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STARTING CONDITIONS OF RELATIVISTIC CARCINOTRONS WITH CONSIDERATION FOR INTERACTION BETWEEN THE ELECTRON BEAM AND THE FORWARD WAVE

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The interaction of an electron beam with the field of the electrodynamic system of a relativistic carcinotron is analyzed in a linear approximation. Consideration is given not only to the interaction of the electron flux with the field of the (−1)th space harmonic of a backward wave, but also to the field of the 0th space harmonic of a forward wave. The presence of this wave is due to the carcinotron design, where the microwave energy is taken out towards the electron collector. The effect of interaction with the forward wave on the starting conditions of the oscillator is demonstrated by an example of a concrete mock-up of a relativistic backward-wave tube.

Relativistic microwave oscillators of the backward-wave-tube type with a retarding system in the form of a cylindrical waveguide with shallow sine-shaped wall corrugation (the waveguide radius varies following the relationship $R(z) = R_W + h \times \sin(k_0 z)$, where $k_0 = 2\pi/d$, with d being the corrugation period), so-called carcinotrons [1–5], are extensively used as sources of powerful high-frequency pulses. Apart from standard assumptions of the monoenergetic and magnetized nature of electrons in the sense that a transverse shift of particles is neglected, a theoretical model of such devices assumes the Cerenkov resonance condition only for the (−1)th space harmonic of the wave whose energy propagates in a direction opposite to the electron beam:

$$\omega \approx k_{z1} t_e = (k_0 + k_{z0-}) v_e, \tag{1}$$

where ω is the wave field frequency, $k_{z0-} = -k_{z0} < 0$ is the wave number of the fundamental (0th) space harmonic of the backward wave; k_{z1} is the wave number of the (−1)th space harmonic of the backward wave; and v_e is the electron velocity along the longitudinal z axis of the system. Such backward wave is usually E_{01} or E_{02}. In further text we shall analyze devices with the wave E_{01}. Note that in relativistic carcinotrons, microwave power is typically taken out towards the electron collector. This is done with the aid of a section of a cylindrical waveguide placed at the oscillator input whose dimensions correspond to the cutoff frequency of the operating wave. The presence of a forward wave in an electrodynamic system may cause certain effects which are analyzed in this study.

It was shown in [2, 4] that an analysis of interaction of an electron flow with the field of a corrugated waveguide in the single-wave approximation of an electrodynamic system is not quite correct in a number of practically important cases when the 0th space harmonic of a forward wave is also slowed down despite the fact that the corrugation amplitude parameter $h/2R_W$ is rather small. Under such devices the Cerenkov resonance requirement is satisfied simultaneously both for the (−1)th space harmonic of the backward wave (interaction of the backward-wave-tube type) and for the 0th space harmonic of the forward wave (interaction of the traveling-wave-tube type), the electron flow sets up a field in the system at a frequency close to the upper boundary of the main (low-frequency) transmission band of the system for E_{01} wave (π mode oscillations), and the electrodynamic system features markedly pronounced resonance properties. Analysis of such systems has a number of peculiarities and often relies on consideration of oscillations in coupled resonator networks [6].

If the corrugation parameter is smaller than the above value, the 0th space harmonic is not slowed down and the Cerenkov resonance is observed far from the boundaries of the transmission band of the electrodynamic system. In this case, interaction of electrons with the 0th space harmonic of the forward wave is usually neglected, because the transit angle of electrons due to phase detuning

$$\theta_0 = L\left(\frac{\omega}{v_e} - k_{z0}\right) \approx L(2k_{z1} - k_0) \gg 2\pi, \tag{2}$$

where L is the length of the corrugated part of the electrodynamic system, is significantly higher than the angle optimum for the interaction ($\sim \pi$).

Consider condition (2) in greater detail. As is known [3], when selecting parameters of a corrugated waveguide for E_{01} operation it is required to eliminate interaction with higher modes, in particular, with E_{02}. This can be done by satisfying the condition

$$\frac{\mu_{02}}{R_W} > k_0 \frac{v_e}{c}, \tag{3}$$

where μ_{0s} is the sth root of the equation $J_0(\mu_{0s}) = 0$, and $J_0(x)$ is the Bessel function of the zero order. To ensure maximum electric strength of the system it is required, as far as possible, to increase the average radius R_W of the corrugated waveguide; therefore, this relationship imposes a constraint on its maximum value. The simplest estimates for the transit angle can be obtained for ultrarelativistic electrons ($\gamma_e^2 = (1-\beta_e^2)^{-1} \gg 1$, $\beta_e = v_e/c$). For example, if the equality sign is put in (3), expression (2) becomes

$$\theta_0 \approx 2\pi \frac{\mu_{01}^2}{\mu_{02}^2} \frac{L}{d}. \tag{4}$$

With corrugated waveguides of length $L \approx (10-15)d$ commonly used in carcinotrons, we have $\theta_0 \approx (4-6)\pi$.

The calculations presented below will be made for the mock-up of an oscillator based on the estimates arrived at in [3]. Accelerating voltage $U_a = 450$ kV, beam current $J_{b0} = 0.5-2$ kA, $R_W \approx 1.14\, d$, $h \approx 0.11\, R_W$, coupling impedance of the electron flow with the field of the (-1)th space harmonic of a backward wave $R_1 \approx -0.3$ ohm (beam radius $r_b \approx 0.6\, R_W$), generator corrugation length $L = 12\, d$.

To answer the question whether it is necessary to consider interaction with a forward wave in relativistic carcinotrons, we shall use a low-current beam approximation ($J_{b0} \ll J_{cv}$), where J_{cv} is maximum vacuum current. Assuming in a linear approximation that variation of all quantities follows the law $\exp\{i[\omega t - (\omega/v_e + \delta k) \times z]\}$, write the system of equations for exciting a forward wave (interaction with the 0th harmonic only) and a backward wave (with the (-1)th harmonic) of the electrodynamic system and waves of the beam space charge:

$$2(b_1 + \delta)\alpha_1 + iI_1^c j = 0, \tag{5}$$

$$2(b_0 + \delta)\alpha_0 + iI_0^c j = 0, \tag{6}$$

$$2(\alpha_0 + \alpha_1) - i(\delta^2 - \sigma) j = 0, \tag{7}$$

where $j = \widetilde{J}_b/J_{b0}$ is a relative amplitude of the variable component of the beam current; $\alpha_s = 2\gamma_e\beta_e^3 e \times E_{zs}/(mc\omega)$ is normalized complex amplitudes of the z components of respective wave fields at the beam location; $I_s^c = 16\pi\gamma_e^3\beta_e^6 R_s J_{b0}/(Z_0 J_0 \beta_{fs}^2)$ is parameters of the electron beam interaction with the field of the respective wave; $Z_0 = 377$ ohms, $J_0 = 17$ kA; $s = 0$ corresponds to the 0th space harmonic and $s = 1$, to the (-1)th space harmonic of the backward wave;

$$R_1 = -\left(\frac{h}{2R_W}\right)^2 \frac{Z_0(k_\perp^2 + (k_0 - k_{z1})k_0)^2}{\pi k_{z1}^2 (k_0 - k_{z1})\omega/c} \frac{I_0^2(p_1 r_b)}{I_0^2(p_1 R_W)}$$

is the coupling impedance of the electron flow with the backward wave; $R_0 = Z_0 \mu_{01}^2 J_0^2(p_1 r_b) \Big/ \left[\pi k_{z1}^3 R_W^4 \frac{\omega}{c} \times J_1^2(\mu_{01})\right]$ is the coupling impedance of the electron flow with the forward wave; $J_1(x)$ is the Bessel function of the first order; $\beta_{fs} = \omega(k_{zs}c)$ is relative phase velocities of waves; $b_s = 2\gamma_e^2\beta_e^2(\beta_{fs} - \beta_e)/\beta_{fs}$ is relative detuning of wave phase velocities as compared to the velocity of electrons; $\sigma = T \times 8\beta_e J_{b0}/(\gamma_e J_0)$ is the space charge parameter; and $\delta = 2\gamma_e^2\beta_e^3 c/\omega \times \delta k$. The coefficient T for a narrow tubular beam has the form

$$T = \frac{I_0(\varkappa r_b)}{I_0(\varkappa R_W)} \left[I_0(\varkappa R_W)K_0(\varkappa r_b) - I_0(\varkappa r_b)K_0(\varkappa R_W)\right], \tag{8}$$

where $\varkappa = [(\omega/v_e + \delta k)^2 - \omega^2/c^2]^{1/2} \approx \omega/(c\beta_e\gamma_e)$, $I_0(x)$ and $K_0(x)$ are the Bessel functions of the imaginary argument.

Solving system (5)–(7) results in the variance equation of the fourth order with respect to the wave number δ:

$$\frac{I_0^c}{b_0+\delta}+\frac{I_1^c}{b_1+\delta}+\delta^2-\sigma=0. \tag{9}$$

When there is no interaction with the forward wave ($I_0^c=0$), Eq. (9) is decomposed into the "three-wave" equation of a relativistic backward-wave tube

$$(\delta^2-\sigma)(b_1+\delta)=-I_1^c \tag{10}$$

and the equation describing the variance law for the 0th space harmonic of the forward wave undisturbed by the electron flow:

$$b_0+\delta=0. \tag{11}$$

To obtain starting conditions for a carcinotron, it is required to set boundary conditions at the input (cutoff narrowing) and the output (matching horn) of the oscillator. Apart from the conventional requirement of no current or velocity modulation of the electron flow at the input [7]:

$$\sum_{m=1}^{4}\frac{\alpha_{1m}+\alpha_{0m}}{\delta_m^2-\sigma}=0, \tag{12}$$

$$\sum_{m=1}^{4}\frac{\delta_m(\alpha_{1m}+\alpha_{0m})}{\delta_m^2-\sigma}=0, \tag{13}$$

and also the condition that there must be no backward wave field at $z=L$ (ideal matching of the output):

$$\sum_{m=1}^{4}\alpha_{1m}\exp\{-i\delta k_m L\}=0, \tag{14}$$

one has to consider the conditions of converting a backward wave into a forward one, which can be written thus:

$$\sum_{m=1}^{4}\alpha_{0m}=\left(\frac{I_0^c}{|I_1^c|}\right)^{\frac{1}{2}}\sum_{m=1}^{4}\alpha_{1m}\exp\{i\psi\}. \tag{15}$$

This condition is tantamount to the absence of microwave power at the system input.

A phase shift depends on the geometry of a transitional section where the corrugated surface changes into the input cutoff narrowing. A possibility of varying the phase shift ψ can be experimentally obtained, for example, by placing, between the cutoff narrowing and the corrugation, a short section of a cylindrical waveguide with the radius equal to the average radius of the corrugated waveguide. Then, the phase shift ψ can be varied by varying the length of the cylindrical waveguide. The variation of the phase shift $\Delta\psi$ depends on changes in the length of the cylindrical waveguide section ΔL: $\Delta\psi=-2k_{z0}\Delta L$. Note that in our statement of the boundary problem, the interaction of waves with the electron flow in this short section of a cylindrical waveguide is actually ignored.

Variance equation (9) was solved with consideration for boundary conditions (12)–(15) by numerical methods. Figure 1 presents results of estimation of the beam starting current J_{bst} depending on the radius on the assumption of no interaction between the electrons and the forward wave of the electrodynamic system ($I_0^c=0$) and a small space charge ($\sigma=0$). In this case the starting conditions are independent of the phase shift ψ. As can be seen from Fig. 1, the values of the starting current calculated by Eqs. (9), (12)–(15) fit well to those computed by Kompfner's method of successive approximations for a three-wave backward-wave tube according to [3].

The situation in a real-life carcinotron differs considerably from the case when the interaction with a forward wave is ignored. For actual parameters of the electron flux interaction with a forward wave, Fig. 2 illustrates the dependence of the current J_{bst} and the relative detuning of the phase velocity $(\beta_{f1}-\beta_e)/\beta_{f1}$ of the (-1)th space harmonic of the backward wave in the starting mode of operation on the phase shift between the waves at the corrugation input with an unchanged beam radius. It can be seen from Fig. 2

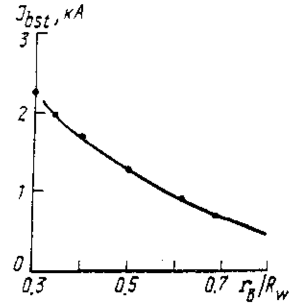

Fig. 1

Carcinotron starting current J_{bst} versus normalized beam radius for the case of small space charge ($\sigma = 0$) and no interaction with a forward wave ($I_0^c = 0$). Solid curve represents results of calculations according to data from [3]; dots stand for the results of numerical solution of variance equation (9) with boundary conditions (12) through (15).

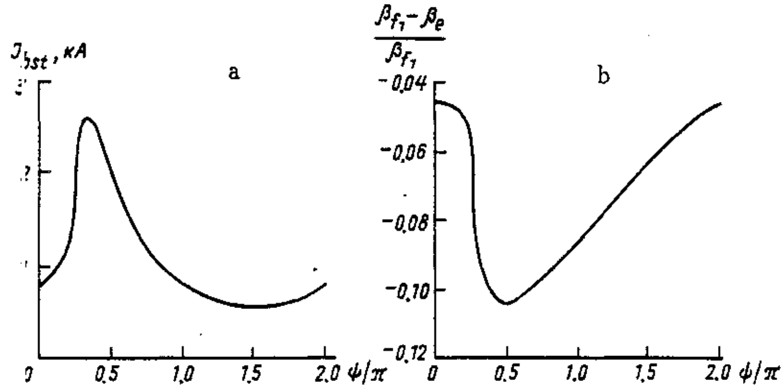

Fig. 2

The starting current J_{bst} (a) and the relative detuning of phase velocity $(\beta_{f1} - \beta_e)/\beta_{f1}$ (b) for the (-1)th space harmonic of the backward wave versus the wave phase shift ψ at the oscillator input for actual values of the current parameter I_0^c.

that the starting current J_{bst} and the detuning depend to a considerable extent on the wave phasing at the system input, with the range of the oscillator starting current being $J_{bst\ max}/J_{bst\ min} \approx 4.3$. Figure 3 shows typical distributions of the normalized amplitudes of the field set up by the 0th and (-1)th space harmonics (curves 2 and 1, respectively) along the oscillator axis at maximum (a) and minimum (b) starting current. Relationships between the normalized amplitude of the beam current variable component j and the longitudinal coordinate for these two cases are presented in Fig. 4. As shown in Figs. 3 and 4, the wave phasing at the oscillator input is less favorable in the former case ($J_{bst} = J_{bst\ max}$) than in the latter. This results in a reduction of the oscillator space effectively used for electron bunching which, in its turn, increases the threshold current at which the generation starts. It should be pointed out that the range of currents required for single-frequency operation of a carcinotron is known [8, 9] to be, as a rule, $(1\text{–}2.5)J_{bst}$. When a carcinotron starts operating in this range one observes first periodic and then stochastic modulation of the output radiation power. If this current exceeds the starting current by a certain value, strong microwave fields may appear in the working space of the oscillator. As a result, a certain number of electrons may be completely stopped and turned back, which may affect significantly the beam transportation stability.

The studies made show that interaction of the electron flow not only with the (-1)th space harmonic of the backward wave, but also with the 0th harmonic of the forward wave should be taken into account in relativistic carcinotrons with electrodynamic systems in the form of corrugated cylindrical waveguides with a small corrugation amplitude. This is particularly important for cases of weak relativity of the beam, when

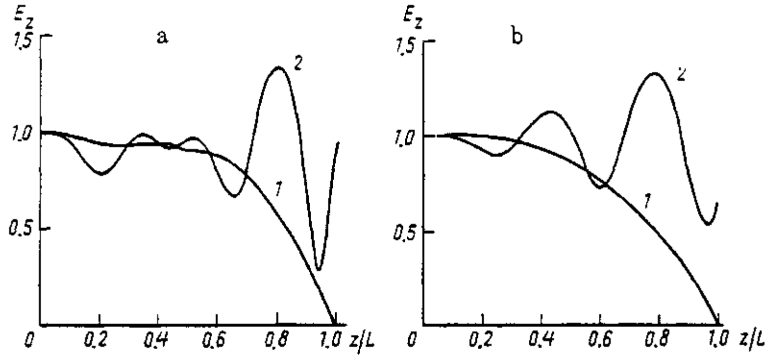

Fig. 3

Distribution of the z component of electrical field over the longitudinal coordinate for the backward (1) and forward (2) waves when (a) $\psi \approx 0.32\pi$ (maximum starting current) and (b) $\psi \approx 1.6\pi$ (minimum starting current).

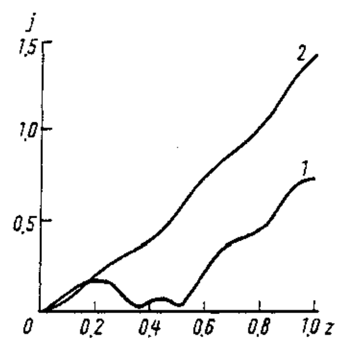

Fig. 4

Distribution of the beam current variable component j over the longitudinal coordinate: $\psi \approx 0.32\pi$ (1) and $\psi \approx 1.6\pi$ (2).

the optimum length of the oscillator is rather small. Consideration for the above interaction results in great changes in the oscillator starting conditions.

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