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DENSITY RESTORATION IN THE VICINITY OF A PLANE SHOCK WAVE FRONT BY THE SHAPE OF THE SCHLIEREN SIGNAL

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The schlieren method of studying density in the vicinity of a plane shock wave with consideration for the diffraction of a laser beam has been developed. An integral equation of the convolution type has been obtained for density, which is solved by means of Tikhonov's regularization method. The density restoration method proposed is employed for processing data obtained in the experiments with shock waves in the low temperature plasma produced by an HF discharge in argon.

1

The schlieren method proposed in 1965 [1] was used in a number of works where kinetics in gases was studied. Density restoration in papers [1, 2] was based on the geometric optics approximation and was restricted to the case of a smooth variation of density. In particular, the method [1, 2] would not do in studying the density profile in the vicinity of a shock wave front, when the characteristic length is much shorter than the diameter of a light beam.

In paper [2] a nonlinear integral equation for restoration of density by the deflection of a laser beam on an arbitrary inhomogeneity has been obtained. Its solution is an incorrectly formulated problem and meets with significant difficulties.

In the present paper a new approach to the problem of restoration of gas density by the shape of a schlieren signal is presented. An integral equation of the convolution type has been obtained for density, which considers the diffraction of a laser beam and the apparatus function of a photodiode. In processing experimental schlieren signals this equation is solved numerically by means of the Tikhonov regularization method [3].

2

We start from the following assumptions.

(1) The gas in a shock tube is considered to be a phase screen, additional phase accumulation $\Delta\varphi$ on the inhomogeneity being less than $\pi/2$.

(2) The gas density behind the shock wave in the coordinate system related to the wave does not depend on time: $\rho = \rho(x_1)$, $x_1 = \eta + v_s t$, where v_s is the velocity of shock wave propagation.

(3) The laser beam is assumed to be Gaussian.

Let us consider the diffraction of a Gaussian beam on a phase screen (Fig. 1). The wave equation in the case of a scalar field has the form

$$\Delta E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0,$$

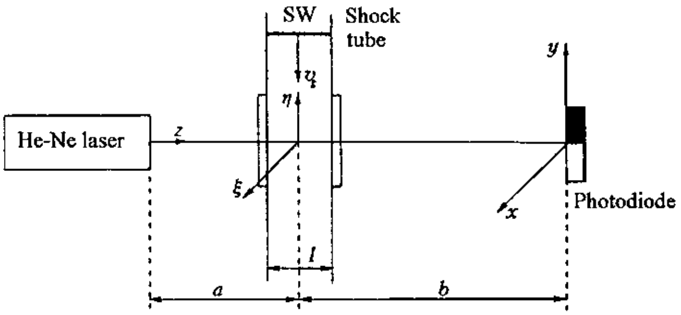

Fig. 1
Experimental lay-out and the coordinate system.

where c is the speed of light. Represent E in the form

$$E(x,y,z,t) = v(x,y,z)\exp(i\omega t),$$

$$v(x,y,z) = u(x,y,z)\exp(-ikz), \quad k = 2\pi/\lambda.$$

Using the approximation of a weakly varying amplitude ($\partial^2 u/\partial z^2 \ll k\partial u/\partial z$), we have [4]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - 2ik\frac{\partial u}{\partial z} = 0. \tag{1}$$

The solution of this parabolic equation can be written by means of the Green function G

$$u(x,y,z) = \iint\limits_{-\infty}^{\infty} G(x,y,z-z_0,\xi,\eta)u_g(\xi,\eta,z_0)\,d\xi\,d\eta,$$

where u_g is the value of the function u in the plane $z = z_0$. In the case of a Gaussian beam incident on a phase screen, u_g is the value of the function u immediately behind the phase screen

$$u_g(\xi,\eta,z_0) = -\sqrt{I_0}\frac{ip}{z_0 - ip}\exp\left(ik\frac{\xi^2+\eta^2}{2(z_0-ip)} + i\Delta\varphi(\xi)\right)\Bigg|_{z_0=a}, \quad p = kw_0^2,$$

where ξ, η are the coordinates in the plane of a phase screen, a is the distance from the laser to the shock tube (see Fig. 1), w_0 is the beam diameter in the necking region, I_0 is the beam intensity, $\Delta\varphi$ is the phase accumulation on the phase screen. The Green function is

$$G(x,y,z,\xi,\eta) = -\frac{ik}{2\pi z}\exp\{ik((x-\xi)^2 + (y-\eta)^2)/2z\}.$$

Thus, the solution of equation (1) in the photodiode plane has the form

$$\begin{aligned} u(x,y,a+b) &= -\sqrt{J_0}\frac{kp}{2\pi(a-ip)b}\iint\limits_{-\infty}^{\infty}\exp\{-A(\xi^2+\eta^2) - B((x-\xi)^2+(y-\eta)^2) + i\Delta\varphi(\eta)\}\,d\xi d\eta \\ &= c(x)\{u(y) + i\varepsilon(y)\}, \end{aligned}$$

$$c(x) = -\sqrt{J_0}\frac{|B|p}{\sqrt{\pi}(a-ip)\sqrt{A+B}}\exp(-ABx^2/(A+B)),$$

$$u_0(y) = \sqrt{\pi A_1 B_1/(A_1+B_1)}\exp(-y^2/(A_1+B_1)),$$

$$\varepsilon(y) = \int_{-\infty}^{\infty} \sin\Delta\varphi(\eta)\exp(-A\eta^2 - B(y-\eta)^2)d\eta,$$

$$A = -ik/2(a-ip), \quad B = -ik/2b, \quad A_1 = 1/A, \quad B_1 = 1/B.$$

The quantity b is the distance between the shock tube and the receiver (see Fig. 1).

Until now we assumed the phase accumulation $\Delta\varphi$ to be a finite quantity less than $\pi/2$. Further for simplicity we assume $|\Delta\varphi| \ll 1$. In this case $\exp(i\Delta\varphi) \cong 1 + i\Delta\varphi$. In our experiments $\Delta\varphi \cong 0.02$. An error we make in expanding in a series and restricting ourselves to the first term of the expansion does not exceed several percents.

The energy flux incident on one of the sections of the photodiode is

$$I_1 = \int_{-\infty}^{\infty} dx \int_{0}^{-\infty} dy |c(x)|^2 |u_0(y) + i\varepsilon(y)|^2.$$

The flux incident on the other section is

$$I_2 = \int_{-\infty}^{\infty} dx \int_{0}^{\infty} dy |c(x)|^2 |u_0(y) + i\varepsilon(y)|^2.$$

The signal received from the photodiode is proportional to the difference of the fluxes

$$V = \text{const}\,(I_1 - I_2). \tag{2}$$

In our case $|u_0 + i\varepsilon|^2 = |u_0|^2 + 2\,\mathrm{Re}(i\varepsilon u_0^*)$, where u_0^* is a complex conjugate quantity, and

$$\int_{-\infty}^{0} |u_0|^2 dy = \int_{0}^{\infty} |u_0|^2 dy$$

due to the fact that the function u_0 is even. The constant entering into expression (2) is found from the standard curve of the photodiode.

With these considerations, we obtain the final expression for the schlieren signal

$$V(X) = \frac{2kk_{GD}l}{\sqrt{\pi}} \frac{\sqrt{(a+b)^2 + p^2}}{a^2 + p^2} \frac{dV}{d\zeta} \int_{-\infty}^{\infty} \Delta\rho(s) L(X - s) ds, \tag{3}$$

$$L(X - s) = \int_{-\infty}^{\infty} L_{\text{opt}}(X - \tau) L_{\text{ap}}(\tau - s) d\tau,$$

$$L_{\text{opt}} = \mathrm{Im}(\exp(-t)^2 (A + A^*)\,\mathrm{cerf}\,(-tv_s(A_1^* - B_1)/A_1^* B_1))), \quad X = v_s t,$$

where L_{ap} is the apparatus function of the photodiode, $dV/d\zeta$ is the calibration constant, ζ is the displacement of the receiver, k_{GD} is the Gladstone–Dale constant, l is the width of the working section, ρ is the density, cerf is the error integral. The apparatus function of the photodiode was determined experimentally.

3

Equation (3) can be solved by means of the classical Fourier transform. However, the values measured are determined in the experiment with certain error which may lead to significant modification of the solution $\rho(x)$. Moreover, equation (3) may prove to have no solution at all. We are dealing with an

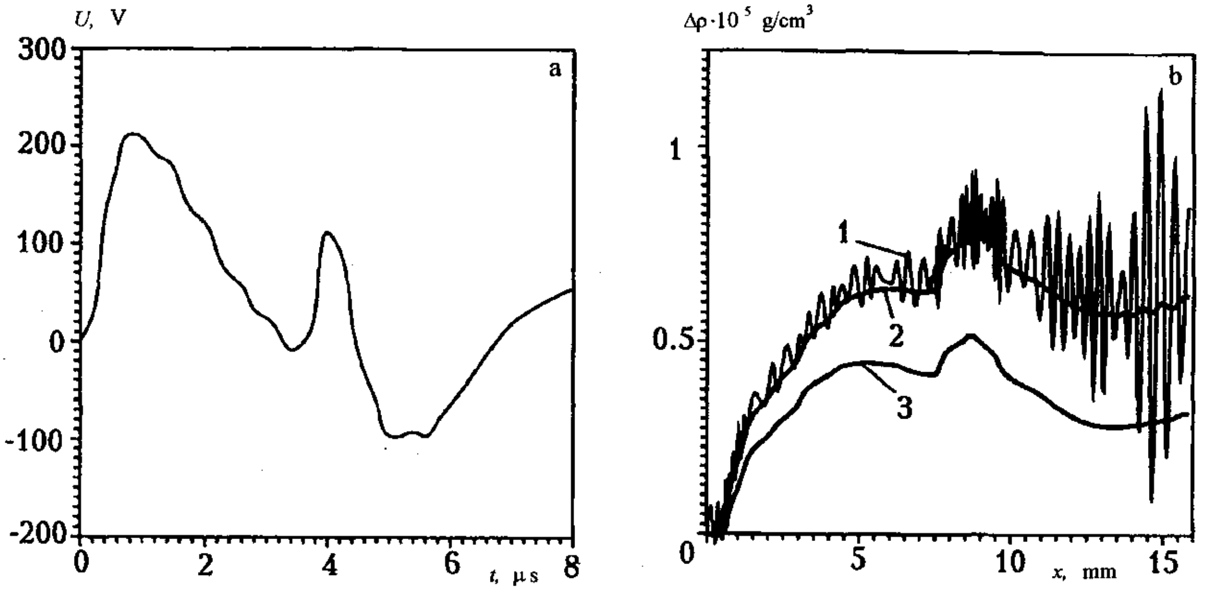

Fig. 2

A typical schlieren signal (a) and restored (basing upon (a)) density distribution in the vicinity of the shock wave front (b); $\alpha = 10^{-8}$ (*1*), 10^{-6} to 10^{-4} (*2*), and 10^{-2} (*3*).

incorrectly formulated problem and to solve it numerically we use the Tikhonov method of regularizing functionals [3].

In the calculation process the Fourier transform is employed. This means that periodic functions should be considered. To avoid distortion of solutions, we produced a buffer zone where values of the schlieren signal were set equal to zero.

In choosing a regularization parameter α the following considerations were used. For small values of α the solution is unstable. With growing α, the solution smoothes out and becomes stable inside a certain interval of α values. The optimal value of α lies inside this interval. Further increase of α leads to a strong smoothing out of the solution. In Fig. 2a a typical schlieren signal is depicted, and in Fig. 2b the restored values of density at various values of the regularization parameter α are shown. At $\alpha = 10^{-8}$ the solution demonstrates irregular spikes, for $10^{-6} \leq \alpha \leq 10^{-4}$ the solution is stable (exactly these values were used in processing the experimental data). At $\alpha = 10^{-2}$ the form of the solution is distorted.

4

Experiments were carried out in a one-diaphragm shock tube with rectangular 40×60 mm cross-section with a dielectric working section provided with optical windows 60 mm in diameter. Two metal plates were mounted in the upper and lower partitions of the section. A transversal HF discharge was produced in-between the plates by means of a HF generator ($f = 13.6$ MHz). The current density was 40 mA/cm^2, the discharge zone was 80 mm long. To measure density in the vicinity of the wave front and behind the wave the laser schlieren method was used. Helium-Neon laser was used as a light source. A laser beam of 1 mm in diameter was incident onto the sectioned photodiode whose signal was fed to an oscillograph of the type C9-8. The shock wave velocity was measured by means of piezotransmitters outside the discharge zone and by means of schlieren signals inside it. The initial translational temperature was found by the density values measured by means of a Fabry–Perot interferometer.

To gauge a photodiode we moved it along the line perpendicular to the axis of a laser beam. In this way a standard curve was obtained, i.e., the dependence of the photodiode voltage on the distance between the axis of the laser beam and the center of the photodiode.

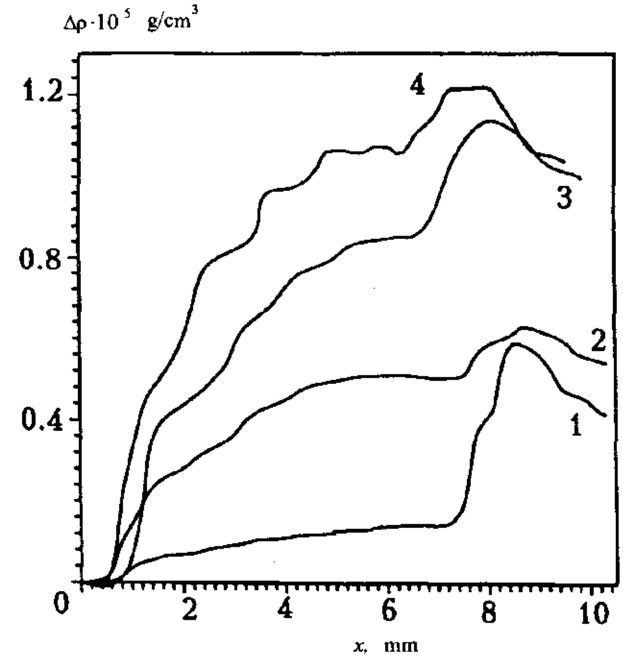

Fig. 3

Density distribution in argon at various values of the Mach number: $M = 2.42$ (*1*), 3 (*2*), 3.25 (*3*), and 3.4 (*4*).

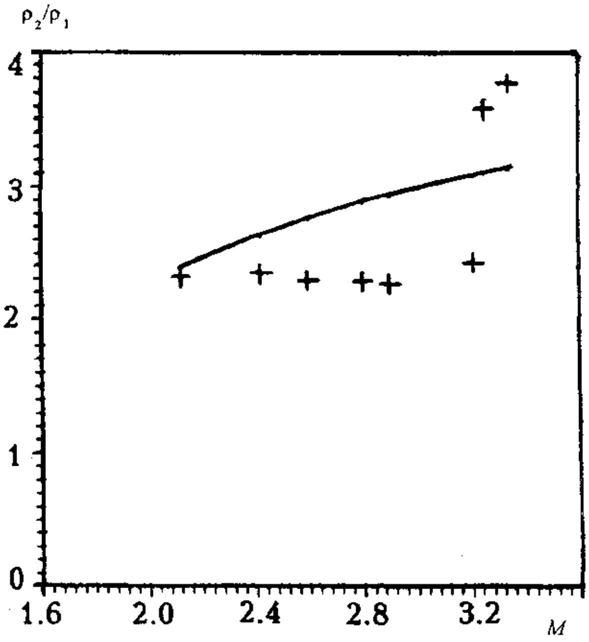

Fig. 4

Ratio of densities in argon at various values of the Mach number (solid line is for the calculated values, and crosses, for the experiment).

In Fig. 3*a* density distribution is depicted in the vicinity of the shock wave front in the low-temperature plasma of the HF discharge at various values of the Mach number. The initial temperature was 1100 K and the initial pressure was 8 torr.

A shock wave in the discharge region has a step-like structure, and the width of the shock transition zone is much greater than the thickness of the wave in the absence of the discharge.

In Fig. 4 the experimental values and the calculated curve of the density ratios when passing through the shock wave are compared. Calculation was carried out using conventional relations for the shock wave disregarding the energy production behind the wave front. The maximum value of density behind the wave was used as an experimental one. We notice that the experiment agrees well with the calculations. In the molecular gas (CO_2) no such agreement is observed [5]: the experimental data prove to be twice lower than the calculation ones.

Thus, a method has been developed for calculating density in the vicinity of the shock wave front by the shape of the schlieren signal. Processing the experimental data by the method presented showed that a shock wave in the low temperature plasma of the HF discharge has a two-step structure. It has been found that the thickness of the shock wave in the discharge zone is much greater than in an unexcited gas.

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