T^{\lambda\nu} = -D_\nu\Big(\frac{\delta L_M}{\delta\Phi_A}F^{B;\nu}_{A;\mu}\Phi_B(x)\Big) - \frac{\delta L_M}{\delta\Phi_A}D_\mu\Phi_A(x). \tag{51}$$

Here $T^{\lambda\nu} = -2\delta L_M/\delta g_{\lambda\nu}$ is the substance tensor in the Riemann space and ∇_λ is the covariant derivative in this space with the metric $g_{\lambda\nu}$.

Identity (51) implies that if equations of substance motion (50) are satisfied, then we have

$$\nabla_\lambda T^{\lambda\nu} = 0. \tag{52}$$

When the number of Eqs. (50) for substance is equal to four, they can be replaced by the equivalent Eqs. (52). As in what follows we shall deal only with these equations for substance, we shall always use equations for substance in the form (52).

Thus, the complete system of equations for substance and gravitational field will have the form

$$\frac{\delta L}{\delta \tilde{g}^{\mu\nu}} = 0, \tag{53}$$

$$\nabla_\lambda T^{\lambda\nu} = 0. \tag{54}$$

The substance will be described by velocity $\mathbf{v}$, substance density ρ, and pressure p. The gravitational field is determined by the ten components of the tensor $\Phi^{\mu\nu}$. We thus have 15 unknowns. For their determination it is necessary to supplement the 14 equations (53), (54) with an equation of state of the substance. If the relations

$$\frac{\delta L_g}{\delta \tilde{g}^{\mu\nu}} = -\frac{1}{16\pi} R_{\mu\nu} + \frac{m^2}{32\pi}(g_{\mu\nu} - \gamma_{\mu\nu}), \tag{55}$$

$$\frac{\delta L_M}{\delta \tilde{g}^{\mu\nu}} = \frac{1}{2\sqrt{-g}}\Big(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\Big), \tag{56}$$

are taken into consideration, the system of equations (53), (54) can be represented in the form

$$\Big(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\Big) + \frac{m^2}{2}\Big[g^{\mu\nu} + \Big(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta}\Big)\gamma_{\alpha\beta}\Big] = \frac{8\pi}{\sqrt{-g}} T^{\mu\nu}, \tag{57}$$

$$\nabla_\lambda T^{\lambda\nu} = 0. \tag{58}$$

By virtue of the Bianchi identity, we have

$$\nabla_\mu \Big(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\Big) = 0,$$

and Eqs. (57) imply

$$m^2 \sqrt{-g}\Big(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta}\Big)\nabla_\mu \gamma_{\alpha\beta} = 16\pi \nabla_\mu T^{\mu\nu}. \tag{59}$$

In view of the expression

$$\Delta_\mu \gamma_{\alpha\beta} = -G^\sigma_{\mu\alpha}\gamma_{\sigma\beta} - G^\sigma_{\mu\beta}\gamma_{\sigma\alpha}, \tag{60}$$

where $G^\sigma_{\mu\alpha}$ is given by formula (26), we find

$$\Big(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta}\Big)\nabla_\mu \gamma_{\alpha\beta} = \gamma_{\mu\lambda} g^{\mu\nu}(D_\sigma g^{\sigma\lambda} + G^\beta_{\alpha\beta} g^{\alpha\lambda}), \tag{61}$$

and, because

$$\sqrt{-g}(D_\sigma g^{\sigma\lambda} + G^\beta_{\alpha\beta} g^{\alpha\lambda}) = D_\sigma \tilde{g}^{\lambda\sigma}, \tag{62}$$

formula (61) takes the form

$$\sqrt{-g}\Big(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta}\Big)\nabla_\mu \gamma_{\alpha\beta} = \gamma_{\mu\lambda} g^{\mu\nu} D_\sigma \tilde{g}^{\lambda\sigma}. \tag{63}$$

Using (63) we can represent (59) as

$$m^2 \gamma_{\mu\lambda} g^{\mu\nu} D_\sigma \tilde{g}^{\lambda\sigma} = 16\pi \nabla_\mu T^{\mu\nu}.$$

This expression can be rewritten in the form

$$m^2 D_\sigma \tilde{g}^{\lambda\sigma} = 16\pi \gamma^{\lambda\nu} \nabla_\mu T^\mu_\nu. \tag{64}$$

With the aid of (64), Eq. (58) can be replaced by the equation

$$D_\sigma \tilde{g}^{\nu\sigma} = 0. \tag{65}$$

Therefore the system of equations (57), (58) reduces to the system of gravitational equations in the form

$$\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right) + \frac{m^2}{2}\left[g^{\mu\nu} + \left(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right] = \frac{8\pi}{\sqrt{-g}}T^{\mu\nu}, \tag{66}$$

$$D_\mu\tilde{g}^{\mu\nu} = 0. \tag{67}$$

Introducing the tensor

$$N^{\mu\nu} = R^{\mu\nu} - \frac{m^2}{2}[g^{\mu\nu} - g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}], \quad N = N^{\mu\nu}g_{\mu\nu},$$

we can write the system of equations (66), (67) as

$$N^{\mu\nu} - \frac{1}{2}g^{\mu\nu}N = \frac{8\pi}{\sqrt{-g}}T^{\mu\nu}, \tag{66a}$$

$$D_\mu\tilde{g}^{\mu\nu} = 0. \tag{67a}$$

It can also be represented in the form

$$N^{\mu\nu} = \frac{8\pi}{\sqrt{-g}}\left(T^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T\right), \tag{68}$$

$$D_\mu\tilde{g}^{\mu\nu} = 0 \tag{69}$$

or

$$N_{\mu\nu} = \frac{8\pi}{\sqrt{-g}}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right), \tag{68a}$$

$$D_\mu\tilde{g}^{\mu\nu} = 0. \tag{69a}$$

It should be stressed particularly that both system (68) and system (69) include the metric tensor of the Minkowski space.

The coordinate transformations under which the metric of the Minkowski space is form-invariant are related by physically equivalent reference frames. The simplest among them are inertial reference frames. Therefore the possible gauge transformations satisfying the Killing conditions,

$$D_\mu\varepsilon_\nu + D_\nu\varepsilon_\mu = 0,$$

do not take us outside the class of physically equivalent reference frames. If we admit that it is possible to measure experimentally the characteristics of the Riemann space and substance motion with an arbitrarily high accuracy, then, based on Eqs. (68a) and (69a), we can determine the metric of the Minkowski space and find the Galilean (inertial) coordinate systems. Hence, in principle, the Minkowski space is observable.

The existence of the Minkowski space is reflected in conservation laws, and therefore their verification in physical phenomena is at the same time the verification of the space-time structure.

The system of gravitational equations can also be written in another equivalent form:

$$\gamma^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\mu\nu} + m^2\tilde{\Phi}^{\mu\nu} = 16\pi t^{\mu\nu}, \tag{70}$$

$$D_\mu\tilde{\Phi}^{\mu\nu} = 0, \tag{71}$$

where $t^{\mu\nu} = -2\delta L/\delta g_{\mu\nu}$ is the density of the energy-momentum tensor of substance and gravitational field in the Minkowski space. This form of equations resembles equations of electrodynamics with photon mass μ in the absence of gravitation:

$$\gamma^{\alpha\beta}D_\alpha D_\beta A^\nu + \mu^2 A^\nu = 4\pi j^\nu, \tag{72}$$

$$D_\nu A^\nu = 0. \tag{73}$$

Whereas in electrodynamics the source of the vector field A^ν is the conserved electromagnetic current j^ν generated by charged bodies, in RGT the source of the tensor field is the conserved total energy-momentum tensor of substance and gravitational field. Therefore the gravitational equations are nonlinear even for the gravitational field proper.

We particularly note that, along with the well-known cosmological term, Eqs. (66) also include an additional term containing the metric $\gamma_{\mu\nu}$ of the Minkowski space. The two terms enter the equations with a common constant that coincides with the graviton mass and therefore is very small. The second mass term in Eqs. (66) containing the metric $\gamma_{\mu\nu}$ gives rise to repulsion forces that are very large in strong gravitational fields. This factor changes the character of the collapse and development of the Universe. As was seen earlier, the existence of the graviton rest mass is of fundamental importance to the construction of field theory of gravitational field. It is owing to the presence of the graviton mass that the theory implies that a homogeneous and isotropic Universe can only be planar.

5. THE CAUSALITY PRINCIPLE IN RGT

As the theories of other physical fields, RGT is constructed within the framework of special relativity theory. According to the latter, any motion of a point test body always occurs inside the causality light cone of the Minkowski space. Consequently, noninertial reference frames realized by test bodies must also be inside the causality cone of the pseudo-Euclidean space-time. This determines the whole class of possible noninertial reference frames. When a particle is acted upon, a local equivalence of inertia and gravitation will take place if the light cone of the effective Riemann space does not extend outside the limits of the causality light cone of the Minkowski space. Only in this case can the gravitational field acting on the test body be locally excluded by passing to an admissible non-inertial reference frame related to the body.

If the light cone of the effective Riemann space extended outside the limits of the causality light cone of the Minkowski space, this would mean that such a "gravitational field" cannot have a non-inertial reference frame in which this "field" can be excluded when acting on the particle. In other words, the local "equivalence" of inertia and gravitation can only be possible when the gravitational field, acting on particles as a physical field, does not drive their world lines outside the limits of the causality light cone of the pseudo-Euclidean space-time.

This condition should be regarded either as a causality principle or as an equivalence principle making it possible to select solutions to the system of equations (66), (67) that have a physical meaning and correspond to gravitational fields. The causality principle does not hold automatically. This is due to the fact that the gravitational interaction enters the coefficients in the second-order derivatives in the field equations, i. e., it changes the original geometry of space-time. This feature is characteristic only of the gravitational field. The interaction of all the other known physical fields usually does not affect the second-order derivatives in the field equations and therefore does not change the original pseudo-Euclidean geometry of space-time.

We now present an analytical statement of the causality principle in RGT. In RGT, the substance motion induced by the gravitational field in the pseudo-Euclidean space-time is equivalent to substance motion in the corresponding effective Riemann space-time. Therefore, for causally related events (world lines of particles and light), on the one hand, the condition

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \geqslant 0 \tag{74}$$

must hold, and, on the other hand, the inequality

$$d\sigma^2 = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu \geqslant 0 \tag{75}$$

must necessarily be satisfied. For the chosen reference frame realized by physical bodies, the condition

$$\gamma_{00} > 0 \tag{76}$$

holds.

In expression (75) we separate the time- and space-like parts:

$$d\sigma^2 = \left(\sqrt{\gamma_{00}}\, dt + \frac{\gamma_{0i}\, dx^i}{\sqrt{\gamma_{00}}}\right)^2 - S_{ik}\, dx^i\, dx^k. \tag{77}$$

Here the Roman indices i and k run over the values 1, 2, and 3,

$$S_{ik} = -\gamma_{ik} + \frac{\gamma_{0i}\gamma_{0k}}{\gamma_{00}}, \tag{78}$$

and S_{ik} is the metric tensor of the three-dimensional space in the four-dimensional pseudo-Euclidean space-time.

The square of the spatial distance is given by the expression

$$d\,l^2 = S_{ik}\,dx^i\,dx^k. \tag{79}$$

We represent the velocity $v^i = dx^i/dt$ in the form $v^i = ve^i$, where v is the magnitude of the velocity and e^i is an arbitrary unit vector in the three-dimensional space:

$$S_{ik}e^ie^k = 1. \tag{80}$$

In the absence of gravitational field, the velocity of light in the chosen coordinate system is readily found from expression (77) by equating it to zero:

$$\left(\sqrt{\gamma_{00}}\,dt + \frac{\gamma_{0i}\,dx^i}{\sqrt{\gamma_{00}}}\right)^2 = S_{ik}\,dx^i\,dx^k.$$

Whence we found

$$v = \sqrt{\gamma_{00}}\Big/\left(1 - \frac{\gamma_{0i}e^i}{\sqrt{\gamma_{00}}}\right). \tag{81}$$

Thus, an arbitrary four-dimensional isotropic vector u^ν in the Minkowski space is expressed as

$$u^\nu = (1,\, ve^i). \tag{82}$$

For conditions (74) and (75) to hold simultaneously it is necessary and sufficient that for any isotropic vector

$$\gamma_{\mu\nu}u^\mu u^\nu = 0 \tag{83}$$

the causality condition

$$g_{\mu\nu}u^\mu u^\nu \leqslant 0 \tag{84}$$

should hold, and it means that the light cone of the effective Riemann space does not extend outside the limits of the causality light cone of the pseudo-Euclidean space-time.

The causality conditions can be written in the following form:

$$g_{\mu\nu}v^\mu v^\nu = 0, \tag{83a}$$

$$\gamma_{\mu\nu}v^\mu v^\nu \geqslant 0. \tag{84a}$$

In GRT only those solutions to the Einstein–Hilbert equations have a physical meaning which satisfy at each point of space-time the inequality

$$g < 0,$$

and also a requirement that is called the energy-dominance condition and is stated in the following way: for any time-like vector K_ν the inequality

$$T^{\mu\nu}K_\mu K_\nu \geqslant 0$$

must hold, and for the given vector K_ν the expression $T^{\mu\nu}K_\nu$ must form a non-space-like vector.

In the proposed theory only those solutions to Eqs. (68a) and (69a) have a physical meaning which, along with the above conditions, also satisfy the causality conditions (83a) and (84a). By virtue of Eq. (68a), condition (84a) can be written in the following form:

$$R_{\mu\nu}K^\mu K^\nu \leqslant \frac{8\pi}{\sqrt{-g}}\Big(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\Big)K^\mu K^\nu + \frac{m^2}{2}g_{\mu\nu}K^\mu K^\nu. \tag{85}$$

To conclude this section we should like to note that although we derived the gravitational equations (66) and (67) with the graviton mass several years ago, the logic of our construction gradually led us to the conclusion that the graviton rest mass does exist because it alone makes it possible to construct the theory of tensor field in the Minkowski space that leads to the effective Riemann geometry of space-time.

6. SOME PHYSICAL IMPLICATIONS OF RGT

The system of RGT equations (66), (67) leads to completely different and qualitatively new physical conclusions as compared to GRT. For example, the concept of collapse is totally changed. It turns out that during the collapse of a spherically symmetric body of arbitrary mass, the contraction process stops in a region close to the Schwarzschild sphere and is then replaced by a subsequent expansion. This means that in nature, there must exist expanding objects along with the contracting ones. Hence, according to RGT, the existence of "black holes" (objects having no material boundaries and "cut off" from the external world) is completely excluded.

Another important physical implication refers to the development of a homogeneous and isotropic Universe. Equations (66) and (67) and also the causality conditions (83) and (84) imply that a homogeneous and isotropic Universe exists infinitely long, and its three-dimensional geometry is Euclidean. The Universe develops cyclically from a maximum finite density to a minimum one, then again to a maximum density (in the case there is no dissipation), etc. The theory predicts the existence of a large "latent" mass of substance in the Universe because, according to Eqs. (66) and (67), at the present time the total density of substance is

$$\rho = \rho_c + \frac{1}{16\pi G}\left(\frac{mc^2}{\hbar}\right)^2. \tag{86}$$

Hence it follows that even for a fairly small graviton mass the density of substance is close to the critical value ρ_c determined by the Hubble constant H:

$$\rho_c = \frac{3H^2}{8\pi G}. \tag{87}$$

RGT explains all the known gravitational experiments in the solar system and, as was seen earlier, makes it possible to introduce the notion of energy-momentum tensor for the gravitational field as in the case of other physical fields. According to Eqs. (66), the energy-momentum tensor $-2\delta L_g/\delta g_{\mu\nu}$ of the gravitational field in the Riemann space vanishes outside the substance. However, this does not mean that there is no gravitational radiation because a gravitational wave transporting energy propagates against an effective gravitation background.

As to the gravitational radiation of massive gravitons, this question was considered in [3], where it was shown that the earlier calculations were based on an incorrectly derived general expression for the intensity. Namely, its derivation did not take into account the important fact that in reality the gravitons propagate not in the Minkowski space but in the effective Riemann space. The inclusion of this fact led the author of [3] to the assertion that the intensity of gravitational radiation of massive gravitons is a positive definite quantity.

The system of RGT gravitational equations (66), (67) opens up new possibilities for both fundamental research and specific studies of various gravitation phenomena.

In conclusion we should like to make some important remarks. Is it possible to equate the graviton mass to zero? Since in our theory the graviton mass removes degeneration with respect to the gauge group, it is not quite correct to equate it to zero directly in Eqs. (66) and (67). In our theory it must not vanish. The system of gravitational equations (66), (67) is hyperbolic, and the causality principle ensures that throughout the space there exists a space-like surface which is intersected by every non-space-like curve in the Riemann space only once, in other words, there exists a global Cauchy surface on which initial physical conditions are set for various problems.

Penrose and Hawking [4] proved existence theorems for singularity in GRT under some definite general conditions. Because, according to Eqs. (68a) and by virtue of causality conditions (85), outside the substance the isotropic vectors of the Riemann space satisfy the inequality

$$R_{\mu\nu}v^{\mu}v^{\nu} \leqslant 0, \tag{88}$$

the conditions of the existence theorems for singularity do not hold in RGT, and their assertions are inapplicable in RGT. In the proposed theory, the events that are space-like in the absence of gravitational field can never become time-like under the action of the gravitational field. By virtue of the causality principle, the effective Riemann space in RGT possesses isotropic and time-like geodesic completeness.

Based on what was presented above, the following general conclusion can be drawn. If, in view of the universality of gravitation, it is assumed that the source of the gravitational field in the Minkowski space is the conserved energy-momentum tensor of substance and massive gravitational field, then the field itself will manifest itself as a second-rank tensor field. By analogy with electrodynamics, it is reasonable to write the field equations in the following form:

$$\Box\Phi^{\mu\nu} + m^2\Phi^{\mu\nu} = \lambda t^{\mu\nu}, \quad \partial_\mu\Phi^{\mu\nu} = 0.$$

However, this system of equations follows from the Lagrangian formalism only in the case when the interaction between substance and gravitational field obeys the geometrization principle, which reduces the action of this field to the effective space-time geometry.

Thus, the assumption of conserved energy-momentum tensor of matter as a universal source of the gravitational field necessarily leads to the effective Riemann geometry. Since the field theory of gravitation requires the introduction of a graviton mass and its structure is close to that of electrodynamics, it is quite probable that the photon rest mass is also nonzero.

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Department of Quantum Theory
and High-Energy Physics