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OBSERVATION OF NONLINEAR EVOLUTION OF ACOUSTIC PULSES IN THE ABSENCE OF DIFFRACTION

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The results of experiments are discussed in which a nonlinear evolution of acoustic pulses was observed under specially chosen conditions, where the effect of diffraction was practically ruled out. Acoustic pulses were generated by an optoacoustic method, which enables one to obtain short powerful pulses of predictable shape. The pulses were picked up by a hydrophone based on a polyvinylidene fluoride film. At high peak pressures the evolution of an almost triangular profile with a clearly pronounced discontinuous front was observed. The pulse amplitude decreased in inverse proportion to the root of the distance to the radiator. The experimental results are in good agreement with the classical theory of such waves.

Powerful acoustic waves, which are the subject of investigation in nonlinear acoustics, have been studied in sufficient detail [1–3]. In the first studies on the theory of finite-amplitude waves, made in the 19th century by classics in mechanics, an exact solution of nonlinear equations of the hydrodynamics of nonviscous liquid was found in the form of a simple plane wave. On the basis of this solution it was concluded that the origination of shock waves in an ideal medium is inevitable. The qualitative specific features of the obtained solution (nonlinear distortion of the wave profile, formation of discontinuities, etc.), repeatedly confirmed in experiments, are generally well known. However, it is difficult to generate true plane waves because of finite sizes of the sources. Therefore, nonlinear distortions in experiments always occur against the background of diffraction effects. In the case of continuous waves the plane wave approximation is violated even in the vicinity of plane radiators: the acoustic field proves to be highly heterogeneous because of edge effects. Diffraction changes the scale of manifestation of nonlinear effects, leads to asymmetric distortion of the wave profile [4, 5]. In the course of pulse propagation diffraction gives rise to additional regions of rarefaction or compression, and this also influences the character of nonlinear evolution. Diffraction is negligible only near the radiator: at a distance much smaller than Fresnel's length. Therefore, a purely nonlinear evolution can be observed only within a small spatial interval [6]. At the same time, it is of interest to investigate the diffraction-free behavior of intensive waves not only near the radiator but also at considerable distances from it, where the wave has a universal triangular or sawtooth profile with shock fronts. In this paper we report the results of experiments in which a nonlinear evolution of acoustic pulses was observed under specially chosen conditions, where the influence of diffraction was practically eliminated.

The near field of pulse radiators, in contrast to the field of continuous sources, has a simple smooth structure. The influence of the radiator edges manifests itself in the form of a relieving wave, time-delayed near the source relative to the radiated pulse [7] (see also below). Therefore, pulse disturbances prove to be convenient for realization of a purely nonlinear evolution of flat waves. For generating acoustic pulses use was made of the optoacoustic method of excitation, adequately studied by now [8]. This method allows one to obtain short powerful pulses of predictable shape. Water was chosen as a nonlinear medium in which the pulses propagated because the acoustic properties of water are well known. Particular difficulties were encountered in recording the acoustic pulses, because of their broad spectrum. A transducer based on a polyvinylidene fluoride (PVDF) film was specially designed for the purpose.

Before presenting the experimental results, we shall consider conclusions that follow from the theoretical analysis of acoustic waves. The propagation of plane acoustic disturbances of finite amplitude in liquid is described by the Burgers equation [2]:

$$\frac{\partial p}{\partial x} = \frac{\varepsilon}{\rho_0 c_0^3} p \frac{\partial p}{\partial \tau} + \frac{b}{2\rho_0 c_0^3} \frac{\partial^2 p}{\partial \tau^2}, \tag{1}$$

where p is the acoustic pressure; x is the coordinate; $\tau = t - x/c_0$; t is time; ε, ρ and c_0 denote the acoustical nonlinearity parameter, density, and the sound propagation velocity in the medium, respectively; b is the dissipative parameter [2]. Analysis is much simpler in the ideal medium ($b = 0$); in this case from (1) we obtain the equation of simple waves

$$\frac{\partial p}{\partial x} - \frac{\varepsilon}{\rho_0 c_0^3} p \frac{\partial p}{\partial \tau} = 0. \tag{2}$$

For this equation to hold true it is necessary that the characteristic ratio of the nonlinear term on the right-hand side of (1) to the linear term (acoustic Reynolds number Re) should be large. Estimates show that in the described experiments this condition was obviously fulfilled: $\mathrm{Re} \simeq 10$.

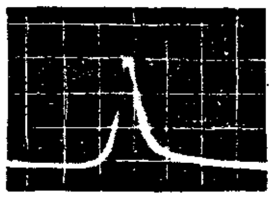

Fig. 1

Shape of an acoustic pulse at the output of the optoacoustic generator.

The solution of equation (2) is known, it is written down in an implicit form as

$$p = \Phi\left(\tau + \frac{\varepsilon x}{\rho_0 c_0^3} p\right), \tag{3}$$

where $\Phi(\tau)$ is the wave profile at the entrance to the nonlinear medium ($x = 0$). In the optoacoustic excitation of acoustic pulses by a short laser pulse there must be: $\Phi(\tau) = p_0 \exp\{-|\tau|/\tau_0\}$, where p_0 is the peak pressure in the pulse; $\tau_0 = (\alpha c_0)^{-1}$; α is the light absorption coefficient [8]. The shape of the wave at the entrance into water, recorded in our experiments, is shown in Fig. 1. As can be seen, it is indeed close to the theoretical one; $\tau_0 \sim 100$ ns.

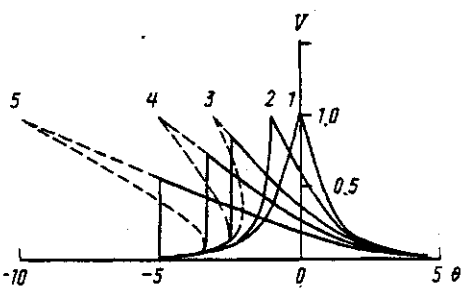

Fig. 2

Nonlinear evolution of the acoustic pulse: $V = p/p_0$, $\theta = \tau/\tau_0$, curves 1–5 correspond, respectively, to distances $x = x_0 \varepsilon p_0/(\rho_0 c_0^3 \tau_0) = 0$, 1, 2, 3, and 4. The dashed curves are constructed according to formula (3); the solid curves are constructed with consideration for discontinuities.

In Fig. 2 we can trace a nonlinear evolution of the wave profile as the wave propagates (the curves are constructed according to formula (3)). A shock front is formed at a certain distance (curve 2); then solution (3) becomes multivalued, and the discontinuity has to be simulated in conformity with the rule of "equality of areas" [2] (curves 3–5). After the formation of the shock front the peak pressure p_m begins to decrease while the pulse duration increases. Analysis for the chosen shape of the initial pulse shows that the dependence of the peak pressure on the distance is specified by the transcendental equation

$$\ln A + \ln\left[\left(1 - \sqrt{(Az+1)^2 - 4z}\right)/z\right] + Az + \sqrt{(Az+1)^2 - 4z} - 1 = 0, \tag{4}$$

where $A = p_m/p_0$, $z = x\varepsilon p_0/(\rho c_0^3 \tau_0)$. The results of a numerical solution of Eq. (4) on a computer are shown in Fig. 3, where A^{-2} is laid off as ordinate. In the sub-discontinuity region ($z < 1$), the peak pressure A does not change in spite of the nonlinear distortion of the wave (see also Fig. 2). At $z > 1$ the value of A begins to decrease because of dissipation of energy at the shock front. At $z > 3$ the shape of the pulse becomes practically triangular, and for a triangular profile $A^{-2} \sim z$ [2]. This relationship can be used to find experimentally the absolute value of the peak pressure and, consequently, to calibrate the hydrophone as well [9].

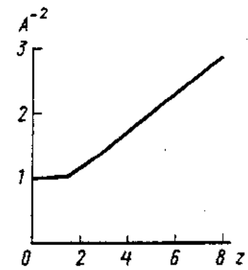

Fig. 3

Inverse square of peak pressure A as a function of distance z.

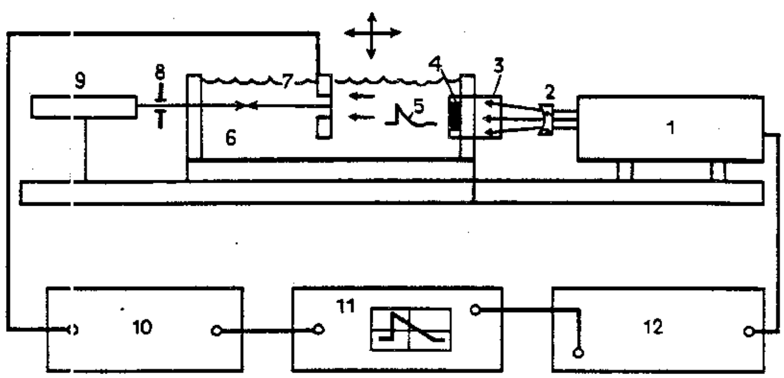

Fig. 4

Schematic diagram of the experimental setup: pulsed laser (1), negative lens (2), glass plate (3), layer of cupric chloride solution (4), acoustic pulse (5), cell with water (6), film hydrophone (7), diaphragm (8), gas laser (9), probe (10), oscillograph (11), pulse generator (12).

Now we shall describe the experimental setup whose layout is shown in Fig. 4. Optical source 1 was a solid-state Nd-glass laser which generated light pulses with duration $\tau = 30$ ns, energy up to 10 J, at a wavelength of 1.06 μm. The laser beam was spread to 5–6 cm in diameter by negative lens 2 and directed to glass plate 3. There was a layer of an aqueous solution of cupric chloride 4 behind plate 3 in whose near-surface region the laser beam was absorbed and an acoustic wave was excited. It is important that the shape of the wave front of the acoustic pulse in such a light-to-sound conversion coincided with the flat shape of plate 3. Acoustic pulses 5 were directed to cell 6 filled with distilled and degassed water.

Waves were detected by hydrophone 7, placed into the cell 6. The hydrophone could move smoothly in different directions. It was made as a membrane of a PVDF film 20 mm in diameter. The sensitivity region of the transducer was the area of intersection of two metallized strips deposited onto the opposite sides of the film. The size across the said area, which determines the scale of the spatial resolution of the hydrophone, was 2 mm. The film thickness was 25 μm, so that the sensitivity of the hydrophone was uniform up to frequencies on the order of 30 MHz [10]. For an acoustic wave to be recorded without distortions, it is necessary that the hydrophone film should be parallel to the wave front. Otherwise the shape of the signal becomes smoothed, since the wave front does not reach all parts of the sensitivity region simultaneously. The hydrophone was so designed that the orientation of the film plane could be varied smoothly. The film plane was matched with the wave front in the following manner. The beam from auxiliary He–Ne laser 9 was directed through diaphragm 8 to plate 3 perpendicular to its surface. Then the hydrophone was placed in the beam region and oriented so that the beam after being reflected from the metallized strip returned into diaphragm 8. It is easy to see that in this way the film of the hydrophone proves to be parallel to the surface of plate 3 and, hence, to the wave front. The accuracy of the setting was limited by the beam scattering upon reflection from the strip and was about 0.01 rad.

The signal from the hydrophone 7 was fed, through a 25 cm long piece of cable, to the input of broadband probe 10 (from an S9-4A oscillograph) that had a large input resistance and a small capacity (1 Mohm and 2.5 pF, respectively). The proper capacity of the hydrophone was 10 pF, therefore the use of the probe with the indicated characteristics helped prevent large attenuation of the signal.

The output of the probe was connected to the input of S8-12 storage oscillograph 11. To compensate for the signal delay with respect to the light pulse, the oscillograph was triggered by G5-54 generator 12 locked with laser 1.

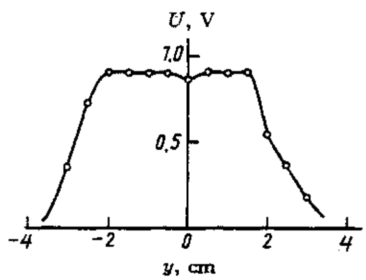

Fig. 5

Transverse distribution of the peak hydrophone signal U at a distance of 1 cm from the optoacoustic generator; y is the transverse coordinate.

The results of observations of powerful acoustic pulses are presented below. As has been pointed out, the experimental conditions were chosen such that the influence of diffraction should be excluded. The diameter of the acoustic beam, equal to the diameter of the corresponding laser beam, was chosen to be sufficiently large. The transverse distribution of the peak value of the hydrophone signal near the optoacoustic oscillator is shown in Fig. 5. It is seen that the distribution is homogeneous and the diameter of the optoacoustic piston radiator $d \simeq 5$ cm. The wave at the beam axis is not sensitive to the finite size of the radiating surface (it remains plane) up to distances at which the wave is catched up by the relieving pulse from the radiator edges.

Let us make corresponding estimates. The relieving pulse from the radiator edge traverses the distance $\sqrt{x^2 + (d/2)^2}$ to the beam axis, so that it lags behind the main signal by the time $\Delta t = \left(\sqrt{x^2 + (d/2)^2} - x\right)/c_0$. If τ_0 is the duration of the radiated pulse, then the interference of the main wave and of the relieving wave begins at $\Delta t \simeq \tau_0$, this corresponding to the distance $x_d \simeq d/(8c_0\tau_0)$. As applied to the experiment being described, the estimate gives $x_d \simeq 1$ m. In the experiment the main compression pulse and the relieving rarefaction pulse were observed separately to a distance $x'_d \simeq 30$ cm. This confirms the given estimate of x_d; the smaller value of x'_d is caused, apparently, by some nonuniformity of the transverse structure of the beam.

Thus, at $x < x'_d$ diffraction should have no influence on the waves being investigated. To verify this condition additionally, we investigated the signal at the beam axis in the linear regime. The energy of the laser pulse decreased to 0.35 J, the nonlinear distortion of the wave shape was not noticeable. The signals at the axis at different distances are shown in Fig. 6 a. One can see that the parameters of the recorded pulse are the same in all the oscillograms. Consequently, the influence of diffraction in our experiments was indeed eliminated.

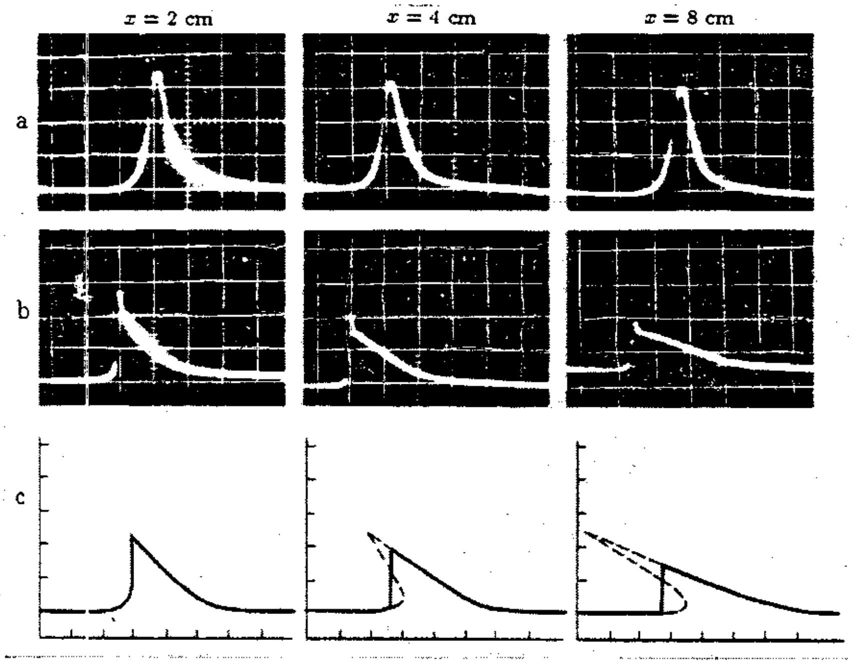

Fig. 6

Evolution of the acoustic pulse shape with variation of distance x at different energy levels of the laser pulse: 0.7 J (a) and 10 J (b). Curves c are theoretical and correspond to oscillograms b. The oscillograph sweep is 200 ns/division. The dashed curves are constructed by formula (3); the continuous curves are constructed with consideration for discontinuities.

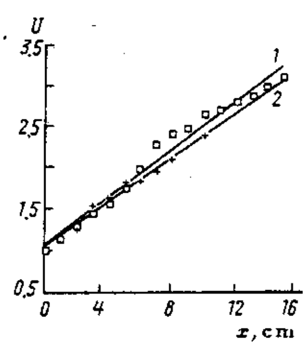

Fig. 7

Inverse square of peak pressure of signal U versus distance $\tilde{x} = x - x_0$ for two values of energy E_l of the laser pulse: $E_l = 8$ J, $x_0 = 3$ cm (1) and $E_l = 6$ J, $x_0 = 10$ cm (2).

Figure 6 b shows wave profiles at different distances from the source under conditions of acoustic nonlinearity. The character of nonlinear evolution described by the theory (see Fig. 2) is confirmed. At moderate peak pressures a gradual "tilting" of the pulse profile to the left was observed that led to the formation of a shock front; at subsequent distances the profile acquired a triangular shape. At high peak pressures (Fig. 6 b) an evolution of the almost triangular profile with a markedly discontinuous front could be observed. As the distance grew, the pulse amplitude decreased, whereas the duration increased. Figure 6 c shows theoretical profiles, corresponding to the oscillograms presented in Fig. 6 b. The theoretical profiles are constructed using presentation (3) of the solution to Eq. (2); the first of the profiles shown in Fig. 6 b is taken as the initial wave front. It should be noted that the theoretical and experimental patterns are in very good agreement.

As pointed out above, the inverse square of the peak pressure after the formation of the shock front must grow in direct proportion to the distance traveled by the wave (Fig. 3). The corresponding experimental dependences for two energy values of the laser pulse, i.e., for two values of the initial peak pressure, are shown in Fig. 7. One can see that the relationship indicated in the theoretical analysis is confirmed.

Thus, in our investigation we succeeded in observing the evolution of powerful plane acoustic waves in a nonlinear medium. The experimental results are in good agreement with the classical theory of such waves.

REFERENCES

1. L. K. Zarembo and V. A. Krasil'nikov, *Introduction to Nonlinear Acoustics* (in Russian), Moscow, 1966.
2. O. V. Rudenko and S. I. Soluyan, *Theoretical Foundations of Nonlinear Acoustics* (in Russian), Moscow, 1975.
3. K. A. Naugol'nykh and L. A. Ostrovskii, *Nonlinear Wave Processes in Acoustics* (in Russian), Moscow, 1990.
4. N. S. Bakhvalov, Ya. M. Zhileikin, and E. A. Zabolotskaya, *Nonlinear Theory of Acoustic Beams* (in Russian), Moscow, 1982.
5. V. G. Andreev, A. A. Karabutov, and O. V. Rudenko, *Vest. Mosk. Univ. Fiz. Astron.*, vol. 25, no. 3, p. 35, 1984.
6. A. M. Galstyan, *Laser Excitation of Nonlinear Acoustic Pulses in Liquid* (in Russian), Ph. D. (Phys.-Math.) Thesis, FIAN, Moscow, 1982.
7. A. A. Kharkevich, *Transient Wave Phenomena* (in Russian), Moscow, Leningrad, 1950.
8. V. E. Gusev and A. A. Karabutov, *Laser Optoacoustics* (in Russian), Moscow, 1991.
9. V. G. Andreev and A. N. Dubrovskii, in: *Abstracts of Papers for the 11th USSR Conference* (in Russian), p. 67, Moscow, 1991.
10. D. R. Bacon, *Proc. Ultrasonic Symp.*, p. 700, 1992.

8 June 1992 Department of Acoustics