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ON INTERPRETATION OF SPIN EFFECTS IN ČERENKOV RADIATION

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Čerenkov radiation power depends on the correlation between electron polarization and photon helicity. This correlation is interpreted in the present paper as an interference between radiation amplitudes of the electron charge and the electron magnetic moment.

Polarization effects in quantum theory of Čerenkov radiation (ČR) [1, 2] (see also [3]) were studied in papers [4–6]. Their results were generalized in [7], where the total power of ČR for arbitrary electron and photon polarizations was obtained. A characteristic polarization effect is the correlation between the electron spin and the photon circular polarization (helicity). In a particular case of pure polarization states, the power distribution of ČR over the frequency ω and the azimuth angle φ (in the plane perpendicular to the electron momentum $\mathbf{p}$) has the form [7]

$$\frac{dW_i}{d\omega\, d\varphi} = \frac{e^2}{4\pi} v\omega \left[\sin^2\theta_0 + \frac{1}{2}(n^2-1)\left(\frac{\hbar\omega}{p}\right)^2 + \zeta\lambda F_1\right], \tag{1}$$

where the subscript $i = l, t$ refers to the electron polarization states, longitudinal (l) or transverse (t) with respect to $\mathbf{p}$; the corresponding functions are as follows:

$$F_l = \frac{\hbar\omega}{p} n \left(1 - \frac{\cos\theta_0}{nv}\right), \tag{2}$$

$$F_t = -\frac{\hbar\omega m}{p^2} \sin\theta_0 \sin\varphi. \tag{3}$$

Here the angle θ_0 between the photon momentum $\hbar\mathbf{k}$ and that of the electron $\mathbf{p}$ is determined by the conservation laws [3]

$$\cos\theta_0 = \frac{1}{nv}\left[1 + \frac{1}{2}(n^2-1)\frac{\hbar\omega}{\varepsilon}\right], \tag{4}$$

$v = p/\varepsilon$ is the velocity of an electron with energy $\varepsilon = \sqrt{m^2 + \mathbf{p}^2}$, $p = |\mathbf{p}|$, $n = n(\omega)$ is the refraction index of the medium. In Eq. (1) the electron spin number $\zeta = \pm 1$, the photon helicity $\lambda = \pm 1$. (Here the system of units is used with $c = 1$, $\alpha = e^2/\hbar \cong 1/137$.)

Note that the result (2) for arbitrary electron polarization was for the first time obtained in [5]. The azimuth asymmetry (see (3)) of power of the circular polarized radiation emitted by a transversely polarized electron [7] is due to the fact that there exists a preferred direction of the electron polarization vector $\boldsymbol{\zeta}$

* Deceased.

($\boldsymbol{\zeta} \perp \mathbf{p}$). We emphasize that in [7] a general case of an electron and a photon, both partially polarized, was also considered.

In the present paper an interpretation of a spin term $\sim \zeta\lambda$ in the power of ČR (1) is proposed which means that it is due to a contribution of interference of the radiation amplitudes of the charge and the magnetic moment of an electron. Restricting ourselves by terms of the first order in the Planck constant $\hbar$, we will show that the result mentioned also follows from the description of a point-like charged particle with an intrinsic magnetic moment equal to the Bohr magneton, in terms of classical electrodynamics.

We start from the general formula for the spectral-angular distribution of ČR power [7] (in what follows we put $c = \hbar = 1$)

$$\frac{dW}{d\omega\, d\Omega} = \frac{e^2}{8\pi}\frac{n\omega^2}{\varepsilon\varepsilon'}\delta(\varepsilon' + \omega - \varepsilon)|\boldsymbol{\alpha}_{fi}\mathbf{e}^*|^2, \tag{5}$$

where $d\Omega$ is the element of a solid angle in the direction $\mathbf{k}$ ($|\mathbf{k}| = n\omega$), $\mathbf{e}$ is the polarization vector of a photon, $\boldsymbol{\alpha}_{fi}$ are the matrix elements of the Dirac matrices,

$$\boldsymbol{\alpha}_{fi} = \overline{u}_f\boldsymbol{\gamma}u_i = u_f^+\boldsymbol{\alpha}u_i. \tag{6}$$

Here u_s ($s = i, f$) are the bispinors of the initial and final electron states whose energy and momentum are related by the conservation law: $\varepsilon' = \varepsilon - \omega$, $\mathbf{p}' = \mathbf{p} - \mathbf{k}$.

Let us consider a reduced radiation amplitude

$$R = \frac{e}{2\varepsilon}\boldsymbol{\alpha}_{fi}\mathbf{e}^* \tag{7}$$

and separate in it the contributions from the charge and the magnetic moment of an electron using the Gordon identity (see e.g. [8]) which is valid for arbitrary positive-frequency solutions of the Dirac equation $u(p)$ and $u(p')$

$$\overline{u}(p')\gamma^\mu u(p) = \frac{1}{2m}\overline{u}(p')[(p' + p)^\mu + i\sigma^{\mu\nu}(p' - p)_\nu]u(p), \tag{8}$$

where $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$ and a pseudo-Euclidean metric with the signature $(+---)$ is used. The first term in the right-hand side of (8) refers to the charge, and the second one to the spin magnetic moment.

Using (6), (8), and the condition $\mathbf{ek} = 0$, we represent amplitude (7) in the form

$$R = \frac{e}{2\varepsilon}\mathbf{e}^*\overline{u}_f\left[\frac{\mathbf{p}}{m} + \frac{i}{2m}(i\omega\boldsymbol{\alpha} + \mathbf{k}\times\boldsymbol{\Sigma}\right]u_i. \tag{9}$$

To make a quasi-classical interpretation of (9) we neglect the electron recoil in the process of photon radiation, putting $u_f \cong u_i$. We express the bispinor u_i in terms of a two-component spinor w_i which determines the polarization state and is normalized according to the condition $w_i^+w_i = 1$, as follows [9]:

$$u_i = \sqrt{\varepsilon_i}\begin{pmatrix} w_i \\ (\boldsymbol{\sigma}\mathbf{p}/\varepsilon_+)w_i \end{pmatrix}, \tag{10}$$

where $\varepsilon_+ = \varepsilon + m$. With regard to (10) we find

$$\overline{u}_i u_i = 2m, \quad \overline{u}_i\boldsymbol{\alpha}u_i = -2i\varepsilon\boldsymbol{\zeta}\times\mathbf{v}, \quad \overline{u}_i\boldsymbol{\Sigma}u_i = 2\varepsilon[\boldsymbol{\zeta} - (1 - \gamma^{-1})(\boldsymbol{\zeta}\mathbf{l})]. \tag{11}$$

Here $\boldsymbol{\zeta} = w_i^+\boldsymbol{\sigma}w_i$ is the vector of the initial electron polarization (in the case of a pure state under consideration $|\boldsymbol{\zeta}| = 1$); $\mathbf{l} = \mathbf{p}/p$, the Lorentz factor $\gamma = \varepsilon/m = (1 - v^2)^{-1/2}$. In (10) and (11) the standard representation of Dirac matrices [9] was used.

We remark that $\boldsymbol{\zeta}$ is the quantum average of the spin operator in the rest frame $\mathbf{O}$ (see e.g. [10])

$$\boldsymbol{\zeta} = \langle\mathbf{O}\rangle = \int d^3x\, \psi_i^+\mathbf{O}\psi_i = \frac{1}{2\varepsilon}u_i^+\mathbf{O}u_i, \quad \mathbf{O} = \gamma^0\left(\boldsymbol{\Sigma} - \frac{\mathbf{p}(\boldsymbol{\Sigma}\mathbf{p})}{\varepsilon\varepsilon_+}\right) - \gamma^5\frac{\mathbf{p}}{\varepsilon}, \tag{12}$$

where $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$. We emphasize that the operator $\mathbf{O}$ and the wave function ψ_i refer to the laboratory system of reference where the electron momentum is equal to $\mathbf{p}$.

Determine the vector of the electron magnetic moment in the rest frame (a mean value of the operator $-\mu_B \mathbf{O}$)

$$\boldsymbol{\mu}_0 = -\mu_B \boldsymbol{\zeta}, \tag{13}$$

where $\mu_B = -e\hbar/2m$ is the Bohr magneton (the electron charge $e < 0$). Then in the laboratory frame, the electron moving with velocity $\mathbf{v} = \mathbf{p}/\varepsilon = v\mathbf{l}$, as is known, (see e.g. [11], p. 397), has the magnetic moment

$$\boldsymbol{\mu} = \boldsymbol{\mu}_0 - (1-\gamma^{-1})(\boldsymbol{\mu}_0 \mathbf{l})\mathbf{l} \tag{14}$$

and acquires the electric dipole moment

$$\mathbf{d} = \mathbf{v} \times \boldsymbol{\mu}_0. \tag{15}$$

Note that in the rest frame $\mathbf{d}_0 = 0$ due to CP-invariance of quantum electrodynamics [9].

In the approximation used, we assume in (9) that $u_f = u_i$ and upon employing (11), (13)–(15) the radiation amplitude is transformed to the form

$$R = e(\mathbf{v}\mathbf{e}^*) - i[\boldsymbol{\mu}(\mathbf{k} \times \mathbf{e}^*) + (\mathbf{d}\mathbf{e}^*)(\mathbf{k}\mathbf{v})]. \tag{16}$$

Here it is considered that, in the classic limit, $\omega = \mathbf{k}\mathbf{v}$. The main formula (5) with due regard for formulas (7), (16), and $\varepsilon' \cong \varepsilon$ is simplified

$$\frac{dW}{d\omega\, d\Omega} = \frac{n\omega^2}{2\pi}\delta(\omega - \mathbf{v}\mathbf{k})|R|^2. \tag{17}$$

In the framework of classical electrodynamics the same result (17) is obtained, where

$$R = \mathbf{e}^* \mathbf{j}_{\mathbf{k}} e^{i(\mathbf{k}\mathbf{v})t}, \quad \mathbf{j}_{\mathbf{k}} = \int d^3x\, e^{-i\mathbf{k}\mathbf{r}} \mathbf{j}(t, \mathbf{r}) \tag{18}$$

is the Fourier transform of the current density produced by a uniformly moving particle with charge e, the magnetic moment $\boldsymbol{\mu}$, and the electric moment $\mathbf{d}$ [11, 12],

$$\mathbf{j}(t, \mathbf{r}) = e\mathbf{v}\delta(\mathbf{r} - \mathbf{v}t) + \nabla \times [\boldsymbol{\mu}\delta(\mathbf{r} - \mathbf{v}t)] + \frac{\partial}{\partial t}[\mathbf{d}\delta(\mathbf{r} - \mathbf{v}t)]. \tag{19}$$

Substituting Eq. (19) into (18) yields an expression for R (18) that coincides with (16) (see also [12]), which is in agreement with the correspondence principle for quantum matrix elements and the Fourier transforms of classical quantities [9].

Let us deal with quasi-classical derivation of formulas (2) and (3). We put

$$\mathbf{p} = p\mathbf{e}_x, \quad \mathbf{n} = \mathbf{k}/k = \cos\theta \mathbf{e}_x + \sin\theta(\cos\varphi \mathbf{e}_y + \sin\varphi \mathbf{e}_z) \tag{20}$$

and choose as vectors of linear polarization of a photon $\mathbf{e}^{(\alpha)}$ ($\alpha = 1, 2$) the following ones [7]:

$$\begin{aligned} \mathbf{e}^{(1)} &= \frac{\mathbf{n} \times \mathbf{p}}{|\mathbf{n} \times \mathbf{p}|} = \sin\varphi \mathbf{e}_y - \cos\varphi \mathbf{e}_z, \\ \mathbf{e}^{(2)} &= \mathbf{n} \times \mathbf{e}^{(1)} = -\sin\theta \mathbf{e}_x + \cos\theta(\cos\varphi \mathbf{e}_y + \sin\varphi \mathbf{e}_z). \end{aligned} \tag{21}$$

In the case of a longitudinally polarized electron $\boldsymbol{\zeta} = \zeta \mathbf{e}_x$ it follows from (13)–(15) that

$$\boldsymbol{\mu} = -\zeta\gamma^{-1}\mu_B \mathbf{e}_x, \quad \mathbf{d} = 0. \tag{22}$$

Upon substituting (21) and (22) into (16), we arrive at the corresponding amplitudes for the photon radiation $R_l^{(\alpha)}$

$$R_l^{(1)} = -i\zeta\gamma^{-1}\mu_B k \sin\theta, \quad R_l^{(2)} = -ev\sin\theta. \tag{23}$$

This means that the charge and the magnetic moment contribute to different components of linearly polarized radiation. We note that in [12] (see p. 159) a general conclusion was made that there is no interference between radiation by a charge and that by electromagnetic dipole moments. However, a correction should be made that this is true only for linear components of radiation polarization when the vector **e** is real (see (16)). For circular polarization described by the complex vectors (see (21))

$$\mathbf{e}_\lambda = \frac{1}{\sqrt{2}}(\mathbf{e}^{(1)} + i\lambda \mathbf{e}^{(2)}), \tag{24}$$

where $\lambda = \pm 1$, the radiation amplitude follows from (23) and (24)

$$R_{l\lambda} = \frac{i}{\sqrt{2}}(\lambda e v - \zeta \gamma^{-1} \mu_B k) \sin\theta. \tag{25}$$

Using (25), the contribution of interference between the charge and the magnetic moment to the ČR power is found from (17)

$$\frac{dW_l^{(e\mu)}}{d\omega\, d\varphi} = \frac{e^2 v}{4\pi}\omega \cdot \lambda\zeta \frac{\hbar n \omega}{p} \sin^2\theta_0, \tag{26}$$

where $\cos\theta_0 = 1/(nv)$. Comparing (1) and (26) yields $F_l = (\hbar n\omega/p)\sin^2\theta_0$, which, in the first order of $\hbar$, coincides with expression (2) (in (4), in the approximation adopted, the quantum correction is neglected).

Consider now the case of transversely polarized electron, assuming, with consideration for (20), that

$$\boldsymbol{\zeta} = \zeta \mathbf{e}_z. \tag{27}$$

From (27) and (13)–(15), we find

$$\boldsymbol{\mu} = \boldsymbol{\mu}_0 = -\zeta\mu_B \mathbf{e}_z, \quad \mathbf{d} = \zeta v \mu_B \mathbf{e}_y. \tag{28}$$

Thus in this case (in contrast to (22)) together with the magnetic moment, an electric moment is induced in the laboratory frame of reference. From (16), (28), and (21) for amplitudes of linearly polarized radiation we obtain

$$\begin{aligned} R_l^{(1)} &= i\zeta\gamma^{-2}\mu_B k \cos\theta \sin\varphi, \\ R_l^{(2)} &= -ev\sin\theta + i\zeta\mu_B k\cos\varphi(1 - v^2\cos^2\theta). \end{aligned} \tag{29}$$

With regard to (29) and the relation

$$R_{l\lambda} = \frac{1}{\sqrt{2}}(R_l^{(1)} - i\lambda R_l^{(2)}),$$

the corresponding interference contribution to ČR power is found

$$\frac{dW_l^{(e\mu)}}{d\omega\, d\varphi} = -\frac{e^2 v}{4\pi}\omega \cdot \lambda\zeta \frac{\hbar m \omega}{p^2} \sin\theta_0 \sin\varphi, \tag{30}$$

which is in agreement with (3).

We emphasize that the quasi-classical method considered can only be applied for the calculation of a correlation contribution proportional to $\zeta\lambda\hbar$. Other quantum effects of the order of $\hbar$ and $\hbar^2$, that are contained in (1), cannot be dealt with completely by this method, as in deriving the main formula of method (16) the recoil in the radiation of a photon was neglected.

In conclusion we note that the spin correlation mentioned above can be used in principle to determine polarization of the high energy electron beam as a result of measuring the degree of circular polarization of ČR. However, since the effect is rather small (see (2) and (3)), the possibility of its direct experimental observation remains to be studied further.

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