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INTRARESONATOR SECOND HARMONIC GENERATION IN LASERS WITH ANISOTROPIC RESONATORS

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A theoretical model is developed that describes generation of solid-state lasers with intraresonator transformation of radiation into the second harmonic in crystals with type II wave synchronism. Regions of existence of stationary and nonstationary generation modes are studied.

INTRODUCTION

Progress in developing small-size solid state lasers end-pumped by semiconductor lasers has given rise to a new generation of miniature solid-state lasers in the near IR range with a narrow emission line, a high efficiency, a good performance, and a high reliability [1–4]. Transformation of radiation of such lasers into the visible range by second harmonic generation in nonlinear crystals substantially extends the scope of their applicability. The highest power and efficiency of radiation transformation into the visible region in solid-state lasers with semiconductor pumping, are achieved by intraresonator second harmonic generation (IRSHG) [5–7].

However, the time dependence of the output power of solid-state lasers in multimode IRSHG regimes has the form of deep chaotic pulsations, which has even come to be known as the "green problem" [8]. A theoretical analysis performed in [6, 9] has shown that the dynamic chaos arises because of competing interaction of modes when summed frequencies are generated in a nonlinear crystal. The number of generated modes should then be three or more. It has been noticed in [10, 11] that chaotic oscillations appear when the modes are not equidistant, whereas with equidistant modes, an important factor is their combination interaction, which, under certain conditions, causes synchronization of the modes and stabilizes generation [10]. According to [12, 13], IRSHG favors stabilization of bidirectional generation regimes in solid-state ring lasers when spatially inhomogeneous burning-out of inversion population of the active medium is removed.

The analysis performed in the works cited did not take into consideration the vector character of light waves. Strictly speaking, it is only applicable to IRSHG in crystals where type I wave synchronism is realized, and the polarizations of all the modes are linear and collinear. At the same time nonlinear KTR crystals with type II wave synchronism have been extensively used in experiments. In describing IRSHG in KTR crystals, it is imperative to take polarization into consideration. So far we know of only one work concerned with the effect of resonator properties on the characteristics of such lasers [9]. However, some of the simplifying assumptions made in [9] very often do not hold in experiment. Specifically, this refers to the assumption that IRSHG losses caused by interaction of differently polarized modes are the same as with modes having the same polarizations. In addition, it has not been taken into account that the cross-saturation coefficients for modes with different polarizations have different values.

The purpose of this work is to develop a theoretical model that would correctly describe IRSHG in solid-state lasers with type II wave synchronism and analyze the class of resonators, most important practically, in which a nonlinear crystal is used in combination with a quarter-wave plate stabilizing the generation regime. The regions of existence of one-, two-, three-, and four-mode regimes are studied. It is shown that chaotic generation regimes in lasers of this type can be obtained even when pumping exceeds the threshold level only slightly.

KEY EQUATIONS

According to [9], in the resonator of a laser including a quarter-wave plate, a KTR crystal and an isotropic active element, modes with two polarizations can exist:

$$E(\omega_i) = E_i \exp\{i\omega_i t\} \begin{bmatrix} B_i \\ C_i \end{bmatrix} \frac{1}{\sqrt{B_i^2 + C_i^2}}, \qquad i = 0, 1.$$

Here E_i and ω_i are the complex mode amplitudes and frequencies; $B_0 = \cos 2\varphi \cdot \cos\delta + (\cos^2 2\varphi \cdot \cos^2\delta + \sin^2 2\varphi)^{1/2}$; $C_0 = \sin 2\varphi$; $B_1 = \{\cos 2\varphi \cdot \cos\delta - (\cos^2 2\varphi \cdot \cos^2\delta + \sin^2 2\varphi)^{1/2}\}/\sin 2\varphi$; $C_1 = 1$; φ is the angle made by the extraordinary KTR crystal axis and the "fast" quarter-wave plate axis; and δ is the phase lag between the ordinary and extraordinary fundamental frequency waves after a single run through the KTR crystal.

We assume that two modes of each polarization can occur in the laser:

$$E(\omega_i^{(k)}) = E_i^{(k)} \exp\{i\omega_i^{(k)} t\} \begin{bmatrix} B_i \\ C_i \end{bmatrix} \frac{1}{\sqrt{B_i^2 + C_i^2}}, \qquad i,\, k = 0,\, 1.$$

Let the numbering of modes of one polarization (k) not to be related to the numbering of modes of the other polarization (we assign the index $k = 0$ arbitrarily to one mode of each polarization and the index $k = 1$ to the other mode). Using the same approach as in [9] yields the expressions for the intensity of the second harmonic signal:

$$I_{\text{sh}} = \langle P \cdot P^* \rangle = \frac{d_{\text{eff}}}{4} \Big\{ g \sum_{i,k} J_i^{(k)^2} + 4g \sum_{i,k} J_i^{(k)} J_i^{(ka)} + 4(1-g) \sum_{i,k,K} J_i^{(k)} J_{ia}^{(K)} \Big\},$$
$$i,\, k,\, K = 0, 1; \qquad ia = 1 - i; \qquad ka = 1 - k.$$

Here $J_i^{(k)} = E_i^{(k)} E_i^{(k)*}$, d_{eff} is the effective coefficient of transformation to the second harmonic in KTR crystal and $g = 4B_1^2 C_1^2 / (B_1^2 + C_1^2)^2$.

Using the expression for I_{sh}, we obtain for $J_i^{(k)}$:

$$\tau_c \frac{dJ_i^{(k)}}{dt} = J_i^{(k)} \Big[N_i^{(k)} - \alpha_i^{(k)} - ge J_i^{(k)} - 2ge J_i^{(ka)} - 2(1-g)\varkappa \sum_k J_{ia}^{(K)} \Big], \tag{1}$$
$$i,\, k,\, K = 0, 1; \qquad ia = 1 - i; \qquad ka = 1 - k.$$

Here $N_i^{(k)}$ is the coefficient of amplification by the active medium of the kth mode of the ith polarization; $\alpha_i^{(k)}$ is linear losses of this mode (per passage); τ_c is the time of passing the resonator around; $\varkappa = d_{\text{eff}}^2/(4\varepsilon_0 \varepsilon D^2)$; and D is the characteristic atomic size for the KTR crystal.

The losses caused by interaction with the modes of the other polarization (the last term) are substantially different from the losses caused by interaction with the modes of the same polarization (the last but one term). This was not accounted for in equations obtained in [9], where it was assumed that interaction with all modes is described by a term similar to the last term in Eq. (1).

Note that at $g = 1$ ($\varphi = \pi/4$), the coupling of the modes of the same polarization is the strongest, whereas the modes of different polarizations are uncoupled; the opposite is true when $g = 0$ ($\varphi = \pi/2$).

Equations (1) should be augmented by the equations for $N_i^{(k)}$:

$$\tau_f \frac{dN_i^{(k)}}{dt} = N_i^{0(k)} - N_i^{(k)} \Big[1 + \sum_{j,K} \beta_{i,k}^{j,K} J_j^{(K)} \Big]; \qquad i,\, k = 0,\, 1. \tag{2}$$

Here $N_i^{0(k)}$ is the coefficient of amplification of a weak signal (unsaturated coefficient of amplification) for the kth mode of the ith polarization; τ_f is the time of longitudinal relaxation of the active medium; $\beta_{i,k}^{j,K}$ are the coefficients of cross-saturation (and self-saturation if $i = j$, and $K = k$).

Set (1), (2) coincides with the set obtained in [9] only if $k = 0$, that is for two-mode generation (one mode for each polarization).

Reducing set (1), (2) to a dimensionless form yields

$$\frac{dI_i^{(k)}}{d\tau} = I_i^{(k)} \big[G_i^{(k)} - a_i^{(k)} - ge I_i^{(k)} - 2ge I_i^{(ka)} - 2(1-g)e(I_{ia}^{(k)} + I_{ia}^{(ka)}) \big], \tag{3}$$

$$\frac{dG_i^{(k)}}{d\tau} = G_i^{0(k)} - G_i^{(k)} \Big[1 + \sum_{j,K} b_{i,k}^{j,K} I_j^{(K)} \Big], \qquad i,\, k,\, j,\, K = 0,\, 1. \tag{4}$$

Here

$$I_i^{(k)} = J_i^{(k)}\beta_{i,k}^{j,K}; \qquad G_i^{(k)} = N_i^{(k)}\frac{\tau_f}{\tau_c}; \qquad G_i^{0(k)} = N_i^{0(k)}\frac{\tau_f}{\tau_c};$$

$$a_i^{(k)} = \alpha_i^{(k)}\frac{\tau_f}{\tau_c}; \qquad b_{i,k}^{j,K} = \frac{\beta_{i,k}^{j,K}}{\beta_{i,k}^{i,k}} \quad (b_{i,k}^{i,k}=1); \qquad \tau = \frac{t}{\tau_c}; \qquad e = \frac{\varkappa}{\beta_{i,k}^{i,k}}\frac{\tau_f}{\tau_c}.$$

Further, we will try to omit one of the indices where possible to make formulas less cumbersome (it will be indicated). In analyzing set (3), (4) we start with the case of one-mode generation.

1. One-Mode Generation

Let i and k be fixed. Consider a regime with $I_i^{(k)} \neq 0$ and $I_j^{(K)} = 0$ if $j \neq i$ or $K \neq k$.
It is easy to find from (3) and (4) that

$$I_i^{(k)} = \sqrt{\frac{1}{4}(1 + a_i^{(k)}/ge)^2 + (G_i^{0(k)} - a_i^{(k)})/ge} - \frac{1}{2}(1 + a_i^{(k)}/ge).$$

Clearly, one-mode generation is only possible if $G_i^{0(k)} > a_i^{(k)}$. An analysis of set (3), (4) for Lyapunov's stability shows that the one-mode regime is stable to the appearance of other modes if

$$\frac{G_i^{0(ka)}}{1 + b_{i,ka}^{i,k} I_i^{(k)}} - 2geI_i^{(k)} - a_i^{(ka)} < 0,$$

$$\frac{G_{ia}^{0(K)}}{1 + b_{ia,K}^{i,k} I_i^{(k)}} - 2(1-g)eI_i^{(k)} - a_{ia}^{(K)} < 0.$$

Here and below, $ia = 1 - i$ and $ka = 1 - k$.

2. Two-Mode Generation

Clearly, the generated modes can have identical or different polarizations. Consider both situations.

a. Modes with identical polarizations. Put $I_i^{(k)} \neq 0$, $I_i^{(ka)} \neq 0$, and $I_{ia}^{(K)} = 0$ for $K = 0, 1$. Let losses and zero signal amplifications for the generated modes be the same: $a_i^{(k)} = a_i^{(ka)} = a_i$ and $G_i^{0(k)} = G_i^{0(ka)} = G_i^0$.

The two-mode regime that we are considering can exist if $G_i^0 > a_i$, and

$$I_i^{(k)} = I_i^{(ka)} = I_i = \frac{1}{6}\sqrt{[a_i/ge + 3/(1+\beta)]^2 + 12(G_i^0 - a_i)/(ge(1+\beta))} - a_i/ge - 3/(1+\beta).$$

Here $\beta = b_{i,ka}^{i,k}$ denotes the coefficient of cross-saturation of the generated modes.

The conditions for stability of this two-mode generation regime to the appearance of new modes are

$$\frac{G_{ia}^{0(K)}}{1 + (b_{ia,K}^{i,k} + b_{ia,K}^{i,ka})I_i} - 4(1-g)eI_i - a_{ia}^{(K)} < 0, \qquad K = 0,\ 1.$$

The condition for stability of this regime to the damping of the generated modes is

$$I_i ge < 1 + (1+\beta)I_i < (a_i/ge + 3I_i)(1-\beta).$$

b. Modes with different polarizations. For definiteness, let the numbers of the modes (the superscripts) be the same.

Put $I_i^{(k)} \neq 0$, $I_{ia}^{(k)} \neq 0$, and $I_j^{(ka)} = 0$ for $j = 0, 1$. We assume that $a_i^{(k)} = a_{ia}^{(k)} = a_k$ and $G_i^{0(k)} = G_{ia}^{0(k)} = G_k^0$.

The conditions for the existence of the regime under consideration are

$$G_k^0 > a_k,$$

$$I_i^{(k)} = I_{ia}^{(k)} \equiv I_k = \frac{1}{2}\sqrt{a_k/[(2-g)e] + 1/(1+\beta_1)]^2 + 4(G_k^0 - a_k)/(2-g)e(1+\beta_1)} - a_k/[(2-g)e] - 1/(1+\beta_1),$$

where $\beta_1 \equiv b_{i,k}^{ia,k}$ is the coefficient of cross-saturation of the generated modes.

The condition for stability to the appearance of new modes (with the indices j, ka) now becomes

$$\frac{G_j^{0(ka)}}{1 + (b_{j,ka}^{i,k} + b_{j,ka}^{ia,k})I_k} - 2eI_k - a_j^{(ka)} < 0, \qquad j = 0, 1.$$

The conditions for stability to the damping of the generated modes have the form

$$g > \frac{2}{3} - \frac{1 + (1+\beta_1)I_k}{3eI_k},$$

$$g > \frac{2}{3} - \frac{(a_k + (2-g)eI_k)(1-\beta_1)}{3e(1 + (1+\beta_1)I_k)}.$$

3. Three-Mode Generation

Of the three generated modes two will obviously have the same polarizations: $I_i^{(k)} \neq 0$, $I_i^{(ka)} \neq 0$, $I_{ia}^{(k)} \neq 0$, and $I_{ia}^{(ka)} = 0$.

As with the two-mode generation (item 2(a)), we assume that $G_i^{0(k)} = G_i^{0(ka)} = G_i^0$, and $a_i^{(k)} = a_i^{(ka)} = a_i$.

Let us introduce the notation $G_{ia}^{0(k)} = G_a^0$, $a_{ia}^{(k)} = a_a$, and $I_{ia}^{(k)} = I_a$.

We also put $\beta_1 = b_{i,K}^{ia,k}$ for $K = 0, 1$. The notation for the coefficients of cross-saturation is illustrated by the chart in Fig. 1.

We then have $I_i^{(k)} = I_i^{(ka)} = I_i$. For convenience, $I_{ia}^{(k)}$ will be denoted by I_a. The set of equations for I_i and I_a has the form

$$G_i^0 - [a_i + 3geI_i + 2(1-g)eI_a][1 + (1+\beta)I_i + \beta_1 I_a] = 0,$$

$$G_a^0 - [a_a + geI_a + 4(1-g)eI_i][1 + I_a + 2\beta_1 I_i] = 0.$$

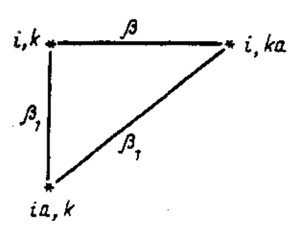

Fig. 1

We solved this set numerically using a combination of the iteration and steepest descent methods.

The intensities I_a and I_i calculated earlier for close values of the parameters β, β_1, g, and e were used as starting approximations. At small e, the initial values of I_a and I_i were taken to be equal to those in the absence of IRSHG ($e = 0$):

$$I_i^{(0)} = \frac{(G_i^0/a_i - 1) - \beta_1(G_a^0/a_a - 1)}{1 + \beta - 2\beta_1^2},$$

$$I_a^{(0)} = \frac{(G_a^0/a_a - 1)(1+\beta) - 2\beta_1(G_i^0/a_i - 1)}{1 + \beta - 2\beta_1^2}.$$

The conditions for stability of the three-mode generation to the appearance of a fourth mode ($I_{ia}^{(ka)}$) have the form

$$\frac{G_{ia}^{0(ka)}}{1+\left(b_{ia,ka}^{i,k}+b_{ia,ka}^{i,ka}\right)I_i+b_{ia,ka}^{ia,k}I_a}-4(1-g)eI_i-2geI_a-a_{ia}^{(ka)}<0.$$

The conditions for stability of this regime to the damping of the generated modes are

$$geI_i<\frac{G_j^0}{G_i}<\frac{G_i}{ge}(1-\beta),\qquad c_0>0,\qquad c_3c_2-c_1>0,\qquad (c_3c_2-c_1)c_1-c_3^2c_0>0,$$

where

$$G_i=a_i+3geI_i+2(1-g)eI_a\quad\text{(stationary value of } G_i^{(k)} \text{ and } G_i^{(ka)});$$
$$G_a=a_a+geI_a+4(1-g)eI_i\quad\text{(stationary value of } G_{ia}^{(k)});$$
$$c_0=I_iI_a\Big[\Big(3ge\frac{G_i^0}{G_i}+G_i(1+\beta)\Big)\Big(ge\frac{G_a^0}{G_a}+G_a\Big)-\Big(2(1-g)e\frac{G_a^0}{G_a}+G_a\beta_1\Big)\Big(4(1-g)e\frac{G_i^0}{G_i}+2G_i\beta_1\Big)\Big];$$
$$c_1=I_a\Big(3geI_i+\frac{G_i^0}{G_i}\Big)\Big(ge\frac{G_a^0}{G_a}+G_a\Big)+I_i\Big(geI_a+\frac{G_a^0}{G_a}\Big)\Big(3ge\frac{G_i^0}{G_i}+G_i(1+\beta)\Big)-4I_iI_a(1-g)e\Big[2(1-g)e\Big(\frac{G_i^0}{G_i}+\frac{G_a^0}{G_a}\Big)+(G_a+G_i)\beta_1\Big];$$
$$c_2=\Big(geI_a\frac{G_a^0}{G_a}+G_aI_a\Big)+3geI_i\frac{G_i^0}{G_i}+I_iG_i(1+\beta)+\Big(3geI_i+\frac{G_i^0}{G_i}\Big)\Big(geI_a+\frac{G_a^0}{G_a}\Big)-8I_iI_a(1-g)^2e^2;$$
$$c_3=G_i^0/G_i+G_a^0/G_a+ge(I_a+3I_i).$$

4. Four-Mode Generation

Let the cross-saturation coefficients be denoted as in Fig. 2.

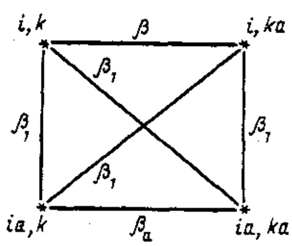

Fig. 2

Put $G_i^{0(k)}=G_i^{0(ka)}=G_i^0$, $a_i^{(k)}=a_i^{(ka)}=a_i$, $G_{ia}^{0(k)}=G_{ia}^{0(ka)}=G_a^0$, and $a_{ia}^{(k)}=a_{ia}^{(ka)}=a_a$. Then $I_i^{(k)}=I_i^{(ka)}=I_i$ and $I_{ia}^{(k)}=I_{ia}^{(ka)}=I_a$, where I_i and I_a are the solutions to the equations

$$G_i^0-[a_i+3geI_i+4(1-g)eI_a][1+(1+\beta)I_i+2\beta_1I_a]=0,$$
$$G_a^0-[a_a+3geI_a+4(1-g)eI_i][1+(1+\beta_i)I_a+2\beta_1I_i]=0.$$

Numerically, I_i and I_a were calculated as with the three-mode generation; at small e values, the I_i and I_a values corresponding to $e = 0$ were used as starting approximations:

$$I_i^{(0)} = \frac{(G_i^0/a_i - 1)(1+\beta_a) - 2\beta_1(G_a^0/a_a - 1)}{(1+\beta)(1+\beta_a) - 4\beta_1^2},$$
$$I_a^{(0)} = \frac{(G_a^0/a_a - 1)(1+\beta) - 2\beta_1(G_i^0/a_i - 1)}{(1+\beta)(1+\beta_a) - 4\beta_1^2}.$$

The conditions for stability of the four-mode generation to the appearance of a fifth mode are

$$\frac{G_i^{0(l)}}{1 + (b_{i\,l}^{i\,k} + b_{i,l}^{i,ka})I_i + (b_{i,l}^{ia,k} + b_{i,l}^{ia,ka})I_a} - 4(1-g)eI_a - 4geI_j - a_i^{(l)} < 0;$$
$$\frac{G_{ia}^{0(l)}}{1 + (b_{ia,l}^{i,k} + b_{ia,l}^{i,ka})I_i + (b_{ia,l}^{ia,k} + b_{ia,l}^{ia,ka})I_a} - 4(1-g)eI_i - 4geI_a - a_{ia}^{(l)} < 0.$$

Here $l = 2, 3, \ldots$.

The conditions for stability of the four-mode regime to the damping of the generated modes are

$$geI_i < \frac{G_i^0}{G_i} < \frac{G_i}{ge}(1-\beta),$$
$$geI_a < \frac{G_a^0}{G_a} < \frac{G_a}{ge}(1-\beta_a),$$
$$c_0 > 0,$$
$$c_3c_2 - c_1 > 0,$$
$$(c_3c_2 - c_1)c_1 - c_3^2c_0 > 0,$$

where

$$G_i = a_i + 3geI_i + 4(1-g)eI_a;$$
$$G_a = a_a + 3geI_a + 4(1-g)eI_i;$$
$$c_0 = I_iI_a\Big[\Big(3ge\frac{G_i^0}{G_i} + G_i(1+\beta)\Big)\Big(3ge\frac{G_a^0}{G_a} + G_a(1+\beta_a)\Big) - \Big(4(1-g)e\frac{G_i^0}{G_i} + 2G_i\beta_1\Big)\Big(4(1-g)e\frac{G_a^0}{G_a} + 2G_a\beta_1\Big)\Big];$$
$$c_1 = I_a\Big(3geI_i + \frac{G_i^0}{G_i}\Big)\Big(3ge\frac{G_a^0}{G_a} + G_a(1+\beta_a)\Big) + I_i\Big(3geI_a + \frac{G_a^0}{G_a}\Big)\Big(3ge\frac{G_i^0}{G_i} + G_i(1+\beta)\Big) - 8I_iI_a(1-g)e\Big[2(1-g)e\Big(\frac{G_i^0}{G_i} + \frac{G_a^0}{G_a}\Big) + (G_a + G_i)\beta_1\Big];$$
$$c_2 = 3ge\Big(I_a\frac{G_a^0}{G_a} + I_i\frac{G_i^0}{G_i}\Big) + I_iG_i(1+\beta) + I_aG_a(1+\beta_a) + \Big(3geI_i + \frac{G_i^0}{G_i}\Big)\Big(3geI_a + \frac{G_a^0}{G_a}\Big) - 16I_iI_a(1-g)^2e^2;$$
$$c_3 = G_i^0/G_i + G_a^0/G_a + 3ge(I_a + I_i).$$

NUMERICAL CALCULATIONS

The derived equations were used to calculate the regions of stability of the stationary regimes considered above. The calculation results obtained with $a_i^{(k)} = 7200$; $G_i^{(k)} = 10^3$; $i, k = 0, 1$; $\beta = \beta_a = 0.7$; and $\beta_1 = 0.85$ are shown in Fig. 3.

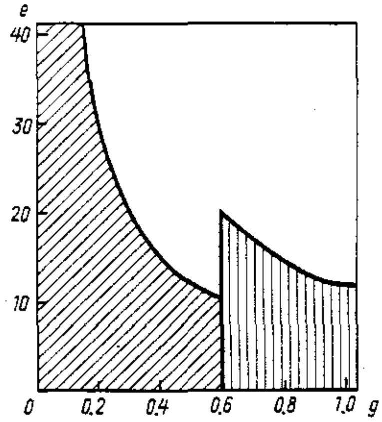
e
40
30
20
10
0
0.2
0.4
0.6
0.8
1.0
g

Fig. 3

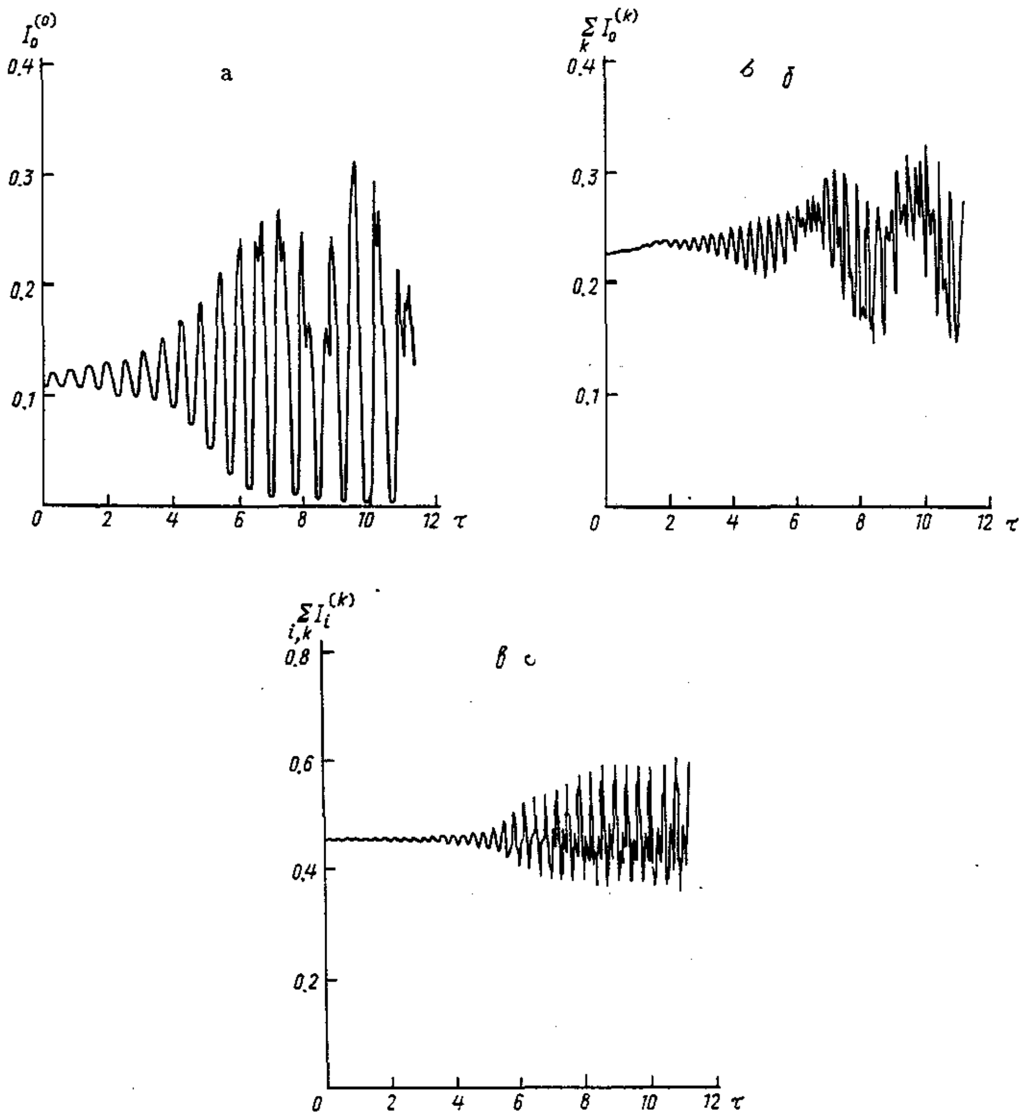
$I_0^{(0)}$
a
0.4
0.3
0.2
0.1
0
2
4
6
8
10
12
τ
$\sum_k I_0^{(k)}$
б
b
0.4
0.3
0.2
0.1
0
2
4
6
8
10
12
τ
$\sum_{i,k} I_i^{(k)}$
в
c
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
τ

Fig. 4

In the obliquely hatched region, the stable regime is stationary generation of two modes with the same polarizations. In the vertically hatched region, the stationary four-mode regime is stable. None of the stationary regimes considered above is stable in the unhatched region.

Numerical solution of set (3), (4) by the Runge–Kutta method confirmed the results of our analysis.

In the region above that of stability of two-mode generation ($g < 0.57$), a two-mode regime with antiphase wave intensity modulation was observed.

In the region above that of stability of four-mode generation ($g > 0.57$), a nonstationary four-mode generation regime was observed. A typical form of oscillations of mode intensities in this regime ($g = 0.6$ and $e = 25$) is illustrated in Fig. 4, where the time dependences of the intensity of one mode (Fig. 4 a), the sum of the intensities of two modes with the same polarization (Fig. 4 b), and the sum of the intensities of all four modes (Fig. 4 c) are shown.

The observed nonstationary intensity oscillations, caused by nonlinear interaction in the KTR crystal, can occur even when the pumping exceeds the threshold level only slightly. It seems, however, that we can easily remove nonstationary oscillations and reach a desired stationary generation mode by varying g and/or e (e can, for instance, be varied by varying the orientation of the KTR crystal, and g by varying φ).

CONCLUSION

The analytic and numerical study on the influence of IRSHG on the character of multimode generation regimes in a laser with an anisotropic resonator can be summarized as follows. Generation of summed frequencies in a nonlinear crystal generally enhances the competition of longitudinal laser modes, thereby increasing the stability of one-mode generation regimes. Then we can observe multistable regimes. Bistability is necessarily present, if the linear losses and the coefficients of zero signal amplification are the same for all modes. In particular, it is easy to see that one-mode regimes with different polarizations can then be stable simultaneously. All two-mode regimes are also bistable: the region of generation stability for two modes of one polarization coincides with the region of generation stability for two modes of the other polarization, and the region of generation stability for two modes with different polarizations and the same longitudinal index (k) coincides with the region of generation stability for two modes with different polarizations and another longitudinal index ($k + 1$). Evidently, the three-mode regimes are also bistable. When stability regions of regimes with different numbers of generated modes (most often, two- and four-mode regimes) overlap, multistability is realized. The largest region of polarization bistability is observed when the efficiency of generation of the summed frequency by modes of different polarizations ($y = 0$) is maximum, whereas generation of the summed frequency by modes with the same polarization is absent (generation of the double frequency from each separate mode is then also absent). In this region, two-mode regimes with the same polarizations of the generated modes can be stable.

It is shown that three-mode generation is the least stable regime.

Stable stationary multimode regimes become nonstationary when the efficiency of transformation into the second harmonic increases. Stationary two-mode generation regimes then turn into alternating switching of generated modes, and stationary four-mode regimes turn into those of radiation intensity pulsations.

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