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MOTION OF THE EARTH'S CENTER OF MASS INDUCED BY GLOBAL CHANGES IN ITS DYNAMIC STRUCTURE AND BY TIDAL DEFORMATIONS

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Using a simple model of a changeable Earth, we studied the displacement of its center of mass with respect to the characteristic point on the Earth's axis for which the coefficient of the third zonal geopotential harmonic is zero. Because of global changes in the dynamic structure of the Earth, the velocity of the secular displacement of its center of mass toward the North pole can be as high as 2 cm/century. The basic periodic motion of the center of mass, resulting from tidal deformation of the Earth, has an amplitude of 8.9 cm and a period of 27.32 days.

A global change in the dynamic structure of the Earth is demonstrated by the secular variations in the parameters of the Earth's gravitational field revealed by laser observations with the *Lageos* and *Etalon* satellites [1–3] (Table 1).

Table 1

Parameters $\dot{J}_2$ and $\dot{J}_3$ and Calculated Values of Velocity $\dot{z}_C$

Authors, year	$\dot{J}_2 \times 10^9$, 1/century	$\dot{J}_3 \times 10^9$, 1/century	$\dot{z}_C$, cm/century
Yoder et al., 1983	-3.0 ± 0.3	-1.0	1.96
Cheng et al., 1989	-2.5 ± 0.1	-0.1 ± 0.3	0.21 ± 0.59
Marchenko, 1992	-2.7 ± 0.7	-0.1 ± 0.6	0.21 ± 1.18
Ibid	-2.74 ± 0.20	-0.91 ± 0.23	1.80 ± 0.46

At present, the secular variation of the coefficient J_2 of the second zonal geopotential harmonic ($\dot{J}_2 < 0$) is being clearly discerned, which is not the case with the secular variations of other coefficients (including that of the third zonal harmonic J_3).

The coefficient J_3 is a measure of asymmetry in the distribution of the Earth's densities with respect to the plane that passes through its center of mass C orthogonally to the polar axis of inertia Cz, therefore the values of the secular variations of $\dot{J}_2$ and $\dot{J}_3$ allow one to evaluate certain effects in the relative motion of the Earth's center of mass.

In the present study we make preliminary evaluations of the secular and periodic effects in the relative motion of the Earth's center of mass, based on a simple model of the Earth as a changeable axially symmetric body (the coefficients J_2 and J_3 are specified functions of time).

In order to describe the relative motion of the Earth's center of mass along its polar axis of inertia, different characteristic points can be used as an origin, e.g., those corresponding to the location of the center of mass at a given epoch or in the nondeformed state of the Earth. In the present study, the proposed origin is the point O located on the Cz axis, for which in the Cartesian coordinate system $Oxyz$ (the axes Ox and Oy are orthogonal to the polar axis Cz) the geopotential coefficient is $J_3{}^* = 0$.

Let z_C be the coordinate of the point C on the Oz axis and R be the mean Earth's radius. The value of z_C is found as the appropriate root of the cubic equation

$$z_C^3 - 3R^2 J_2 z_C - J_3 R^3 = 0$$

and is given by the formula

$$z_C = -2R\sqrt{J_2}\cos\left[\frac{1}{3}\cos^{-1}\left(\frac{J_3}{2\sqrt{J_2^3}}\right) - \frac{2\pi}{3}\right] \simeq -\frac{RJ_3}{3J_2}. \tag{1}$$

The motion velocity of the center of mass z_C, derived from Eq. (1), is given by

$$\dot{z}_C = (-3z_C\dot{J}_2 + R\dot{J}_3)\Big/\left[3\left(\frac{z_C^2}{R^2} - J_2\right)\right] \simeq -z_C\left(\frac{\dot{J}_3}{J_3} - \frac{\dot{J}_2}{J_2}\right). \tag{2}$$

For the Earth, $J_2 = 1.082628 \times 10^{-3}$, $J_3 = -2.5280 \times 10^{-6}$, $R = 6378$ km [4]. From Eq. (1) we find $z_C = 4.984904$ km, that is, the point O is displaced toward the South pole with respect to the Earth's center of mass. The values of $\dot{z}_C$, calculated by Eq. (2) for different sets of values of $\dot{J}_2$ and $\dot{J}_3$, are listed in Table 1. Thus, the velocity of the secular displacement of the Earth's center of mass toward the North pole can be as high as 2 cm/century.

Note also that the condition $\dot{z}_C > 0$ and Eq. (2) impose a restriction on the possible values of $\dot{J}_3 < (J_3/J_2)\dot{J}_2 = 0.059 \times 10^{-12}$ 1/century. Under this condition, the Earth's center of mass is displaced in the northern direction.

Tidal deformations of the Earth give rise to periodic variations of the coefficients J_2 and J_3, which in turn lead to periodic changes in the relative positions of the points C and O. The principal tidal variations of J_2 and J_3, resulting from the deformations of the elastic mantle, are given by [5]

$$\begin{aligned}\delta J_2 =& 10^{-9}(2.0124\cos 2(F+D) + 0.9342\cos 2(F-D+\Omega) \\ &- 0.8433\cos\Omega + 0.8342\cos(2F+\Omega)), \\ \delta J_3 =& 10^{-11}(2.46309\sin(F - l_(+ \Omega) \\ &+ 4.51742\sin(F+\Omega) + 0.712936\sin F),\end{aligned}$$

where F, D, $l_($, and Ω are the classical arguments of the theory of the Moon's orbital motion. The corresponding periodic variations of z_C (in centimeters) are given by

$$\begin{aligned}\delta z_C =& -4.838\sin(F - l_(+ \Omega) - 8.873\sin(F+\Omega) - 1.400\sin F \\ &- 0.927\cos 2(F+D) - 0.430\cos 2(F-D+\Omega) \\ &+ 0.388\cos\Omega - 0.384\cos(2F+\Omega).\end{aligned}$$

Thus, because of tidal deformations of the Earth's mantle, its center of mass can deviate from the point O by a distance of 18 cm, and the principal oscillation of the center of mass has an amplitude of 8.9 cm and a period of 27.32 days.

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REFERENCES

1. C. F. Yoder, I. G. Williams, I. O. Dickey, et al., *Nature*, vol. 303, p. 757, 1983.
2. M. K. Cheng, R. I. Eanes, C. K. Shum, et al., *Geophys. Res. Lett.*, vol. 16, no. 5, p. 393, 1989.
3. A. N. Marchenko, *Artificial Satellites. Planetary Geodesy*, vol. 28, no. 1, p. 17, Warsaw, 1992.
4. E. P. Aksenov, *Theory of Motion of Earth's Artificial Satellites* (in Russian), Moscow, 1977.
5. I. M. Ferrandiz and I. Getino, *AAS*, no. 3121, p. 915, 1993.

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