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FREQUENCY OF GEOMAGNETIC FIELD INVERSIONS AS A FUNCTION OF SOLAR SYSTEM POSITION IN THE GALAXY

K. V. Semenkov

The connection between the position of the Sun in the Galaxy and geodynamic phenomena is studied. The problem of the Sun's motion in the Galaxy has been solved without taking the influence of spiral arms into account. The Sun's orbit has been calculated for a particular form of the Galaxy potential. The Fourier analysis of the frequency of inversions of the magnetic field of the Earth as a function of time has been carried out. It has been shown that the frequency of inversions of the magnetic field of the Earth increased when the Sun left the spiral arm; the geomagnetic activity increased when the Sun either intersected the Galaxy plane or was at minimum separation from its center.

INTRODUCTION

This article is devoted to a very interesting problem, the time dependence of global geodynamic phenomena on the Earth position in the Galaxy. In particular, the researchers studying the Earth geological and geomagnetic history noticed that the Earth geodynamic activity shows certain periodicity which corresponds to the solar system motion with respect to the Galaxy center and plane.

1. Studies on the residual rock magnetization have shown that the Earth magnetic field reversed its polarity many times. These reversals occur irregularly, but on the average they are separated in time by 200 thousand years, and the reversal lasts for 10 to 25 thousand years. Actually, the intervals between the reversals can vary from 30 thousand to 30 million years (for instance, 235–290 million years ago the field had the reversed polarity for almost 60 million years). During the inversion, the field strength decreases by a factor of 5 to 7 [1, 2].

The general pattern of inversions exhibits certain regularities which can be observed in the graph of the normalized Fourier spectrum of the ratio of the number of intervals with direct polarity to the number of intervals with reversed polarity [3]. This ratio was calculated from the intervals of 15 million years duration. The authors of [3] have distinguished two basic harmonics with the periods of 260–340 and 70–90 million years.

2. Similar periods can also be traced in the geological history of the Earth. A period of 215 million years consisting of four cycles of 30, 50, 85, and 50 million years duration, is observed. The first cycle is characterized by global cooling off, a general uplift of continents, and seismic and volcanic activity increases. The third cycle involves global warming up, lithospheric platform collisions, and vibrational motions on stable blocks of the crust. The second and fourth cycles are intermediate [4].

The authors of the above publications have not found any interior or terrestrial sources of this periodicity, but assumed that it is associated with the Sun motion in the Galaxy, because the period of the Sun revolution around the Galaxy center is 240 million years, and the period of its oscillations in the direction perpendicular to the disk is 80 million years.

The goal of the present paper is to verify and refine the above-mentioned regularities in the light of modern data on the Galaxy structure.

POSITION OF THE SUN IN THE GALAXY

Our Galaxy belongs to a spiral type. Its mass is $M_G = 4 \times 10^{44}$ g. The Galaxy is usually regarded as consisting of three components: the disk, the bulge, and the halo. The bulge is the central thickening 1 kparsec in size. The halo is the spheroidal component. The disk is the most massive component, its mass is $M_D \cong 0.9 M_G$. The disk thickness is 0.5 kparsec in the neighborhood of the Sun and its radius is 20 kparsec. The disk has two spiral arms which rotate as a solid body with angular velocity $\Omega_p = 25$ km/s kparsec. Diffuse matter (dust, gas) is concentrated mainly in the arms. The disk rotates rapidly around the Galaxy center. The Sun is located in the galactic disk at a distance of 8.2 kparsec from its center and is 25 parsec apart from the Galaxy plane. The circular velocity of the centroid of the circumsolar stars is $v_0 = 220$ km/s. The Sun velocity with respect to the centroid is $\mathbf{v} = \{v_x, v_y, v_z\} = \{10, 10, 6\}$ km/s [5].

In order to clarify the question of the Galaxy influence upon terrestrial processes, the Sun orbit in the Galaxy must be calculated. We shall do it in several stages. First we calculate the orbit in a regular field, i.e., disregarding the effects of the arms and various nonuniformities. Then we take the arms into account, and after that we try to account for the influence of massive close bodies, possible vibrations of the galactic nucleus, and other effects.

MOTION OF THE SUN IN REGULAR GALAXY FIELD

The problem will be solved in a cylindrical system of coordinates (r, θ, z). Let us suppose that the galactic potential does not depend on θ and is symmetrical about the Galaxy plane, $U = U(r, z)$.

The Lagrange equations under these assumptions are

$$\begin{aligned} \ddot{r} - r\dot{\theta}^2 &= \frac{\partial U}{\partial r}, \\ \frac{d}{dt}(r^2\dot{\theta}) &= \frac{\partial U}{\partial \varphi} = 0, \\ \ddot{z} &= \frac{\partial U}{\partial z}. \end{aligned} \tag{1}$$

We seek a solution to this system in the form of a nearly circular orbit in the linear approximation. This can be done because $|\mathbf{v}|/v_0 \ll 1$.

As the zero approximation, we take a circular orbit

$$\begin{aligned} r &= r_0, \\ \theta &= \omega(t - t_0), \\ z &\equiv 0, \end{aligned} \tag{2}$$

where ω is the angular velocity of the Sun motion in the circular orbit, which corresponds to the period of revolution $T = 230$–240 million years. Substituting (2) into (1), we have

$$\begin{aligned} -r_0\omega^2 &= \left(\frac{\partial U}{\partial r}\right)_0, \\ r_0^2\omega &= L, \\ \left(\frac{\partial U}{\partial z}\right)_0 &\equiv 0, \end{aligned} \tag{3}$$

where the derivatives are computed at the point $(r_0, \theta, 0)$, $L = \text{const}$ is the projection of the Sun angular momentum onto the Z-axis, and $(\partial U/\partial z)_0 \equiv 0$ by virtue of the potential symmetry. Let us assign small perturbations to the coordinates and the angular velocity: $r = r_0 + \tilde{r}$; $\dot{\theta} = \omega + \tilde{\omega}$; $z = \tilde{z}$. Let us expand the derivatives of the potential into a Taylor series in the vicinity of the circular orbit accurate to within the linear terms in relation to $\tilde{r}$, $\tilde{\theta}$, $\tilde{z}$. Then system (1) becomes

$$\begin{aligned} &\ddot{r} - \omega^2 r_0 - \omega^2\tilde{r} - 2r_0\omega\tilde{\omega} = \left(\frac{\partial U}{\partial r}\right)_0 + \left(\frac{\partial^2 U}{\partial r^2}\right)_0 \tilde{r}, \\ &r_0^2\omega + 2r_0\omega\tilde{r} + r_0^2\tilde{\omega} = L, \\ &\ddot{z} = \left(\frac{\partial^2 U}{\partial z^2}\right)_0 z. \end{aligned}$$

Taking (3) into account, we obtain

$$\ddot{\tilde{r}} - \omega^2\tilde{r} - \left(\frac{\partial^2 U}{\partial r^2}\right)_0 \tilde{r} - 2r_0\omega\tilde{\omega} = 0,$$
$$2r_0\omega\tilde{r} + r_0^2\tilde{\omega} = 0,$$
$$\ddot{z} = \left(\frac{\partial^2 U}{\partial z^2}\right)_0 z.$$

Substituting the second equation of the system into the first one, we obtain the final system for the perturbations $\tilde{r}$, $\tilde{\omega}$, $\tilde{z}$

$$\ddot{\tilde{r}} + \left(3\omega^2 - \left(\frac{\partial^2 U}{\partial r^2}\right)_0\right)\tilde{r} = 0,$$
$$\tilde{\omega} = -2\frac{\tilde{r}}{r_0}\omega,$$
$$\ddot{z} = \left(\frac{\partial^2 U}{\partial z^2}\right)_0 z,$$

where, according to (3), $\omega^2 = -\frac{1}{r_0}\left(\frac{\partial U}{\partial r}\right)_0$.

The solution to this system can be represented in the form

$$r = r_0 + a\cos(\varkappa_1 t + \varphi_1),$$
$$\theta = \omega t - \frac{2\omega a}{r_0\varkappa_1}\sin(\varkappa_1 t + \varphi_1), \tag{4}$$
$$z = b\cos(\varkappa_3 t + \varphi_3),$$

where $\varkappa_1^2 = -\left[\frac{3}{r_0}\left(\frac{\partial U}{\partial r}\right)_0 + \left(\frac{\partial^2 U}{\partial r^2}\right)_0\right]$, $\varkappa_3^2 = -\left(\frac{\partial^2 U}{\partial z^2}\right)_0$. The quantities a, b, φ_1, φ_3 are determined from the initial conditions.

MOTION RELATIVE TO CIRCULAR ORBIT

According to (4), harmonic oscillations occur along the Z-axis. In order to determine the projection of the perturbed orbit onto the Galaxy plane, let us pass to a coordinate system which has its origin in the circular orbit and rotates around the Galaxy center with angular velocity ω.

In this system,

$$\tilde{r} = a\cos(\varkappa_1 t + \varphi_1),$$
$$\tilde{\theta} = -\frac{2\omega a}{r_0\varkappa_1}\sin(\varkappa_1 t + \varphi_1), \tag{5}$$

where $\tilde{\theta} = \theta - \omega t$. Hence it follows that $\frac{\tilde{r}^2}{a^2} + \frac{\tilde{\theta}^2}{(2a\omega/r_0\varkappa_1)^2} = 1$. Consequently, the perturbed motion in the Galaxy plane occurs along an ellipse. This ellipse is historically referred to as an epicycle.

NUMERICAL RESULTS

The Galaxy model for the calculations was taken from [6]. The potential has the form $U(r, z) = \Phi_0\varphi(\xi)$, where Φ_0 is the potential at the Galaxy center;

$$\varphi(\xi) = \frac{\alpha}{\beta + w(\xi)} \tag{6}$$

is the dimensionless potential; $\xi(\rho, \zeta, \gamma, \varepsilon)$ is the equipotential equation; α, β, γ, ε are the parameters of a particular Galaxy; $w(\xi) = \sqrt{1+\varkappa\xi^2}$; $\rho = r/R_0$; $\zeta = z/R_0$; R_0 is a scale parameter. The equipotential equation can be written as

$$\xi^2 = (p-\varepsilon)^2 + (1-\gamma)\rho^2 - 1, \tag{7}$$

where $p^2 = \gamma(1-\varepsilon^2)\rho^2 + (1+q)^2$, $q^2 = \gamma\varepsilon^2\rho^2 + \zeta^2 + \varepsilon^2$. Substituting the parameters of our Galaxy $\gamma = 0$; $\varkappa = 1$; $\alpha = 2$; $\beta = 1$; $\varepsilon = 0.1$ [7], we obtain $\xi^2 = (1+\sqrt{\zeta^2+\varepsilon^2}-\varepsilon)^2 + \rho^2 - 1$, $\varphi = 2/(1+\sqrt{1+\xi^2})$. It is obvious that

$$\frac{\partial U}{\partial r} = \frac{\Phi_0}{R_0}\frac{\partial\varphi}{\partial\rho}; \quad \frac{\partial U}{\partial z} = \frac{\Phi_0}{R_0}\frac{\partial\varphi}{\partial\zeta}; \quad \frac{\partial^2 U}{\partial r^2} = \frac{\Phi_0}{R_0^2}\frac{\partial^2\varphi}{\partial\rho^2}; \quad \frac{\partial^2 U}{\partial z^2} = \frac{\Phi_0}{R_0^2}\frac{\partial^2\varphi}{\partial\zeta^2}.$$

Let $\xi_0 \equiv \xi(\rho_0, 0)$, where $\rho_0 = r_0/R_0$. Then $\xi = \rho_0$ is the point in the circular orbit. Differentiating (6) and (7) and substituting $\xi = \xi_0$, $\rho = \rho_0$, $\varphi_0 = \varphi(\xi_0)$, $\zeta = 0$, we have

$$\begin{aligned}
&\left(\frac{\partial\varphi}{\partial\xi}\right)_0 = -\frac{1}{2}\frac{\varphi_0^2\rho_0}{\sqrt{1+\rho_0^2}}.\\
&\left(\frac{\partial^2\varphi}{\partial\xi^2}\right)_0 = \frac{1}{2}\left[\varphi_0^2\left(\varphi_0\frac{\rho_0}{\sqrt{1+\rho_0^2}} - \frac{1}{\sqrt[3]{1+\rho_0^2}}\right)\right],\\
&\left(\frac{\partial\xi}{\partial\rho}\right)_0 = 1; \quad \left(\frac{\partial^2\xi}{\partial\rho^2}\right)_0 = \left(\frac{\partial\xi}{\partial\zeta}\right)_0 = 0; \quad \left(\frac{\partial^2\xi}{\partial\zeta^2}\right)_0 = \frac{1}{\varepsilon\rho_0}.
\end{aligned} \tag{8}$$

Let us now find Φ_0 and R_0. In this case we start from the following grounds:

(1) for $\sqrt{r^2+z^2} \to \infty$ the potential becomes Newtonian;

(2) the rotation around the Galaxy center with angular velocity ω is determined from the derivative $(\partial U/\partial r)_0$.

We have

$$\begin{aligned}
U &= \frac{2\Phi_0}{1+\sqrt{\rho^2+(1+\sqrt{\zeta^2+\varepsilon^2}-\varepsilon)^2}} \xrightarrow[\zeta\to\infty]{\rho\to\infty} \frac{2\Phi_0}{\sqrt{\rho^2+\zeta^2}}\\
&= \frac{2\Phi_0 R_0}{\sqrt{r^2+z^2}} = \frac{GM}{\sqrt{r^2+z^2}},
\end{aligned} \tag{9}$$

where G is the gravity constant and M is the Galaxy mass;

$$\omega = \frac{v_0^2}{r_0} = -\left(\frac{\partial U}{\partial r}\right)_0 = -\frac{\Phi_0}{R_0}\left(\frac{\partial\varphi}{\partial\rho}\right)_0 = \frac{4\Phi_0\rho_0}{2R_0\sqrt{1+\rho_0^2}(1+\sqrt{1+\rho_0^2})^2}. \tag{10}$$

Let us express Φ_0 from (8) and substitute into (9)

$$\Phi_0 = \frac{GM}{2R_0}.$$

$$\frac{v_0^2}{GM} = \frac{r_0}{R_0^2}\frac{\rho_0}{\sqrt{1+\rho_0^2}(1+\sqrt{1+\rho_0^2})^2}, \tag{11}$$

$$\frac{v_0^2 r_0}{GM} = \frac{\rho_0^3}{\sqrt{1+\rho_0^2}(1+\sqrt{1+\rho_0^2})^2}. \tag{12}$$

Let us introduce new variables $A \equiv \sqrt{1+\rho_0^2}$, and $D \equiv v_0^2 r_0/GM$. Then, substituting D into (8) and taking into account the fact that $r_0/R_0 = \rho_0$, we obtain

$$\begin{aligned}
&2\Phi_0 = \frac{GM}{R_0},\\
&(D^2+1)A^3 - (D^2+3)A^2 + 3A - 1 = 0.
\end{aligned} \tag{13}$$

Substituting the experimentally found values $G = 6.67\times10^{-8}$ dyne $\cdot$ cm^2/g^2; $v_0 = 220$ km/s; $r_0 = 8.2$ kparsec; $M = 4\times10^{44}$ g, we find that $D = 0.225$. Solving the second equation, we find $A = 3.02$. Given A and D, we can easily determine that $\Phi_0 = 1.45\times10^{15}$ cm^2/s^2; $R_0 = 2.9$ kparsec; $\rho_0 = 2.85$. Substituting these values into (8), we have

$$\left(\frac{\partial\varphi}{\partial\xi}\right)_0 = -0.017,$$
$$\left(\frac{\partial^2\varphi}{\partial\xi^2}\right)_0 = 0.053, \tag{14}$$
$$\left(\frac{\partial\xi}{\partial\rho}\right)_0 = 1; \quad \left(\frac{\partial^2\xi}{\partial\zeta^2}\right)_0 = \frac{1}{\varepsilon\rho_0} = 3.509.$$

Substituting (14) into (4), we obtain $\varkappa_1 = 1.1\times10^{-15}$ s^{-1}, $\varkappa_3 = 2.7\times10^{-15}$ s^{-1}, resulting in $T_1 = 180$ million years, $T_3 = 75$ million years, where T_1 is the period of revolution along the epicycle and T_3 is the period of oscillations along the Z-axis.

These values agree with the results of star kinematics investigation, which is reasonable because any model of the Galaxy must comply with the observations in the vicinity of the Sun.

Let us determine the amplitudes and phases of oscillations of the Sun with respect to the circular orbit taking $t = 0$ as the present time.

1. Motion along the z coordinate (relative to the Galaxy plane). Using the experimental data [5] $z(0) = 25$ parsec, $\dot z(0) = 6$ km/s, we obtain from (4)

$$z(0) = b\cos\varphi_3, \quad \tan\varphi_3 = -\frac{\dot z(0)}{\varkappa_3 z(0)},$$
$$\dot z(0) = -\varkappa_3 b\sin\varphi_3, \quad \frac{z(0)}{\cos\varphi_3} = b.$$

Hence $b = 69$ parsec, $\varphi_3 = -1.2$ rad, which corresponds to $\Delta T = -14$ million years. Consequently (see (4)), the Sun crossed the Galaxy plane 4, 41, 78, 115, ... million years ago, and was at the greatest distance from the Galaxy plane 23, 60, 97, 134, ... million years ago.

2. Motion in r and φ (along the epicycle). Using the experimental data $\dot{\tilde r}(0) = -10$ km/s, $r_0\dot\theta = 10$ km/s, we obtain from (4)

$$\dot{\tilde r}(0) = -\varkappa_1 a\sin\varphi_1, \quad \tan\varphi_1 = -\frac{2\dot{\tilde r}(0)\omega}{\dot{\tilde\theta}(0)\varkappa_1 r_0},$$
$$r_0\dot{\tilde\theta}(0) = 2a\omega\cos\varphi_1, \quad a = -\frac{\dot{\tilde r}(0)}{\varkappa_1\sin\varphi_1}.$$

Since $\varphi_1 = 2.1$, $a = 360$ parsec, $2\omega a/\varkappa_1 = 550$ parsec. The value $\varphi_1 = 2.1$ rad corresponds to $\Delta T = 60$ million years. Consequently (see (4)), the Sun was at maximum separation from the Galaxy center 60, 240, 420, 600, ... million years ago, and it was at minimum separation 150, 330, 510, 690, ... million years ago.

Let us now determine the point in time at which the Sun crossed the spiral arms. Let us simulate them with a logarithmic spiral $r_j = A_j e^{-\alpha\theta}$, where $j = 1, 2$ is the arm number, $A_1 = 6.1$ kparsec, $A_2 = 10.3$ kparsec, $\alpha \equiv \tan i = \tan 9.6°$ [8]. The arms rotate as a solid body around the Galaxy center with angular velocity $\Omega_p \cong 24$ km/(s $\cdot$ kparsec). The instant the Sun crosses the arms is determined from the system of equations $r_{\rm arm} = r_{\rm Sun}$, $\theta_{\rm arm} = \theta_{\rm Sun}$.

In the reference system rotating together with the arm

$$Ae^{-\alpha\theta} = r_0 + a\cos(\varkappa_1 t + \varphi_1),$$
$$\theta = (\omega - \Omega_p)t - \frac{2\omega a}{r_0}\sin(\varkappa_1 t + \varphi_1),$$
$$A\exp\left[-\alpha\left\{(\omega - \Omega_p)t - \frac{2\omega a}{r_0}\sin(\varkappa_1 t + \varphi_1)\right\}\right] = r_0 + a\cos(\varkappa_1 t + \varphi_1).$$

Solving this equation for the first arm, we obtain 3 roots

$$t_1 = -540 \quad \text{million years,}$$
$$t_2 = -600 \quad \text{million years,}$$
$$t_3 = -680 \quad \text{million years.}$$

In order to refine these moments, the arm gravity fields must be taken into account.

CONNECTION WITH PALEOMAGNETISM

Having obtained some information on the Sun orbit, let us make an effort to detect a connection between its motion in the Galaxy plane and the geomagnetic phenomena. In Fig. 1 the authors of [9] show the share of mixed-polarity states as a function of time. They plot the time dependence versus the reversal frequency. A significant increase in the frequency of reversals about 500 million years ago may possibly be associated with the Sun leaving the arm.

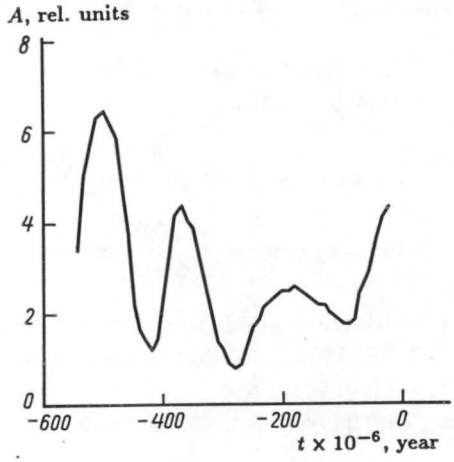

Fig. 1

A share of mixed polarity states as a function of time. The zero on the time scale corresponds to the present time.

Table 1

$\Phi(\nu)$, rad	$\Phi(\nu)$ mil. years	$T(\nu)$ mil. years	$W(\nu)$
−1.6	97	380	5.50
−1.5	55	230	3.00
0.9	−21	150	5.25
1.3	−25	120	2.85
0.55	−8	94	1.25
0	0	79	1.20
−0.2	2	69	1.20

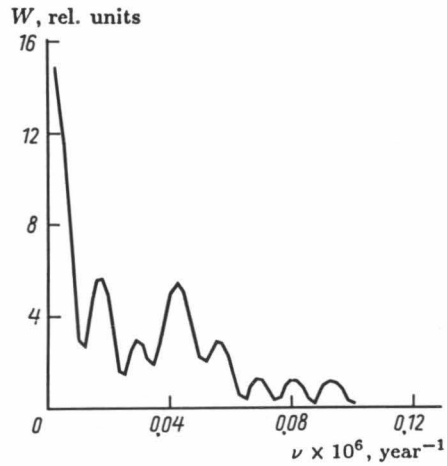

Fig. 2

Amplitude of Fourier image dependence in Fig. 1.

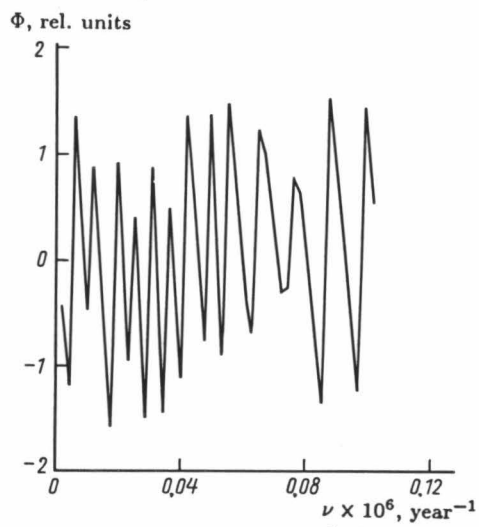

Fig. 3

Phase of Fourier image dependence in Fig. 1.

In order to open up finer effects, we carried out a Fourier analysis of this dependence (Figs. 2 and 3). The results obtained (the amplitudes and phases of the maxima) are given in Table 1. Here ν is the frequency corresponding to the amplitude maximum, $\Phi(\nu)$ is the maximum phase, $T(\nu)$ is the period of the harmonic with frequency ν, and $W(\nu)$ is the amplitude of the harmonic.

The period of 230 million years corresponds to the revolution of the Sun around the Galaxy center. The harmonic with a period of 79 million years, which corresponds to the period of oscillation along the Z-axis, took the extreme values when the Sun crossed the Galaxy plane. The harmonic with a period of 150 million years corresponding to the period of motion along the epicycle approached its maximum when the Sun was at minimum separation from the Galaxy center, and vice versa.

Our calculations are approximate. However, we know that the errors in the original data (information on the Galaxy structure, on paleomagnetic data) are 10–20%, hence the accuracy of data obtained is of about the same order.

MAIN CONCLUSIONS

On comparing the paleomagnetic data with the results of calculations of the Sun orbit in the Galaxy the following correspondences have been revealed. The frequency of geomagnetic field inversions increased when the Sun left the Galaxy spiral arms. The frequency of the Earth magnetic field inversions increased when the Sun either crossed the Galaxy plane or was at minimum separation from the Galaxy center.

Further research is needed to refine the discovered regularities and their causes.

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