The problem of tsunami generation by traveling sea-floor shoves was investigated by laboratory modeling and theoretically within the framework of linear potential theory. The relationships were derived between the maximum amplitude and energy of the waves and the sea-floor shove propagation velocity. The experimental data are in good agreement with the theoretical relationships.

In the majority of studies on the process of tsunami generation by sea-floor shoves it is assumed that the sea-floor movement occurs simultaneously over the entire active region. This, however, is not always the case in nature. In this connection, of great interest for the problem of tsunami are perturbations of the ocean floor, which received the name of traveling shoves in the literature [1-4]. Consecutive dislocation of sea-floor blocks, a crack propagating along the sea-floor [5], or a dispersing packet of seismic waves [6] may serve as a natural analog of the traveling shove. Moreover, the traveling shove may also be associated with a submarine landslide [7, 8]. It should be pointed out that almost all the above-cited investigations are theoretical and were carried within the framework of long-wave approximation. The latter may, in the case of traveling shoves, substantially distort the result.

In this work we studied wave generation by the method of laboratory modeling and theoretically using the linear potential theory.

EXPERIMENTAL

We carried out laboratory modeling in a rectangular open hydraulic channel whose size was 15 x 15 x 330 cm. As the source of waves we used three similar pneumatic bottom wave generators (with length \( l = 30 \) cm) [9] that were arranged at the center of the hydraulic channel and actuated in succession (Fig. 1a). The movement of each generator simulated a vertical shove of the floor and was controlled by a transducer designed as an inductance coil with a movable core connected with the floor. The arrangement of the floor displacement transducers is indicated in Fig. 1a by arrows 1, 2, and 3. Each of the generators performed individual vertical displacements with an amplitude not exceeding 2 mm. The duration \( \tau \) of the movement of each generator was chosen so as to provide pulsed shoves: \( \tau \ll (gH)^{-1/2} \), where \( g \) is the gravitational acceleration. The depth of water \( H \) in the experiments was 3, 5, 7, and 10 cm.

Perturbation of the free surface of water was measured by means of two IR wave-height recorders [10] placed at the boundaries of the generation region (their position is indicated in Fig. 1a by arrows 4 and 5). The motion of the bottom wave generators and the signals coming from the wave-height recorders were recorded using a multichannel high-speed automatic recorder Type N 3030. The records were used to determine (1) the maximum amplitude of the wave perturbation traveling in and opposite to the direction of the shove propagation, (2) the vertical displacement of each bottom wave generator \( \eta_i \), and (3) the shove propagation velocity \( v = (1/t_{12} + 1/t_{23})/2 \), where \( t_{12} \) and \( t_{23} \) are the periods of time between switching on the first and second generator and between the second and third generator, respectively. The total number of the experiments was over 100.
In physical modeling of the process of generation and propagation of tsunami waves, it was suggested in [11] that the following three criteria should be considered for better similarity:

$$\Pi_1 = H/\lambda; \quad \Pi_2 = A/H; \quad \Pi_3 = T(g/H)^{1/2},$$

where $\lambda$, $A$, and $T$ denote the wavelength, the amplitude, and the wave period, respectively.

For actual tsunami waves the values of these criteria, according to [11], fall into the following range:

$$\Pi_1 \sim 10^{-2}, \quad \Pi_2 \sim 10^{-3}, \quad 12 \leq \Pi_3 \leq 360.$$

In our experiments, the criteria assumed the following values:

$$10^{-2} < \Pi_1 < 10^{-1}, \quad 10^{-2} < \Pi_2 < 10^{-1}, \quad 5 < \Pi_3 < 40.$$

Taking into account the fact that for the case of wave generation the value of the criterion $\Pi_2$, as shown in [12], can be increased appreciably, the waves obtained in the experiment can be regarded as being similar to actual short-period tsunami waves in the open ocean.
The results of our experiments are presented in Fig. 2 as plots of the maximum amplitude of wave perturbation $A_{\text{max}}$ versus the velocity of the floor shove propagation $v$. The plot is built in dimensionless coordinates: the wave disturbance amplitude is normalized to the mean floor shove amplitude for the particular experiment $A_0 = (\eta_1 + \eta_2 + \eta_3)/3$, and the velocity $v$ is normalized to the propagation velocity of long waves $v_0 = (gH)^{1/2}$. Data on the maximum amplitude of the wave traveling in the direction opposite to the shove propagation correspond to negative values of the dimensionless velocity. The large scatter of experimental data due to the fact that the amplitudes of the floor shoves $\eta_i$ were not strictly equal made it impossible to separate the experimental curves for different water depths; therefore, the experimental points in Fig. 2 represent data averaged over all the specified water depths $H$.

**MATHEMATICAL MODEL**

We shall consider a layer of an ideal incompressible homogeneous liquid in the field of gravity. The layer is unbounded along the axis $OX$ and has a constant depth $H$ (see Fig. 1b). Let the origin of the rectangular system of coordinates $OXY$ be disposed on the undisturbed free surface of the liquid, and the axis $OZ$ be directed vertically upward. In order to find the wave perturbation on the liquid surface induced by floor movements occurring in accordance with the law

$$z = -H + \eta(x, t),$$

one has to solve the problem with respect to the potential of the liquid flow velocity $F(x, z, t)$:

$$\nabla F = 0,$$

$$F_{tt} = -gF_z, \quad z = 0,$$

$$F_z = \eta_t, \quad z = -H.$$

The displacement of the liquid free surface is expressed in terms of the flow velocity potential in the following manner [13]:

$$\xi(x, t) = -g^{-1}F_t(x, 0, t).$$

As the model of the traveling shove, we shall choose the following form of the floor movement:

$$\eta(x, t) = \eta_0[\theta(x) - \theta(x - b)][1 - \theta(x - vt)],$$

where $\theta(x)$ is the Heaviside step function.

We shall seek a solution of the stated problem by the method of Laplace and Fourier transformations with respect to the time and space coordinates, respectively:

$$F(x, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{s+i\infty} dp \exp \{pt - ikz\}(A(p, k)\cosh kz + B(p, k)\sinh kz).$$

To solve the problem we used methods adopted in the theory of functions of one complex variable (no details of the procedure will be given here). As a result, we obtained the following expression for the displacement of the liquid free surface:

$$\xi(x, t) = \frac{\eta_0}{4\pi i} \int_{-\infty}^{\infty} \exp (-ikz) \left\{ \frac{\exp [ib(k + P/v)] - 1}{k + P/v} \exp (-iPt) \right. 

\left. + \frac{\exp [ib(k - P/v)] - 1}{k - P/v} \exp (iPt) \right\} dk,$$

where $P = (gktanh(kH))^{1/2}$, which holds on condition that $t \geq b/v$. The imaginary part of expression (1), in view of the oddness of the corresponding part of the integrand function, is equal to zero. The integral in the real part of Eq. (1), preliminarily reduced to the dimensionless form in accordance with the formulas

$$t^* = t(g/H)^{1/2}, \quad x^* = x/H, \quad \alpha = b/H, \quad k^* = kH, \quad v^* = v(gH)^{-1/2},$$

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was calculated numerically.

The results of the calculations are given in Fig. 2 as curves of the maximum amplitude of the wave at the boundaries of the generation region, at the points \( x = 0 \) (\( v/v_0 < 0 \)) and \( x = b \) (\( v/v_0 > 0 \)). In view of a certain difference in the types of the floor movement in the laboratory model and in the theoretical model, the experimental and theoretical dependences cannot coincide completely. This is particularly apparent at \( v/v_0 < 0 \). The latter circumstance is also connected with the fact that the difference in the amplitudes of displacements of the bottom wave generators could reach \( \sim 30\% \), whereas the maximum amplitude of the wave traveling in the direction opposite to the shove propagation is determined by the amplitude of the maximum shove. Consequently, owing to the normalization of the wave amplitude to the value \( A_0 = (\eta_1 + \eta_2 + \eta_3)/3 \), dimensionless amplitude will be a fortiori overestimated compared with the case when the amplitudes of the floor displacements are the same. Nevertheless, both the experiment and the theory demonstrate that at floor shove velocities close to the velocity of long waves the amplitude of the wave traveling in the direction of the shove propagation sharply increases. It is known that the amplitude of the tsunami wave in the open ocean cannot exceed the amplitude of the piston-like shove of the floor. In the case of the traveling shove (Fig. 2) the wave amplitude can substantially exceed the floor displacement amplitude, and this becomes more pronounced as the size of the generation region \( b \) increases.

The maximum wave amplitude, no doubt, is of great interest, but in the dispersing wave in the course of its propagation the amplitude experiences rapid changes; therefore, it is more convenient to use energy to characterize the intensity of wave generation by the traveling shove of the sea-floor. The wave energy was calculated by the following formula:

\[
W(t) = \rho a \int \xi^2(x, t) \, dx,
\]

where \( \rho \) is the water density, \( a \) is the channel width, and \( W(t) \) is the doubled potential energy of the wave. The energy was calculated for the time \( t^* = 50 \), when the equilibrium between the potential and kinetic energy was attained and the value of \( W \) was no longer time-dependent. The results of the calculations are presented in Fig. 3 in dimensionless coordinates. The energy values are normalized to \( W_0 = \rho g a b \eta_0^2/2 \). The dependence of the energy has a resonance character, which begins to manifest itself when the size of the generation region \( b \) exceeds the depth \( H \) more than two times. At shove propagation velocities close to the velocity of long waves, the energy of the wave traveling in this direction has a maximum, whose value increases almost linearly as the size of the generation region \( b \) grows. When the velocities substantially exceed the velocity of long waves, the energies of the waves traveling in and opposite to the direction of the shove propagation level out, and the directivity of energy radiation is lost.

![Fig. 3](image_url)

Calculated energy of the wave excited by the traveling shove of the sea-floor vs. shove propagation velocity. Positive \( v/v_0 \) values correspond to waves propagating in the positive direction of the \( OX \) axis; negative \( v/v_0 \) values correspond to waves propagating in the negative direction of the \( OX \) axis. Curves 1–4 correspond to the values of the parameter \( \alpha = 1, 2, 5, \) and 10.
CONCLUSIONS

Our investigation has demonstrated that traveling sea-floor shoves can lead to excitation of tsunami waves which possess appreciably greater amplitudes and energies than in the case of the conventional piston mechanism. The main part of the energy and large amplitudes belong to the waves traveling in the direction of shove propagation. The most important parameter is the ratio of the generation region to the depth of the water basin. As this parameter increases, the energy and amplitude of the excited waves grow rapidly.

All these effects are important at shove propagation velocities \( v^* \) ranging from 0.3 to 5 and \( \alpha > 1 \). Outside these ranges of the parameters the resonance effect and the directivity of energy radiation (in the two-dimensional case) almost disappear.

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