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FOCUSING OF WEAK ACOUSTIC PULSES

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The process of focusing of acoustic pulses is investigated theoretically and experimentally. The analysis is carried out in the parabolic approximation. It is shown that the pulse profile at an arbitrary point of a Gaussian beam can be found using Green's function. The calculation of the acoustic field is performed for different focusing conditions. The numerical results are in good agreement with the experimental data. The proposed approach makes it possible to calculate pulse acoustic fields sufficiently accurately and take account of the effects of focusing and diffraction.

By now the focusing of sinusoidal waves has been thoroughly studied, but the interest in the focusing of pulses has been rather recent. A pulse possesses a wide frequency spectrum, and therefore its focusing proceeds differently compared to that of sinusoidal waves.

In the present paper a technique is suggested and used for describing the pulse focusing, and the numerical calculation results are compared with the data of an experiment in which the method of laser optoacoustic generation has been employed. A laser pulse of duration 30 ns at wavelength 1.06 μm was absorbed in a copper chloride solution. The thermal expansion of the solution, whose surface was shaped by a spherically concave quartz plate, produced an acoustic pulse of duration 0.2 μs with peak pressure 1–30 atm [1]. The focusing radius was 20 cm with the initial beam diameter of 5 cm. Analysis of wave beams can be substantially simplified by the use of the parabolic (quasi-optic) approximation [2]. It is also advisable to apply this approach for studying pulses.

Let a focused acoustic pulse with exponential time profile $\Psi(t)$ be set at the entrance to the medium and let its peak pressure be distributed in the transverse direction according to the Gaussian law:

$$P(z=0,\, r,t) = P_0 \exp\{-(r/a)^2\} \exp\{-|t/t_0 + r^2/(2RC_0t_0)|\}.$$

Here t is time, t_0 is the characteristic pulse duration, P_0 is the peak pressure at the axis; a is the acoustic beam radius; z and r are the longitudinal and transverse coordinates, respectively; C_0 is the sound velocity; and R is the radius of curvature of the wave front. The exponential approximation of the time profile is the best for acoustic pulses resulting from the laser optoacoustic generation in liquid. By virtue of the linearity of the problem, the focusing of pulses in the parabolic approximation can be calculated based on the well-known results on the focusing of sinusoidal waves [3]:

$$P/P_0 = \int_{-\infty}^{\infty} (\partial/\partial t)[\Psi(t-t')]g(z,r,t')\,dt', \tag{1}$$

$$g(z,r,t) = [1/|1-z|]\Theta(F_1)\exp(-F_2)I_0([2r/(1-z)][F_1]^{1/2}),$$

$$F_1 = [z/(1-z)][r^2/(1-z)+Dt]; \quad F_2 = [r/(1-z)]^2 + F_1,$$

$$g(z=1,\, r,t) = (4\pi r^2)^{-1/2}\exp\{-[(Dt-r^2)/(2r)]^2\}.$$

Here Θ is the Heaviside function, I_0 is the modified Bessel function, and g is Green's function. The dimensionless parameter $D = R/x_d = 2RC_0t_0/a^2$ characterizes the diffraction effect and reflects the ratio of the focusing radius to the characteristic diffraction length x_d. The acoustic pressure P was calculated by formulas (1) using the Simpson method for different D, r, and z. On the beam axis ($r = 0$) the solution can be expressed in an analytical form, which made it possible to check the calculation accuracy: the numerical calculation data coincided with the analytical results to an accuracy of 10^{-3}.

We consider the dynamics of the pulse shape and amplitude. In its propagation, the initial unipolar pulse turns into a bipolar one owing to the arrival of a rarefaction pulse from the edges. Both the amplitude of compression phase P^+ and that of rarefaction phase P^- increase as the wave approaches the focus. At the focus we have $P^+ = P^- = P_0/D$, and subsequently P^+ and P^- decrease.

The magnitude of the peak pressure at the focus is not infinitely large because it is restricted by the diffraction, and it increases in direct proportion to P_0, as it should in the absence of nonlinear effects. The variation of the peak pressure, normalized to P_0, along the axis for different values of D is illustrated in Fig. 1. As the parameter D increases, the diffraction manifests itself more strongly: the pressure at the focal point decreases and the focal region expands. The value of $D = 0.2$ corresponds to the performed experiment (see Fig. 1). As is seen, the experimental results are well described by the corresponding curve 2. The measured diameter of the neck at the level $1/e$ was 6 mm, which is close to the calculated value of 5 mm.

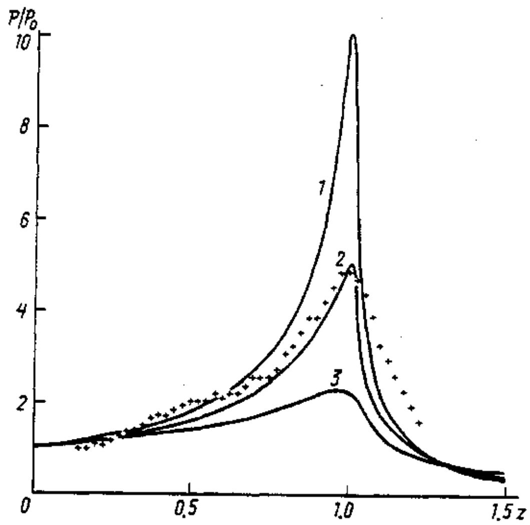

Fig. 1

Peak pressure, normalized to P_0, along the axis for $D = 0.1$ (1), 0.2 (2), and 0.4 (3). The crosses represent experiment at $P_0 = 4$ atm and $D = 0.2$.

Of great importance is that in the focusing of weak pulses the gain factor $K = P^+/P^-$ of the peak pressure at the focus is constant, equal to $1/D$, and does not depend on the initial pulse amplitude. This theoretical result was confirmed experimentally: for $P_0 = 2$, 5, and 7 atm $K = 5$. However, beginning with $P_0^* = 8$ atm the factor K decreases: at $P_0 = 15$ atm and 30 atm the factor K was 3.7 and 2.6. This decrease in K is due to the manifestation of another mechanism that differs fundamentally from diffraction: the nonlinearity. At P_0^* in the converging pulse there appears a discontinuity at the focus, and at $P_0 > P_0^*$ this occurs before the focus. The pulse energy is intensively dissipated at the discontinuity, and the pulse arriving at the focus is already substantially weakened.

A model that does not take into account the nonlinearity cannot be used to describe the focusing of pulses with $P_0 > P_0^*$. The pressure P_0^* can be regarded as an upper bound (with respect to the initial pressures) of the range of applicability of the linear theory. For $D < 0.5$ we have [3]

$$P_0^* = \rho_0 C_0^3 t_0 / (\varepsilon R \ln(1/D)),$$

where ρ_0 and ε are the density and nonlinearity of the medium, respectively. For example, in our experiment the pressure P_0^* calculated by this formula is 7 atm, which agrees with the measurement data.

The model under consideration makes it possible to calculate the time profile of the pulse at any point of the acoustic field. The scheme of isobaric lines for P^+ and P^- in the coordinates (r, z) is presented in Fig. 2. These lines are closed pear-shaped curves, not symmetric relative to the focal plane. This means that, as the focus is approached, the rates of growth and decay of peak pressures are different. For instance, as the focus is approached, the rate of growth of the pressure P^+ is less than the rate of decay when receding from the focus. By contrast, P^- grows faster before the focus and decays much more slowly behind it. It should also be noted that the lines for P^+ and P^- with the same numbers trace figures of different areas. In other words, the volume where the pressure P^+ exceeds a prescribed level is less than the analogous volume corresponding to the pressure P^-. Besides, Fig. 2 demonstrates inversion of the monopolar signal after it passes the focus: at $z > 1$ the value of P^- exceeds P^+, and the compression pulse turns into a rarefaction pulse.

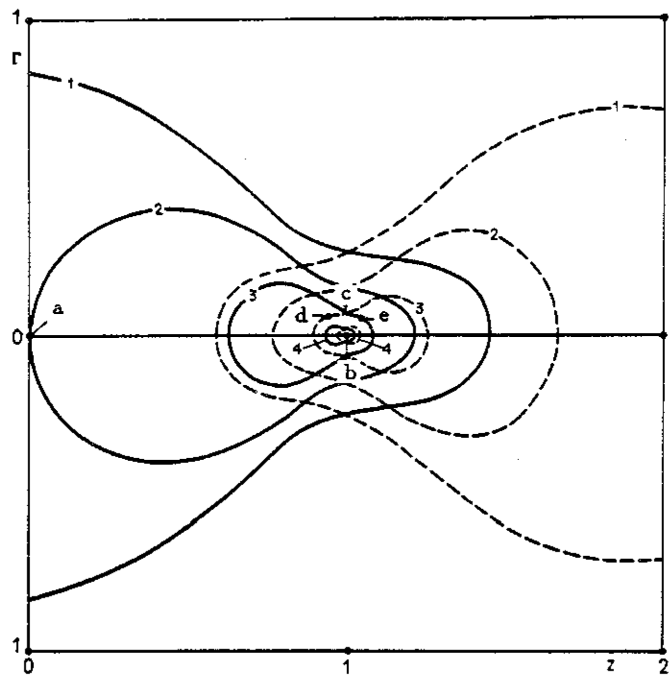

Fig. 2

Scheme of isobars for P^+ (solid lines) and P^- (dashed lines) in the coordinates (r, z). The numbers 1–4 correspond to the extremal values $0.5\,P_0$, P_0, $2\,P_0$, and $4\,P_0$, respectively.

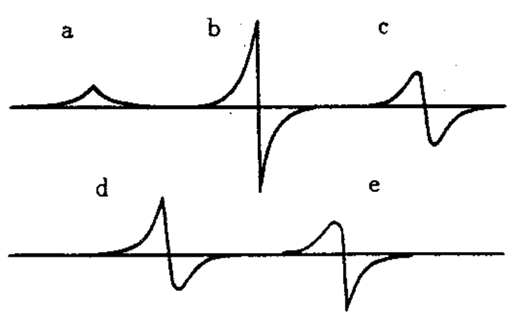

Fig. 3

Pulse profiles at the points *a*–*e* (see Fig. 2).

The time profiles of the pulses at the points *a*–*e* (see Fig. 2) are shown in Fig. 3. The initial pulse (*a*) is monopolar and exponential. At the focal point (*b*) $P^+ = P^- = 1/D$ attain maximum values and the shape of the pulse is related to its initial profile. When displaced in the focal plane (*c*), the profile remains symmetric but becomes smoother and somewhat more elongated. Each pulse behind the focus corresponds to another pulse before the focus and is, to a high accuracy, an inverted copy of the latter (d and e).

Thus, in contrast to the focusing of sinusoidal waves (when the wave shape does not change and remains sinusoidal and the isobaric surfaces are symmetric ellipsoids), the focusing of pulses has a number of distinctions. The pulse profile changes fundamentally, and the isobaric surfaces are pear-shaped. Such pulsed acoustic fields can be calculated with consideration for the effects of focusing and diffraction using the proposed method.

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