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A METHOD OF STUDYING THE FREQUENCY PROPERTIES OF A TWO-COMPONENT SIGNAL IN THE IONOSPHERE

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A method of determining the stochastic properties of the instantaneous frequency of a two-component short-wave signal in the ionosphere is set forth. An unconventional approach using non-integer statistical moments is employed.

Many problems of radio communications in the short-wave range require data on the signal phase derivative, i. e., on the instantaneous frequency, for their solution. The effect of such a randomly-inhomogeneous medium as the ionosphere leads to the appearance of fluctuation properties in the instantaneous frequency. These properties complicate substantially the operation of radio systems, so the study of the probabilistic properties of the signal phase derivative is of practical interest. A widely known solution of this problem for a model of the short-wave signal consists in a superposition of the determinate sinusoidal component and a narrow-band noise which has a Gaussian probability distribution.

The objective of this work is to develop a procedure for determining the frequency properties of ionospheric signals for another two-component model which, as is demonstrated, e. g., in [1], is more adequate to the ionospheric conditions.

In this model the signal consists of a quasi-regular (mirror) component $E_0(t)$ and a narrow-band scattered component $E_p(t)$:

$$E(t) = E_0(t) + E_p(t) = A\cos(\omega_0 t - \varphi(t)) + a(t)\cos(\omega_0 t - \psi(t)) = R(t)\cos(\omega_0 t - \theta(t)). \qquad (1)$$

The amplitude of the quasi-regular component $A = \text{const}$. The initial phase $\varphi(t)$ has a uniform probability distribution on the interval $[-\pi, \pi]$, and the derivative of the initial phase $\dot\varphi = \dot\varphi(t)$ has a Gaussian probability distribution $N(0, \sigma_0^2)$. The probability distribution of the scattered component $E_p(t)$ is assumed to be normal: $N(0, \sigma^2)$. The constants $E_0(t)$ and $E_p(t)$ are statistically independent. Under these conditions random process (1) is stationary and ergodic [2].

Within the framework of this model of signal (1), physically justifiable for the ionosphere, it was obtained in [3] that the derivative of the phase $\dot\theta = \dot\theta(t)$ is a random process with the probability density

$$W_1(\dot\theta_1) = 2^{-3/2}\pi^{-1/2}\sigma_1^{-1}(1+\dot\theta_1^2)^{-3/2}\exp\{-\beta^2\}\int\limits_{-\infty}^{\infty}\exp\{-\dot\varphi_1/2\sigma_1^2\}\exp\{P\}[(1+Q)I_0(P)+QI_1(P)]\,d\dot\varphi_1. \qquad (2)$$

Here $\dot\theta_1 = \dot\theta/\sqrt{-\rho_0''}$, where $\rho_0'' = \rho''(0)$ is the second derivative of the coefficient of autocorrelation $\rho(\tau)$ of low-frequency phase-quadrature components of the scattered component $E_p(t)$; β^2 is the signal-to-noise parameter (the ionospheric disturbance parameter) which characterizes the ratio of the powers $E_0(t)$ and $E_p(t)$; $\dot\varphi_1 = \dot\varphi/\sqrt{-\rho_0''}$; $\sigma_1^2 = \sigma_0^2/\sqrt{-\rho_0''}$, where $\dot\varphi = \dot\varphi(t)$ is the derivative of the initial phase of the quasi-regular component $E_0(t)$ and σ_0^2 is the variance of the derivative of the initial phase $E_0(t)$;

$$P = \frac{\beta^2(1+2\dot\varphi_1\dot\theta_1 - \dot\varphi_1^2)}{2(1+\dot\theta_1^2)}, \qquad Q = \frac{\beta^2(1+\dot\varphi_1\dot\theta_1)^2}{1+\dot\theta_1^2};$$

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$I_0(P)$, $I_1(P)$ are modified Bessel's functions.

The probability distribution of the derivative of the phase $\dot{\theta}$ has some specific features. This distribution is symmetrical, i. e., $W_1(\dot{\theta}_1) = W_1(-\dot{\theta}_1)$, and has no statistical moments, except the first one; $\langle\dot{\theta}_1\rangle = 0$.

The absence of variance forces us to introduce the mean absolute value of the phase derivative in order to characterize probability density (2):

$$\langle|\dot{\theta}_1|\rangle = \pi^{-1}\int_0^{\pi}(1 + 2\sigma_1^2\beta^2\cos^2\alpha)^{1/2}\exp\{-\beta\sin^2\alpha\}\,d\alpha. \tag{3}$$

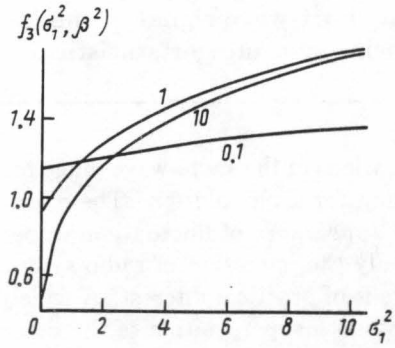

Fig. 1

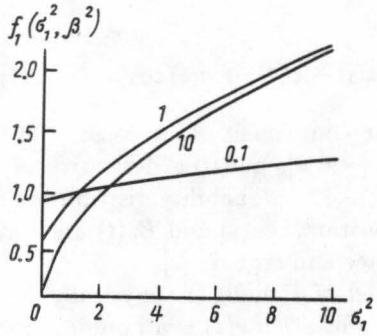

Fig. 2

The parameters of the probability density $W_1(\dot{\theta}_1)$ and of the statistical moment $\langle|\dot{\theta}_1|\rangle$ (the quantities which determine the particular form of distribution (2) and moment (3)), are ρ_0'', σ_0^2, and β^2. The parameters can also be of interest as such. They characterize in a definite manner the frequency properties of the components of signal (1): ρ'' characterizes the scattered component $E_p(t)$, because it can be shown that the width of the power spectrum $\Delta\omega \sim \sqrt{-\rho_0''}$; σ_0^2 characterizes the quasi-regular component $E_0(t)$, because by definition the variance characterizes the amplitude of fluctuations of the instantaneous frequency.

When determining these parameters, we shall assume that the parameter β^2 can be found independently, for instance, in the standard manner through the statistical moments of the envelope $R(t)$ of signal (1). Then the remaining two parameters ρ_0'' and σ_0^2 will be defined by one equation, namely, by expression (3), which can be written in a general functional form as

$$\langle|\dot{\theta}_1|\rangle = f_1(\sigma_1^2, \beta^2) \tag{4}$$

or

$$\langle|\dot{\theta}|\rangle = \sqrt{-\rho_0''}f_1(\sigma_1^2, \beta^2). \tag{5}$$

We shall obtain the second equation by introducing the non-integer moment for $\dot{\theta}_1$:

$$\langle|\dot{\theta}_1|^{1/2}\rangle = \int\limits_{-\infty}^{\infty} |\dot{\theta}_1|^{1/2} W_1(\dot{\theta}_1)\, d\dot{\theta}_1 = 2\sqrt{\pi}\exp\{\beta^2\}\Gamma^{-2}(1/4)$$

$$\times \int\limits_0^{\pi} (1 + 2\sigma_1^2\beta^2\cos^2\alpha)^{1/4}\, {}_1F_1\left(\frac{3}{4}, \frac{1}{2}; \beta^2\cos^2\alpha\right) d\alpha = f_2(\sigma_1^2, \beta^2),$$

where $\Gamma(x)$ is the gamma-function, ${}_1F_1(a, b; x)$ is the degenerate hypergeometrical function. Since

$$\langle|\dot{\theta}|^{1/2}\rangle = (-\rho_0'')^{1/2} f_2(\sigma_1^2, \beta^2)$$

using (5), we have

$$\frac{\langle|\dot{\theta}|\rangle}{\langle|\dot{\theta}|^{1/2}\rangle} = \frac{f_1(\sigma_1^2, \beta^2)}{f_2(\sigma_1^2, \beta^2)} = f_3(\sigma_1^2, \beta^2). \tag{6}$$

If the statistical moments $\langle|\dot{\theta}|\rangle$ and $\langle|\dot{\theta}|^{1/2}\rangle$ are determined experimentally, then σ_1^2 is found from Eq. (6), then ρ_0'' is found from Eq. (5), and further $\sigma_0^2 = -\rho_0''\sigma_1^2$. It is most simple to find the answer to this problem graphically, using the plots of functions (4) and (6) presented in Figs. 1 and 2, respectively. The plots represent $f_1(\sigma_1^2, \beta^2)$, $f_3(\sigma_1^2, \beta^2)$ as functions of the argument σ_1^2 at different values of β^2 as the parameter (the numerals at the curves).

Thus, within the discussed model of the signal, having at our disposal the statistical characteristics of the phase derivative (instantaneous frequency) of the whole signal (1), we can obtain information on the frequency properties of the signal components: the quasi-regular (mirror) component $E_0(t)$ and the scattered component $E_p(t)$.

The found parameters ρ_0'' and σ_0^2 can also serve as a source of information on the signal-forming processes in the ionosphere. We would like also to emphasize here that the unconventional approach with the use of non-integer statistical moments allowed us to solve new problems.

REFERENCES

1. A. G. Vologdin, S. F. Mirkotan, and S. M. Savel'ev, *Geomagnet. Aeronom.*, no. 2, p. 226, 1972.
2. A. G. Vologdin and V. D. Gusev, *Vest. Mosk. Univ. Fiz. Astron.*, vol. 32, no. 3, p. 42, 1991.
3. A. G. Vologdin and V. D. Gusev, *Radiotekhnika*, no. 2, p. 69, 1994.

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